# Spectra and Statistics of Velocity and Temperature Fluctuations in Turbulent Convection

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Direct measurements of the velocity in turbulent convection of $SF_6$ near its gas-liquid critical point by light scattering on the critical density fluctuations were conducted. The temperature, velocity, and cross frequency power spectra in a wide range of the Rayleigh and Prandtl numbers show scaling behavior with indices, which are rather close to the Bolgiano-Obukhov scaling in the wave number domain. The statistics of the velocity fluctuations remain Gaussian up to the Reynolds numbers of $10^5$.

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Turbulent convection exemplifies many startling aspects of turbulent flows. The discovery of scaling laws in the heat transport and temperature statistics sheds new light on the nature of the convective turbulence [1,2]. Bursts of thermal plumes from thermal boundary layers and a coherent large-scale circulation, which modifies the boundary layers via its shear, are found to coexist in a closed convection cell [1–3]. These two salient features, which reflect the competition between buoyancy and shear, were used in theoretical models to explain the observed scalings in the temperature field [1,4,5]. The predictions which follow from these models for the velocity field are still far from experimental verification [5].

Therefore, direct measurements of the velocity field would become a crucial step to test predictions of various theoretical models, to understand the mechanism of energy vs entropy transfer, to determine the relative contribution of the thermal plumes and the large-scale circulation to the heat transport, and to study the mechanism of the onset and generation of the large-scale flow by intermittent thermal bursts.

Lack of velocity field measurements is explained by the fact that both conventional methods, such as hot-wire anemometry and laser Doppler velocimetry (LDV), are not suitable for turbulent convection, as was commonly agreed [6]. Indeed, a hot-wire anemometry is useless due to strong temperature fluctuations which accompany velocity fluctuations, and thus do not permit one to correctly measure velocity. Fluctuations in a refractive index due to the large temperature fluctuations corrupt the signal and drastically reduce the signal-to-noise ratio of LDV measurements because of wandering and defocusing of laser beams. Various attempts to measure the velocity field were rather limited due to severe experimental difficulties in directly measuring the fluid velocity in convective flows in a wide dynamical range [6,7]. Novel direct velocity measurements by the dual- and single-beam two-color cross-correlation spectroscopy technique [6] in the rather limited range of the Rayleigh numbers, Ra, still do not provide information about the velocity spectra, even in the frequency domain, and direct information about statistics in a sufficiently wide dynamical range.

Another problem, which is still controversial and requires further experimental tests, is whether the scaling laws in the inertial range of the temperature and velocity power spectra in homogeneous turbulent convection are either those suggested by Kolmogorov or those suggested by Bolgiano and Obukhov (BO) [4,8]. It was argued [4] that only the Kolmogorov scaling $k^{-5/3}$ for both the temperature and velocity power spectra, where $k$ is the wave number, is consistent. It means that the temperature is a passive scalar which follows just the velocity field. Other theories predict that, depending on the parameter range, either the Kolmogorov or BO scalings can be observed [8]. The latter gives a power law $k^{-11/5}$ for the velocity fluctuation spectrum and $k^{-7/5}$ for the temperature fluctuation spectrum. Here the temperature is not a passive scalar but provides a potential energy on all scales. The parameter which determines the transition from the BO to Kolmogorov scaling is the Bolgiano length $l_B$ [8,9]. From a physical point of view, $l_B$ is the scale at which a potential energy is injected into a system, and then buoyancy transforms it into kinetic energy. Thus the buoyancy force should be relevant on scales $l > l_B$, where the BO scaling takes place, while at $l < l_B$ the Kolmogorov scaling is expected [8,9]. As shown in Refs. [8,9], at $\mathrm{Pr} > 1$, $l_B$ is smaller than a characteristic scale $L$ of a system, which means that two scaling regimes can coexist in the temperature spectra. Moreover, $l_B$ decreases with increasing Ra and Pr, which results in a wider range of the BO scaling [9].

