Effects of Pinning on the Flux Flow Hall Resistivity

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We demonstrate that pinning strongly renormalizes both longitudinal and Hall resistivity in the flux flow regime. Using a simple model for the pinning potential we show that the magnitude of the vortex contribution to the Hall voltage decreases with increase in the pinning strength. The Hall resistivity ρ_{xy} scales as ρ_{xx}^2 only for a weak pinning. On the contrary, a strong pinning breaks the scaling relation and can even result in a sign reversal of ρ_{xy}.

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Experiments show that the Hall resistivity in high-T_c superconductors depends at least on two factors: the doping level and the intensity of vortex pinning in the superconducting sample. By varying the doping level, it is possible to change not only the magnitude of the Hall voltage but also its sign: The sign reversal of the Hall angle is usually observed for underdoped compounds while it is not seen for overdoped superconductors [1]. On the other hand, the Hall voltage is also a function of the pinning strength. By tuning the concentration of columnar defects, the Hall anomaly was shown to become less pronounced with the increase in the density of the defects [2–4]. The microscopic theory of vortex dynamics in clean type II superconductors explains the Hall angle sign reversal as the effect of a complicated electronic spectrum in the superconductor [5,6]. The effect of pinning on the Hall anomaly is not well understood. There are several contradicting views. According to Ref. [7], pinning affects the counterflow in the vortex core which may result in a more pronounced sign anomaly as the pinning strength increases. This prediction, however, disagrees with the experiments [2,4] where the opposite trend has been observed. On the contrary, Ref. [8] demonstrates that pinning does not affect the *Hall conductivity* and thus the *Hall resistivity* scales as $\rho_{xy} \propto \rho_{xx}^{\beta}$ with $\beta = 2$ for small Hall angles. Such scaling was observed in several experiments [3,4,9]. However, the scaling exponent was not always universal and varied from $\beta \approx 2$ to $\beta \approx 1$ depending on the applied magnetic field and concentration of defects.

The scaling [8] rests on a perturbation expansion of the vortex response in the pinning strength. However, recent experiments raise questions as to whether the arguments supporting the scaling are general enough and call us to revisit this problem. In the present Letter, we use a simple model for the pinning potential to demonstrate that the pinning intensity can affect drastically the current dependences of both the longitudinal and the transverse voltages in the flux flow regime. The relevant phenomena take place on a macroscopic scale larger than the vortex core size: While the microscopic parameters for each vortex do not change, the trajectory of the vortex motion changes giving rise to a pronounced variation in the overall response of the superconducting sample. We show that the magnitude of the vortex contribution to the Hall resistivity decreases with an increase in the pinning strength which agrees with the general observations of Refs. [2,4]. Nevertheless, pinning can affect the Hall-angle sign anomaly: the vortex contribution may reverse sign for a large pinning strength. For weak pinning, however, the Hall conductivity remains almost intact as was shown in Ref. [8] and the scaling $\rho_{xy} \propto \rho_{xx}^2$ for small Hall angles is restored.

Symmetry of the pinning force.—We start with the equation of the vortex dynamics in the form

$$\frac{\Phi_0}{c}[\mathbf{j} \times \mathbf{z}] + \mathbf{F}_{\rm pin}(\mathbf{r}_L) + \mathbf{F}_{\rm el}(\mathbf{r}_L) = \eta \mathbf{v}_L + \eta' [\mathbf{v}_L \times \mathbf{z}]. \tag{1}$$

Here $\mathbf{j}$ is the transport current, $\Phi_0 = \pi\hbar c/|e|$ is the magnetic flux quantum, and $\mathbf{z}$ is the unit vector in the direction of the magnetic field. The first term in Eq. (1) is the Lorentz force, and the next two terms are the pinning and elastic forces which depend on the vortex position with respect to both the pinning centers and other vortices in the vortex lattice; the terms on the right-hand side are the friction and the transverse forces on the moving vortex, respectively.

