# Lambda Point in the $^4$He-Vycor System: A Test of Hyperuniversality

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We have performed a high resolution specific heat measurement on $^4$He completely filling the pores of Vycor glass. Within 10 mK of the superfluid transition we observe a peak in the heat capacity which is only 0.02% the size of the background. The peak is fitted with a rounded version of the "logarithmic singularity" observed in bulk $^4$He. This adds strong support to the suggestion that disorder imposed by the Vycor is irrelevant to the 3DXY superfluid phase transition. The size of the peak, while in agreement with that found in other experiments in dilute superfluid $^4$He, is considerably larger than that predicted by the theory of hyperuniversality.

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One of the foundations of the modern theory of critical phenomena is that all phase transitions are divided into a handful of universality classes. Experimentally, one groups phase transitions on the basis of the power laws in the reduced temperature $t = 1 - T/T_C$ which thermodynamic properties follow very near to the transition temperature $T_C$. The critical exponents of these power laws, which can be predicted to high accuracy thanks to the computational machinery of the renormalization group theory (RGT), are the same for all systems within a universality class [1].

Because of its high purity, lack of strain, and the relative ease of precision measurement, the superfluid transition of $^4$He is an unrivaled testing ground not only for determining the values of critical exponents, but also for answering questions about universality. Very precise measurements of the specific heat of $^4$He to within a few nK of $T_\lambda$ have confirmed the power law

$$C = \begin{cases} A(-t)^{-\alpha} + B & \text{for } T_\lambda\,, \\ A' t^{-\alpha} + B' & \text{for } T_\lambda\,, \end{cases} \tag{1}$$

where $\alpha = -0.012\,45$, $A = 5.594$ J/mol K, and $A/A' = 1.054$. $B$ and $B'$ are nonuniversal parameters [2,3].

The superfluid density has also been measured precisely under the conditions of saturated vapor pressure and is found to obey the power law

$$\rho_S = \rho_{S0} t^{-\nu} \quad \text{for } t > 0\,, \tag{2}$$

with $\nu = -0.6702$ and $\rho_{S0} = 0.351$ g/cm$^3$ [4]. The "logarithmic singularity" in the specific heat of Eq. (1) and the "2/3 power law" of Eq. (2) identify the superfluid transition of $^4$He as a member of the 3DXY universality class.

Another pillar in the theory of critical phenomena is the theory of hyperuniversality [5]. Hyperuniversality is a statement that the amount of energy (measured in units of $k_B T$) per fluctuation is the same for all systems within a universality class. This is expressed by defining the universal constant $R$,

$$R = \left(\frac{A}{k_B}\right)^{1/3} \xi_0\,, \tag{3}$$

where $\xi_0$ is the critical amplitude of the correlation length. Recent RGT calculations find $R = 0.815 \pm 0.015$ [5]. While $\xi_0$ is not accessible directly, it can be calculated from the superfluid density by using the Josephson relation [6]

$$\xi_0 = \frac{k_B T_C m_{\mathrm{He}}^2}{\hbar^2 \rho_{S0}}\,. \tag{4}$$

The relation in Eq. (3) was tested by Ahlers and co-workers, who performed specific heat and superfluid density measurements near $T_\lambda$ in samples at a series of pressures between saturated vapor pressure (SVP) and 30 bars [3]. $^4$He samples along this $\lambda$ line are believed to all be in the 3DXY universality class. Ahlers and co-workers adjusted $\xi_0$ from 3.1 Å to 3.5 Å by varying the pressure, and in this range they found a pressure-independent value of $R = 0.84 \pm 0.02$, in reasonable agreement with the theory of hyperuniversality [5].