First measurements of the frequency spectra of the temperature fluctuations [1] and many subsequent experiments [3,5] reveal the BO power law, although the inertial range was very limited. The applicability of the Taylor hypothesis in this case is not completely justified, particularly close to the middle of the cell where a large-scale circulation is rather weak. The only measurements of the velocity spectra in a real space by a homodyne spectroscopy method in a rather narrow range of Ra also confirm the BO predictions [10]. On the other hand, numerical experiments on Fourier-Weierstrass approximation of the Boussinesq equations [11], various numerical studies of the turbulent convection [12], and, particularly, recent remarkable ones

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[13] support the Kolmogorov scaling. However, analysis of the numerical data on Rayleigh-Bénard convection based on the extended self-similarity [14] reveals BO scaling, and calls for further experiments.

Our experimental studies during the past several years convincingly demonstrated that the Rayleigh-Bénard convection in a gas near the gas-liquid critical point (CP) is a very appropriate system to study high Ra turbulent convection [15,16]. Singular behavior of the thermodynamic and kinetic properties of the fluid near $T_c$ provides the opportunity to reach both extremal values of the control parameter (Ra up to $5 \times 10^{14}$ has been reached in our experiment on $SF_6$) and to scan Prandtl (Pr) number over an extremely wide range (from one up to several hundred). All of these features make the system unique in this respect [15,16]. However, the most relevant aspect of the system to the results presented in this Letter is our finding that the presence of the critical fluctuations provides us with the possibility to conduct LDV measurements in a rather wide range of proximity to CP between $3 \times 10^{-4}$ and $10^{-2}$ in the reduced temperature $\tau = (\overline{T} - T_c)/T_c$, where $\overline{T}$ is the mean cell temperature. It corresponds to variation in Pr from about 20 to 200. The upper limit in $\tau$ is defined by a scattered light intensity. It is experimentally found that at $\tau > 10^{-2}$ the photomultiplier current is too low to recover the Doppler frequency shift. The lower limit is determined by multiple scattering in a large convection cell, as a result of which the light intensity at the detector is drastically reduced. The signal-to-noise ratio of the LDV measurements is increased with the closeness to CP until it drops rather abruptly at $\tau < 3 \times 10^{-4}$ [15,17].

Critical density fluctuations, which move with the flow, scatter light as effectively as seeding particles in a conventional LDV: the corresponding signal from a photomultiplier is of excellent quality [15,17]. The light scattering occurs on the short-scale thermal density fluctuations advected by the large-scale turbulent fluctuations. In the range of $\tau$ used in the experiment, the scattering takes place mostly on the thermal density fluctuations. Indeed, due to the smallness of the Mach number at maximum flow velocities, the density fluctuations due to turbulent flow were negligible.

In this Letter we studied Pr dependence of spectra and statistical properties of the vertical velocity fluctuations at one location in the frequency domain using this novel LDV technique. In addition we also obtained the temperature fluctuations spectra in a wide range of Ra and Pr.

The experiment we present here was done with a high purity gas $SF_6$ (99.998%) in the vicinity of $T_c$ and at the critical density ($\rho_c = 730$ kg/m$^3$). This fluid was chosen due to the relatively low critical temperature ($T_c = 318.73$ K) and pressure ($P_c = 37.7$ bars) and the well-known thermodynamic and kinetic properties far away from and in the vicinity of CP. The setup was described elsewhere [15,16]. The cell is a box of a cross section $76 \times 76$ mm$^2$ formed by 4 mm Plexiglas walls, which are sandwiched between a Ni-plated mirror-polished copper bottom plate and a 19-mm-thick sapphire top plate at $L = 105$ mm apart, i.e., the cell aspect ratio is 0.7.

The cell is placed inside the pressure vessel with two side thick plastic windows to withstand the pressure difference up to 100 bars. The cell had optical accesses from above, through the sapphire window, and from the sides. They were used for both shadowgraph flow visualization and for LDV. The pressure vessel was placed inside a water bath that was stabilized with rms temperature fluctuations at a level of 0.4 mK. The gas pressure was continuously measured with 1 mbar resolution by the absolute pressure gauge. Together with a calibrated 100 $\Omega$ platinum resistor thermometer, they provide us with the thermodynamic scale to define the critical parameters of the fluid and to use the parametric equation of state recently developed for $SF_6$ [18]. Errors in the experimental determination of the critical parameters together with inaccuracy in the values of the thermodynamical and kinetic properties of $SF_6$ give errors of about 20% in Pr and about 40% in Ra in a whole range.