To find the I-V curve one needs to solve the differential equation (1) for the averaged in time vortex velocity $\bar{\mathbf{v}}_L$ in a given realization of the pinning potential. If the pinning potential is random, one can solve this equation within a finite region inside the superconductor and then average the obtained vortex velocity $\bar{\mathbf{v}}_L$ over the disorder. We denote the time- and disorder-averaged velocity $\langle \bar{\mathbf{v}}_L \rangle \equiv \mathbf{v}$. After the final averaging, the pinning force will depend on the vortex velocity $\mathbf{v}$ only,

$$\langle \bar{\mathbf{F}}_{\rm pin} \rangle = F_1(v)\mathbf{v} + F_2(v)[\mathbf{v} \times \mathbf{z}]. \tag{2}$$

We argue that both coefficients F_1 and F_2 are generally nonzero. Indeed, Eq. (1) does not possess a definite symmetry under the time-reversal transformation $t \to -t$

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and $\mathbf{H} \to -\mathbf{H}$ because of a finite dissipation $\eta \neq 0$. Therefore, the vortex trajectory in the simultaneously inverted magnetic field and transport current is different from its trajectory for the initial orientations of $\mathbf{j}$ and $\mathbf{H}$. Since the pinning force is determined by the vortex trajectory, it will contain an explicit dependence both on the direction of the magnetic field and on the direction of $\mathbf{v}_L$ after averaging.

There are two limiting cases which can be considered for illustration. Let us take first the limit of zero Hall angle, $\eta' = 0$. If we simultaneously invert both $\mathbf{j} \to -\mathbf{j}$ and the direction of magnetic field $\mathbf{H} \to -\mathbf{H}$, the equation of vortex motion remains unchanged and $\mathbf{v}_L \to \mathbf{v}_L$. The trajectory of a vortex and its direction of motion remain the same; thus the average pinning force does not change: $\bar{\mathbf{F}}_{\text{pin}} \to \bar{\mathbf{F}}_{\text{pin}}$. This is possible only if $F_2 = 0$. Another example is the absence of dissipation, $\eta = 0$. In this case, the time-reversal symmetry is restored; thus the vortex trajectory remains the same under the transformation $\mathbf{j} \to -\mathbf{j}$, $\mathbf{v}_L \to -\mathbf{v}_L$ together with $\mathbf{H} \to -\mathbf{H}$. Now the vortex velocity changes its sign: the vortex travels along the trajectory in the opposite direction. However, the pinning force averaged over time is independent of the direction of the vortex motion along its trajectory. Indeed, it is determined only by vortex positions being even in $t \to -t$ for a given vortex position $\mathbf{r}$. Therefore, $\bar{\mathbf{F}}_{\text{pin}} \to \bar{\mathbf{F}}_{\text{pin}}$ under such transformation. According to Eq. (2), it is possible only if $F_1 = 0$: No dissipation can arise from a potential force.

We thus arrive at the conclusion that pinning may affect both longitudinal and transverse flux flow conductivity through the modified force parameters

$$\eta \to \eta - F_1; \qquad \eta' \to \eta' - F_2 .$$

It was shown in Ref. [8] that, for a weak pinning, the factor F_2 is small: it vanishes faster than F_1 as the pinning strength decreases. We shall also see this using our model for the pinning potential.

The model.—To quantify our discussion, let us replace the true pinning relief which is seen by vortices moving along their trajectories with some model potential. To this end, we consider the sample as being subdivided into different regions (grains) such that, within each grain, pinning is due to the washboard potential. Pinning planes in different grains are randomly oriented with respect to each other. The washboard potential is characterized by the "depinning current" $\mathbf{j}_p$ such that the pinning force is

$$\mathbf{F}_{\text{pin}} = -\frac{\Phi_0}{c}[\mathbf{j}_p \times \mathbf{z}]. \tag{3}$$

In the coordinate frame associated with the "pinning planes" and having the local x axis along the planes,

$$\mathbf{j}_p = (-j_c \sin(2\pi y/d), 0, 0) . \tag{4}$$

The pinning strength j_c is assumed constant; the interplanar distance d will not appear in the final results. The local coordinate frame (x, y) is randomly oriented over the superconductor (see Fig. 1). This model describes well