Since the range of pressures over which superfluid $^4$He exists is restricted from 0.05 bar (SVP) to 30.2 bar, one can adjust the correlation length by only 13%. Another approach, which allows a dramatic increase in $\xi_0$, is to place the $^4$He in a suitable porous material, such as Vycor. Vycor is a porous glass with a highly interconnected network of pores of average diameter 70 Å, providing an open volume of 30%. Scattering measurements show that Vycor is uncorrelated on all length scales above 100 Å [7,8]. As shown in previous work with $^4$He in porous Vycor glass [9], the $^4$He-Vycor system affords a test of the theory for values of $\xi_0$ in excess of 100 Å. Because the disorder imposed by the Vycor is uncorrelated on length scales greater than $\xi_0$, and because for the superfluid transition in bulk $\alpha < 0$, the Harris criterion [10] suggests that the superfluid critical behavior should not be altered by the presence of the Vycor. These arguments are supported by the "2/3" power laws observed in the superfluid density of both films and filled-pore samples [9]. The superfluid density of porous gold [11], which has a structure very similar to that of Vycor except with a pore size of hundreds of Å, also exhibits the "2/3" power

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law. Furthermore, cusps in the specific heat coincident with superflow onset were seen for *films* of $^4$He on Vycor [12] and for porous gold filled with $^4$He [11]. The picture we derive from these studies, consistent with the Harris criterion, is that the primary effect of these porous media is to dilute the superfluid without changing the nature of the superfluid phase transition. In other words, the $^4$He-Vycor system is a dilute superfluid in the same universality class as the pure $^4$He system.

What was missing from this picture, until recently, is a $\lambda$ peak in the specific heat at $T_C$ in filled-pore Vycor. The failure of several attempts [12–14] to see this feature is often explained through the theory of hyperuniversality, which argues that the cusp would have been too small to see. The lack of such a peak would be inconsistent with our regarding the $^4$He-Vycor system as another member of the 3DXY universality class. In this paper we report on the observation of the cusp near the $\lambda$ point in the $^4$He-Vycor system, demonstrate that this system is a dilute 3D superfluid in the 3DXY universality class, and discuss its implications to the theory of hyperuniversality.

To this end, we have performed a high resolution measurement of the specific heat of the $^4$He-Vycor system. Our best measurements have a resolution of about 5 ppm, compared to the measurements of Finotello *et al.* [12], who have attained a resolution of 0.2%. This much-increased resolution is accompanied by a higher density of data points per temperature interval. While we give a brief summary of our novel technique here, a more thorough discussion of our calorimeter appears in another article [15]. In short, our measurement takes the high resolution thermometers of Lipa and co-workers [16] as a point of departure. These authors used the paramagnetic salt $Cu(NH_4)_2Br_4$ (CAB), which has a Curie temperature of about 1.8 K, as a thermometric element. The magnetic flux contained in the superconducting pickup coil around the CAB changes rapidly as a function of temperature around 2 K. The SQUID-based detection system of Lipa and co-workers, through measuring this magnetic flux, allowed a temperature resolution of 0.1 nK at $T_\lambda$.

Our technique is to ramp up the temperature of the experiment with the application of a constant heater power $\dot{Q}$ to the cell. The cell warms through the temperature range from 1.8 to 2.2 K which contains the known $T_C$ for the superfluid transition in this sample. The heat is generated by Joule heating a low temperature resistor on the cell with a high-precision current source. For these measurements our heating rates vary from 0.2 $\mu$W to 5 $\mu$W, which corresponds to temperature drift rates at 2 K between 1.5 $\mu$K/s and 40 $\mu$K/s. We calculate the heat capacity from

$$C = \frac{\dot{Q}}{\dot{T}}, \tag{5}$$

where $\dot{T}$ is the time rate of change of temperature.

In these experiments we measured the heat capacity under saturated vapor pressure of $^4$He filling the pores of the Vycor. The two Vycor samples are disks of diameter 2.5 cm and thickness 1 mm, giving a volume of 0.478 cm$^3$. We ground our specimens from a plane equidistant from the surface and the leach plane in the center of 0.953 cm thick plates. The low aspect ratio of the Vycor enhances the thermal contact providing an internal time constant of the cell of around 40 seconds below $T_\lambda$. Rounding of features in the heat capacity due to the drift ranges from 0.06 to 1.6 mK.