Local temperature measurements in gas were made by three 125 $\mu$m thermistors suspended on glass fibers in the interior of the cell (one at the center, and two about halfway from the wall). Local vertical component velocity measurements at about $L/4$ from the bottom plate were conducted by using LDV on the critical density fluctuations [15,17].

A typical time series of the vertical velocity component of turbulent convection at Pr = 93 and the frequency power spectra at Ra from $3 \times 10^{11}$ to $8 \times 10^{13}$ is shown in Fig. 1. The spectra for other values of Pr look similar. At higher Ra they are characterized by well-defined low frequency peaks related to the large-scale circulation by surprisingly large inertial range and by a relatively

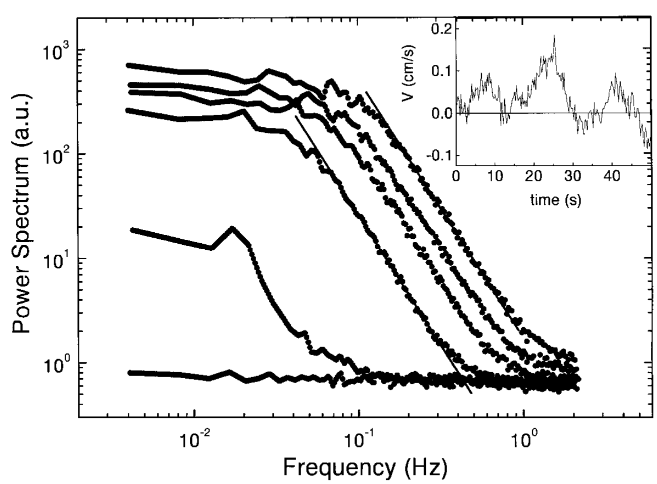


FIG. 1. Frequency power spectra of velocity fluctuations for Ra from $3 \times 10^{11}$ to $8 \times 10^{13}$ at Pr = 93. The solid line is a power law fit with a scaling index $-2.43$. Inset: Vertical velocity time series at Ra = $3 \times 10^{12}$.

high level of white noise, which significantly limits the dynamical range of the spectra. The noise level does not depend on the flow velocity and remains at the same level regardless of the heating power, which drives the convection flow, but does depend on $\tau$. Therefore, the signal-to-noise ratio increases when approaching the CP and when the velocity increases. As shown, the source of the white noise is the phase noise due to the critical density fluctuations. The results of these studies will be published elsewhere [15,17].

The random phase of the scattering signal causes an additional noise in the velocity measurements, and thus limits obtaining the reliable data. Indeed, the measured Doppler frequency shift can be presented as $\dot{\phi} = \vec{V} \cdot \vec{k} + \dot{\phi}_{\text{th}}$, where $\vec{V} \cdot \vec{k}$ is the velocity component along the wave vector $\vec{k}$, and $\dot{\phi}_{\text{th}}$ is the frequency shift due to the thermal phase noise. Then the corelation function of the measured frequency shift, averaged over a long time, is $\langle \dot{\phi}\dot{\phi} \rangle = \langle (\vec{V} \cdot \vec{k})^2 \rangle + \langle \dot{\phi}_{\text{th}}\dot{\phi}_{\text{th}} \rangle + 2\langle (\vec{V} \cdot \vec{k}) \cdot \dot{\phi}_{\text{th}} \rangle$. This expression at adiabatically slow temporal variations of $\vec{V} \cdot \vec{k}$ reduces to $\langle \dot{\phi}\dot{\phi} \rangle = \langle (\vec{V} \cdot \vec{k})^2 \rangle + \langle \dot{\phi}_{\text{th}}\dot{\phi}_{\text{th}} \rangle$, which leads to the same additive relation in the corresponding frequency spectrum. Therefore, at $\vec{V} \cdot \vec{k} \gg \dot{\phi}_{\text{th}}$, one reproduces the velocity from the measured Doppler frequency shift. Besides, the smallest energy containing eddies should be larger than both the correlation length and beam waist. If these conditions are satisfied, one can expect that the frequency shift of the scattered light is equal to the fluid velocity in the wave number units.