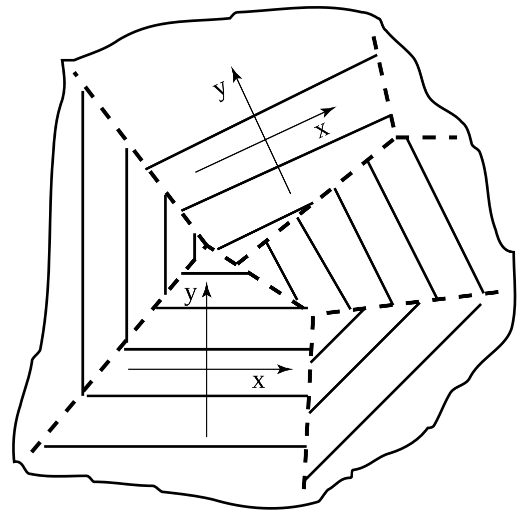

FIG. 1. Grains with washboard pinning potential and local reference frames randomly oriented through the sample.

vortex pinning by twin boundaries. However, the qualitative picture should also hold for pinning by columnar and even by point defects.

We find the vortex velocity and the induced electric field within each grain and then average the electric field over random orientations of pinning planes for a given current density. The approximation of a constant current density implies that the moving vortex array maintains its arrangement on average as each individual vortex performs a nearly random motion from one grain to another. This requires that the grain size is much smaller than the intervortex distance. We consider vortices individually neglecting their interaction with each other. This can be done if the pinning potential is stronger than the elastic energy of the vortex array.

From Eqs. (1), (3), and (4), we have for the vortex velocity in the local frame

$$v_{Lx} = \frac{\Phi_0}{\eta c} j_y - \frac{\eta'}{\eta} v_{Ly} , \tag{5}$$

$$v_{Ly} = -\frac{c}{B} R_\parallel j_c \left[a + \sin\left(\frac{2\pi y}{d}\right)\right], \tag{6}$$

where we denote $a = [j_x - (\eta'/\eta) j_y]/j_c$ and introduce the resistivity in the absence of pinning,

$$R_\parallel = \frac{B\Phi_0}{c^2} \frac{\eta}{\eta^2 + \eta'^2}; \qquad R_\perp = \frac{B\Phi_0}{c^2} \frac{\eta'}{\eta^2 + \eta'^2} . \tag{7}$$

For large currents such that $a^2 > 1$, the solution of Eq. (6) determines the time t_0 needed to cover the distance d along the y axis:

$$\int_0^d \frac{dy}{a - \sin(2\pi y/d)} = \frac{cR_\parallel t_0}{B} .$$

The average velocity $\bar{v}_{Ly} = d/t_0$, thus

$$\bar{v}_{Ly} = -\frac{c}{B} R_{\parallel} j_c \mathrm{sgn}(a)\sqrt{a^2 - 1},$$

$$\bar{v}_{Lx} = \frac{\Phi_0}{c\eta} j_y + \frac{c}{B} R_{\perp} j_c \mathrm{sgn}(a)\sqrt{a^2 - 1}.$$

The induced electric field is

$$E_x = -(B/c)\bar{v}_{Ly}; \qquad E_y = (B/c)\bar{v}_{Lx}.$$

For low currents, $a^2 < 1$, we find from Eqs. (5) and (6) $v_{Ly} = 0$ and $\bar{v}_{Lx} = j_y(\Phi_0/c\eta)$: Vortices perform a guided motion along the pinning planes.

The longitudinal and transverse components of the electric field can be found from

$$\frac{\mathbf{j} \cdot \mathbf{E}}{j^2} = R_{\parallel}\Theta(a^2 - 1)\left[1 - \frac{j_c}{j}\frac{F(a)\cos(\phi - \alpha)}{\cos\alpha}\right] + \bar{R}_{\parallel}\Theta(1 - a^2)\sin^2\phi \tag{8}$$

and

$$\frac{[\mathbf{j} \times \mathbf{E}] \cdot \mathbf{z}}{j^2} = R_{\perp}\Theta(a^2 - 1)\left[1 + \frac{j_c}{j}\frac{F(a)\sin(\phi - \alpha)}{\sin\alpha}\right] + \bar{R}_{\parallel}\Theta(1 - a^2)\sin\phi\cos\phi, \tag{9}$$