A cold valve on the cell allows it to be filled and closed off without leaving $^4$He in the fill capillary. Two regulated, weakly coupled isothermal stages absorbed heat leaks before they could make their way to the cell, to which heat flows from the final regulation stage with a thermal time constant of about $10^5$ seconds. The thermal filtering provided by these isolation stages reduces heat leaks to $10^{-6}$ the size of the applied heat. We measured the temperature of the experiment with a carbon glass thermometer which we checked using $T_\lambda$ of the bulk helium in our cell as a reference. Our experiment was mounted on a $^3$He refrigerator.

The total heat capacity of the filled cell near 2 K is shown in Fig. 1. We can see clearly the broad peak in the heat capacity at 2.1 K, well above $T_C$, which was first observed by Champeney and Brewer [14]. Also clearly shown is the sharp cusp at the bulk $T_\lambda$. The $^4$He in the Vycor pores accounts for 35% of the heat capacity of the cell at 2 K. The superfluid bulk $^4$He is about 45%, and the remainder is the heat capacity of the cell construction materials, including copper, brass, niobium, and the CAB salt. From carefully filling the Vycor during superfluid density measurements, we conclude that $5.33 \times 10^{-3}$ moles of $^4$He fill the Vycor.

It is very difficult to make out a tiny feature at 2 K from looking at Fig. 1. In Fig. 2 we subtract a straight line from the total cell heat capacity in the region of

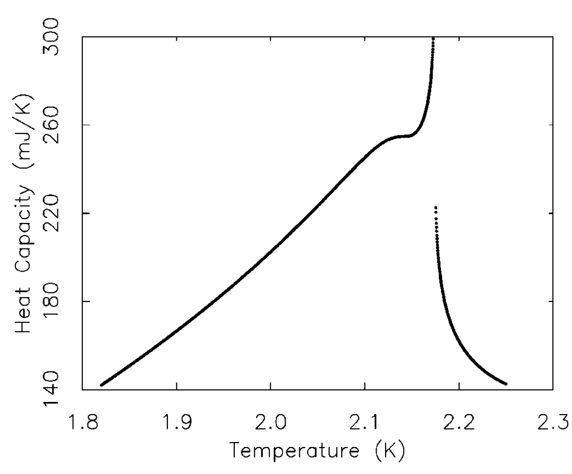


FIG. 1. The total heat capacity of the cell.

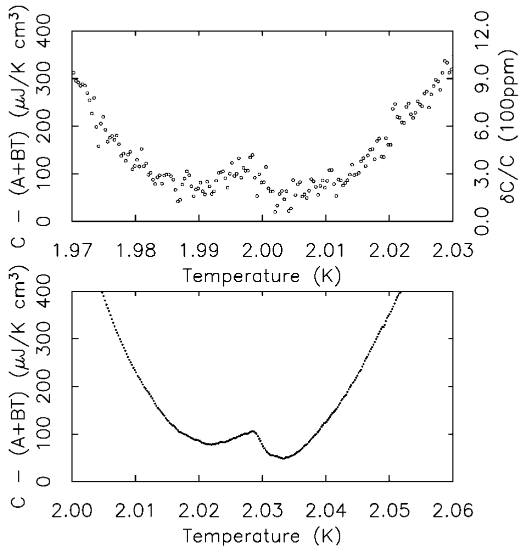


FIG. 2. The total specific heat in the region of interest for both Vycor samples, with a straight line subtracted so as to emphasize the cusp occurring near $T_C$. The upper panel shows the heat capacity from a sample with $T_C = 1.995$, while the lower shows a later, higher resolution measurement on a different sample with $T_C = 2.031$.