From the fit in the inertial range (Fig. 1), one finds well-defined power law behavior. In fact, the LDV measurements do not include the whole inertial range due to the noise limitation. So one cannot reach the dissipation region. The scaling index obtained by the linear fit, similar to that shown in Fig. 1, and averaged over all values of Ra and Pr, gives $-2.4 \pm 0.2$, which is rather close to the BO scaling in the wave number domain [8].

From the velocity time series, the probability density function (PDF) of the velocity fluctuations normalized by the rms velocity was also constructed. The PDF of the velocity fluctuations as well as the velocity differences at a finite time lag were Gaussian (Fig. 2) for all Ra and Pr studied. The observation of the Gaussian PDF for the velocity differences is possibly related to the problem of noise on small scales, as discussed above. We also investigated the dependence of the rms vertical velocity fluctuations and mean vertical velocity on Ra and Pr. The latter scaling behavior was reported elsewhere [15,16]. The former is presented in Fig. 3 in Re$(= V_s L/\nu)$ vs Ra coordinates. Here $V_s$ is the rms of vertical velocity fluctuations, and $\nu$ is the kinematic viscosity. The fit to the data for three values of Pr = 27, 45, and 93 suggest the scaling Re $= 0.005\text{Ra}^{0.43\pm0.02}$ with no Pr dependence. The data for Pr = 190 deviate from this scaling behavior and can be better fitted with the scaling exponent 0.34.

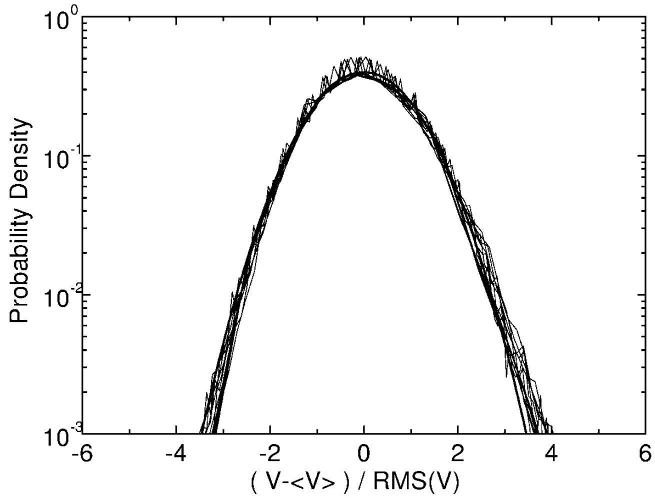


FIG. 2. PDF of the velocity time series for Ra from $3 \times 10^{11}$ to $8 \times 10^{13}$ at Pr = 93. Each PDF is normalized by the rms of each time series. The thick line is a Gaussian PDF.

The former scaling exponent of Re vs Ra for lower Pr is rather close to the value obtained for water (Pr = 7) and for the theoretical prediction $3/7$ of both the mixing zone [1] and the large-scale flow theories [4]. We would like to emphasize that Pr dependence of Re in the bulk is very different from that found for the large-scale circulation [15,16].

The frequency power spectra of the temperature fluctuations were measured both at the cell center and on the side ($L/4$ from the wall) in a wide range of Ra and Pr. We also used the data which we took in the compressed gas $SF_6$ far away from CP at $P = 20$ bars, $\overline{T} = 303$ K, and the density $\rho = 0.18$ g/cm$^3$, which corresponds to Pr = 0.9, and at $P = 50$ bars, $\overline{T} = 323$ K, and $\rho = 1.07$ g/cm$^3$, which corresponds to Pr = 1.5.