where

$$F(a) = a - \mathrm{sgn}(a)\sqrt{a^2 - 1}$$

and $\bar{R}_{\parallel} = \Phi_0 B/\eta c^2$. We put $j_x = j\cos\phi$, $j_y = j\sin\phi$, and introduce the "ideal Hall angle" through $\tan\alpha = R_{\perp}/R_{\parallel} = \eta'/\eta$. Next we put $\phi = \pi/2 - \alpha - \beta$ and denote

$$\sin\beta_0 = (j_c/j)\cos\alpha; \qquad 0 \le \beta_0 \le \pi/2$$

so that $a = \sin\beta/\sin\beta_0$.

The next step is to average Eqs. (8) and (9) over the directions of the pinning planes. This is equivalent to averaging over the directions of the current with respect to the local frame.

Before getting to calculations of the averages let us set the idea of our final results by considering first the simplest case $j < j_c\cos\alpha$ so that $\beta_0 = \pi/2$. Now $a < 1$ for all orientations of the pinning planes, and vortices can glide only along the planes. Therefore the average dissipation is finite while the average Hall voltage is equal to zero for random orientations of the planes. The latter follows from Eq. (9) where the second term on the right-hand side vanishes after averaging over ϕ.

Note that the factor $\cos\alpha$ in the inequality $j < j_c\cos\alpha$ shows that the region of guided motion decreases as the ideal Hall angle α approaches $\pi/2$. Indeed, for the limit $\alpha = \pi/2$, Eq. (6) yields $v_{Ly} = \Phi_0 j_y/c\eta'$; the velocity along the x axis is determined by Eq. (5),

$$v_{Lx} = \frac{\Phi_0}{c\eta'}\left[j_x + j_c\sin\left(\frac{2\pi y}{d}\right)\right].$$

Therefore, the motion along the y axis, i.e., the motion across the planes, is not restricted and the guided motion does not take place for any current if $\alpha = \pi/2$; the pinning force and the Lorentz force in the y direction are balanced by the reactive force generated by the x component of the vortex velocity.

Vortex traps: the I-V curve.—There is no complete pinning in the washboard potential within a grain: the vortex can always glide along the pinning planes. However, interfaces between adjacent grains can provide traps for vortices. We can model the traps in the following way. Consider a vortex after it enters a new grain. If $a^2 > 1$ in this grain, the vortex can move in any direction and thus is not trapped at the boundary. On the contrary, vortex would perform a guided motion along the pinning planes if $a^2 < 1$. In this case, the trap appears if gliding in this grain is directed towards its interface with the preceding grain. Therefore, only the vortex moving away from the interface contributes to the flux flow voltage. It means that, when calculating the average voltage, we should take the contribution to the integral from the regions of a guided motion $a^2 < 1$ with the weight $\frac{1}{2}$. Vortices can also be much easier trapped by other defects when their motion is restricted to one dimension by the pinning planes. In general, we can introduce a "transparency" γ such that a vortex which performs a guided motion within a grain is immobilized with the probability $1 - \gamma$. For the example considered above, $\gamma = 1/2$. According to this definition, the region with $a^2 < 1$ contributes with the weight factor γ.

For $j > j_c\cos\alpha$, the averaging over β gives for the longitudinal voltage

$$E_{\parallel}/jR_{\parallel} = \gamma(1 + \cos^2\beta_0 + \sin^2\beta_0\tan^2\alpha)/2 + (1 - \gamma)[y_1(\beta_0) + y_2(\beta_0)\tan^2\alpha]/2$$

where

$$y_1(\beta_0) = 1 - \frac{2\beta_0}{\pi} + \cos^2\beta_0 - \frac{1}{\pi}\sin(2\beta_0),$$

$$y_2(\beta_0) = 1 - \frac{2\beta_0}{\pi} - \cos^2\beta_0 + \frac{1}{\pi}\sin(2\beta_0).$$

One can check that both terms are positive, of course. The Hall voltage becomes

$$E_{\perp}/jR_{\perp} = \left[\cos^2\beta_0 - \frac{(1 - \gamma)}{\pi}\sin(2\beta_0)\right].$$

The Hall angle is $\tan\theta_H = \rho_{\perp}/\rho_{\parallel}$.