interest for both samples. The measurement on the first Vycor sample, shown on the upper panel of Fig. 2, results from our implementation of a thermometry system of the design of Ref. [16]. We have briefly reported on these measurements in Ref. [17]. We can clearly resolve a peak about three times larger than the noise level. Insertion of a 1 mm diam by 3 mm long cylinder of Vycor taken from the same plate as the first sample into one of the channels of a double-channel Helmholtz resonator, a technique described in more detail in Ref. [18], allows us to determine the temperature of superflow onset. The double-channel Helmholtz resonance is associated with the ac flow of $^4$He driven through a superleak in parallel with an open channel of similar size. A large shift in frequency is observed at the onset of superflow in the superleak. In this experiment a different set of thermometers was used, but we nonetheless determined superflow onset to be at 1.987 K, while the cusp in the specific heat is near 1.995 K.

The lower panel of Fig. 2 shows the most recent measurement of the second sample of Vycor. For this sample we modified our measurement technique, after realizing that we could obtain a signal directly proportional to $\dot{T}$ by inserting a small resistance into the superconducting flux loop of the high resolution thermometer containing the SQUID input coils, the leads, and the CAB pick-up coils. Our detection system is then sensitive not to the magnetic flux in the pickup coils, but rather to the changes in the flux with respect to time which is directly proportional to $\dot{T}$. Again using the continuous warming technique, we obtained the data in the lower panel of Fig. 2, which show greatly improved resolution.

This second Vycor sample was mounted in a torsional oscillator similar to that used in earlier work [8]. The onset temperature for superflow was then determined while using the same carbon-glass thermometer from the heat capacity measurement. We found superflow onset to occur at 2.023 K. We can conclude that both peaks occur within 10 mK of the superflow onset temperatures as measured in the separate experiments.

The more recent data are of sufficient quality to allow comparison to the bulk $^4$He $\lambda$ peak and thereby test the theory of hyperuniversality. Because we expect the peak to have a nearly logarithmic shape, the singular contribution from the cusp contributes significantly to the specific heat even 50 mK away from $T_C$. This precludes a direct background subtraction.

So we take a different approach and instead subtract a rounded and reduced version of the bulk specific heat of the functional form of Eq. (1). We use the universal parameters $A$, $A'$, and $\alpha$ measured in bulk and the bulk value for the nonuniversal constants $B = B' = 0.4335\ \mathrm{J/K\,cm^3}$ [3]. We round the bulk specific heat by convolving it with a normalized Gaussian function of width $\sigma$ centered at $T_C$. We reduce the amplitude of the specific heat cusp by the factor $\Lambda$.

We perform the fit by varying the parameters $\sigma$, $\Lambda$, and $T_C$ and subtracting the rounded bulk cusp from the raw specific heat. The remainder is the residual curve. Our goal is to obtain a residual curve with no trace of the observed cusp, as judged by the deviation of the residual from a third-order polynomial fit over the 50 mK range. We demonstrate the fitting technique in Fig. 3. This figure shows our best fit with $T_C = 2.0318 \pm 0.0007$ K, $\sigma = 2.3 \pm 0.8$ mK, and $\Lambda = 1.4 \pm 0.2 \times 10^{-4}$. The $T_C$ and $\Lambda$ are in line with our expectations. The 2.3 mK rounding is very reasonable given the rounding in the superfluid density observed in this sample.

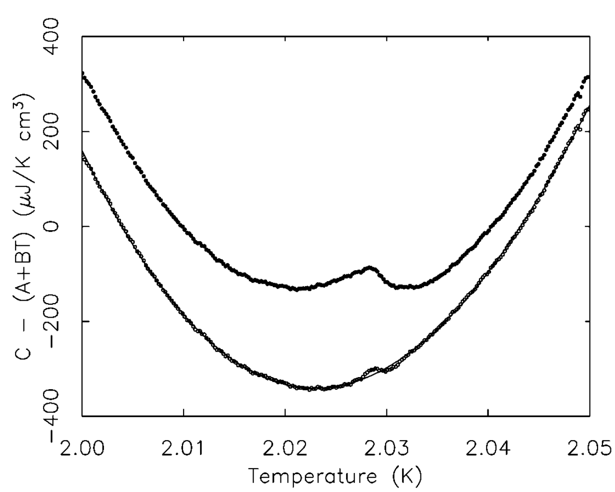


FIG. 3. The specific heat close to $T_C$ with a linear term subtracted. The top curve shows the raw data, while the lower curve is the residual after subtraction of the rounded cusp.