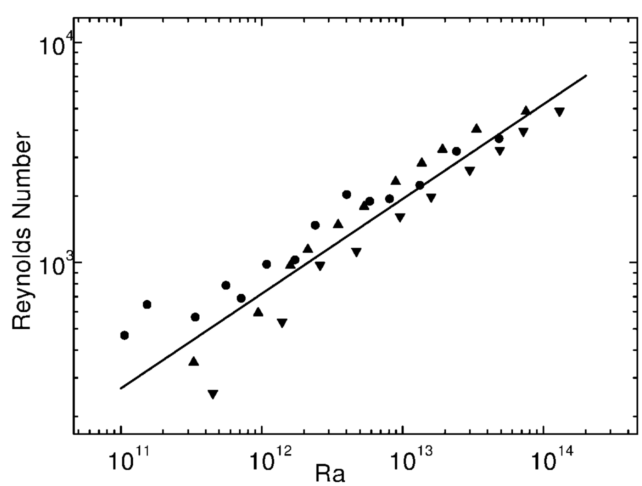


FIG. 3. Re for the rms vertical velocity fluctuations as a function of Ra for three values of Pr: 27 (circles), 45 (up triangles), 93 (down triangles). The solid line is a power law fit.

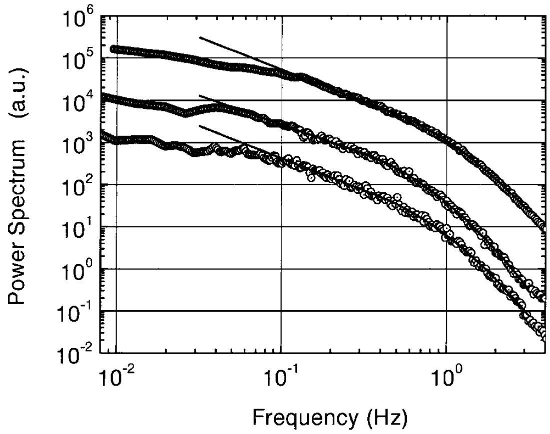


FIG. 4. Frequency power spectra of temperature fluctuations at the cell center at Pr = 93, Ra = $8.4 \times 10^{13}$ (top); Pr = 190, Ra = $1.5 \times 10^{14}$ (middle); Pr = 300, Ra = $3 \times 10^{14}$ (bottom). In each plot a line of the best fit by the model described in the text is shown.

The data far away from CP cover the range of Ra between $10^9$ and $5 \times 10^{12}$. Most of the power spectra did not show any reasonable range of a scaling law. At Pr above 50 and the highest Ra, the power law behavior became noticeable. At Pr $> 90$ and the highest values of Ra, the scaling region of about one decade in frequency $f$ was observed (Fig. 4). The scaling index was obtained by fitting the spectra by [2]

$$P(f) = \begin{cases} Bf^s & \text{if } f < f_c\,, \\ Bf^s(f/f_c)^{\ln(f/f_c)^{-\alpha}} & \text{if } f > f_c\,, \end{cases}$$

where $s$, $f_c$, and $\alpha$ are the fit parameters. The scaling index obtained by the fit and averaged over all spectra is $s = -1.45 \pm 0.1$, which is close to the BO power law for the temperature fluctuation spectra in the wave number presentation.

One can expect that the characteristic frequency $f_c$ is directly related to the Bolgiano length via the large-scale flow velocity $V_{\rm ls}$, namely, $f_c = V_{\rm ls}/l_B$. Values of $l_B$, obtained from $f_c$ and the measured $V_{\rm ls}$, are at least an order of magnitude larger than the values of $l_B$, computed for the corresponding Ra and Pr up to the prefactor [9].

Thus the measured scaling indices for both the velocity and temperature fluctuations in the frequency domain are rather close to the theoretically predicted BO scaling in the wave number presentation. As commonly accepted, the Taylor hypothesis can be used to pass from the wave number to the frequency domain. Since we measured both the velocity and temperature spectra in the reqion with nonzero mean velocity, the application of the Taylor hypothesis is better justified. From the velocity and temperature time series taken at the same spatial location, we constructed cross spectra which also showed scaling behavior with the scaling index $-1.85 \pm 0.1$, which is close to the theoretically predicted value in the BO region [8].

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