If $j < j_c\cos\alpha$, we have $\beta_0 = \pi/2$ and $\langle\mathbf{j} \cdot \mathbf{E}\rangle = j^2\gamma\bar{R}_{\parallel}/2$ while $\langle[\mathbf{j} \times \mathbf{E}] \cdot \mathbf{z}\rangle = 0$. The Hall angle vanishes.

The I-V curve becomes nonlinear in the presence of pinning both for the longitudinal and for the transverse voltages. Moreover, we see that pinning not only affects the longitudinal flux flow resistivity: it changes drastically the Hall resistivity, as well. With an increase in the depinning strength for a fixed measuring current, the magnitude of the vortex contribution to the Hall voltage decreases.

It changes sign for $\tan\beta_0 = \pi/[2(1-\gamma)]$ and then vanishes at all for a high enough pinning intensity $\beta_0 = \pi/2$ when the vortex motion is completely guided by the pinning profile and thus averages out for randomly oriented trajectories. In our model, the longitudinal resistivity remains finite at this point unless $\gamma = 0$: vortices always have a possibility to glide. The disappearance of the Hall voltage prior to full immobilizing of vortices for $\gamma \neq 0$ is, probably, model dependent. Complete trapping corresponds to $\gamma = 0$. In this case, all vortices which can move only along the pinning planes get trapped at the interfaces between different grains or by other defects. The longitudinal and Hall voltages vanish simultaneously.

The limit of weak pinning corresponds to $\gamma = 1$ and $\beta_0 \ll 1$. We have for a small Hall angle

$$\rho_{xx} = R_{\|}\left(1 - \frac{1}{2}\sin^2\beta_0\right), \qquad \rho_{xy} = R_{\perp}(1 - \sin^2\beta_0).$$

This results in a scaling relation $\rho_{xy} \propto \rho_{xx}^2$ as a function of the pinning strength which holds within the first order terms in j_c^2/j^2 in agreement with Ref. [8]. However, the scaling behavior is usually seen in experiments when the resistivity is several hundred times smaller than the normal-state value, i.e., close to the flux creep regime when pinning cannot be regarded as weak. The present model does not consider the flux creep and more studies are needed to investigate the Hall resistivity in this regime.

The microscopic theory of the vortex dynamics in clean superconductors [5,6] predicts that, in the absence of pinning, the vortex contribution to the Hall current described by the coefficient η' in Eq. (1) may have the sign different from the sign of the Hall effect in the normal state. This is possible if the Fermi surface of the metal in the normal state has both holelike and electronlike pockets. The Hall anomaly may thus depend on the doping level. Moreover, since different characteristics of the Fermi surface are responsible for the vortex contribution at different temperatures (for a given magnetic field) the sign of the vortex Hall effect may alternate up to 2 times.

The pinning can remove or introduce the Hall anomaly: if there was an anomaly without pinning, it could disappear for high pinning intensity, and vice versa. Indeed, assume that, without pinning, the vortex and the normal-state Hall effects have different signs. The Hall anomaly appears as temperature or magnetic field are decreased below T_c or H_{c2}, respectively, because the vortex contribution overtakes the normal state Hall value. For increasing pinning, the vortex contribution gets smaller and can reach values which are insufficient to counteract that of the normal state. The vortex contribution may even change its sign which also affects the Hall anomaly. Pronounced effects of pinning on the Hall anomaly have been observed experimentally [4].

The fact that pinning affects both longitudinal and transverse forces acting on a moving vortex may have an implication for interpreting measurements of the transverse force experienced by moving vortices [10]. Indeed, the pinning contribution to the transverse force does not drop out if the direction of the magnetic field is reversed; thus a substantial pinning component may be present in the total force measured in Ref. [10].

In conclusion, we demonstrate that pinning strongly renormalizes both longitudinal and Hall flux flow resistivity. Using a simple model for pinning we show that the magnitude of the vortex contribution to the Hall voltage decreases with increase in the pinning strength or with a decrease in transport current. The Hall resistivity ρ_{xy} does not scale generally as ρ_{xx}^2. A strong enough pinning can even result in a sign reversal of ρ_{xy}.

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