We see from Fig. 3 that there is in fact a large difference between the data trace and the residual even far away from $T_C$. Also we see that most of the bump appears to be gone, but there are still some wiggles near $T_C$. Subtracting the best-fit third-order polynomial as a background function from the data trace leaves us the singular part of the specific heat. These data are plotted out to a range of 0.015 in reduced temperature in Fig. 4. This is the approximate range over which the "2/3" power law fits the superfluid density in Vycor. Except for the slight deviations just above $T_C$, the fit is fairly reasonable. We also plot the superfluid density measured in this sample and see that it goes to zero very close to the $T_C$ obtained from the fit, but still below it. In both samples the displacement of the specific heat peak above superflow onset is somewhat larger than our expectations, a fact for which we currently have no explanation.

Figure 4 lends the most striking support to the assertion that the $^4$He-Vycor system is in the 3DXY universality class along with bulk $^4$He. We use the predictions of hyperuniversality [5] to relate the specific heat and the correlation length. We find that the critical amplitude $\rho_{S0} = 11.9 \pm 0.1$ mg/cm$^3$, which via Eq. (4) corresponds to $\xi_0 = 93 \pm 1$ Å. Inserting our values for $A$ and $\xi_0$ into Eq. (3) we find that $R = 1.2 \pm 0.2$.

This is considerably larger than that found from measuring the specific heat and superfluid density in bulk $^4$He samples along the $\lambda$ line [3]. However, this discrepancy is supported by applications of hyperuniversality to thin films of $^4$He on Vycor [12] and also to $^4$He filling the pores of porous gold [11]. We must note that particularly in the Vycor systems we are expecting hyperuniversality to work over a factor of 30 in correlation length.

In this Letter, we have shown the results of a calorimetric measurement of the $^4$He-Vycor system over 2 orders of magnitude more precise than the prior measurements. The primary result of this work is the resolution of a heat capacity cusp very near to superflow onset. We fit this cusp to a rounded and diminished cusp of the sort found at $T_\lambda$ in bulk $^4$He through which we test the theory of hyperuniversality. We find that our peak, although larger than that predicted by the theory, is in fair agreement with peaks measured in other dilute $^4$He systems. The theory may capture much of the physics, but either there is some fine tuning necessary, or there is some subtle aspect of the imposition of disorder which has been ignored. We hope to see a resolution to the remaining discrepancy between the theory and our observations.

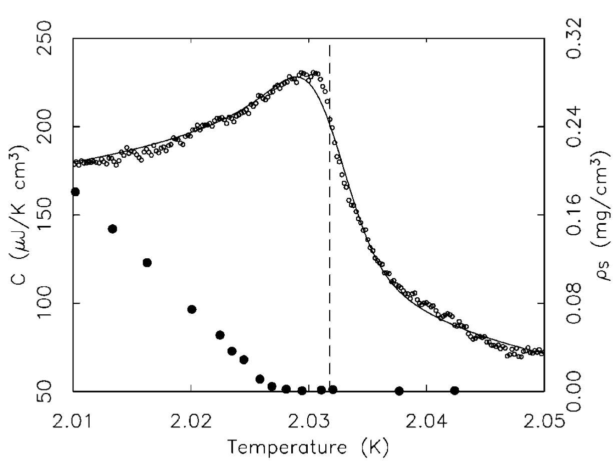


FIG. 4. The data near $T_C$ and the rounded cusp fit plotted with the superfluid density.

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