First-Order Chiral Phase Transition May Naturally Lead to a "Quenched" Initial Condition and Strong Soft-Pion Fields

O. Scavenius[1,2] and A. Dumitru[1]
[1]*Physics Department, Yale University, P.O. Box 208124, New Haven, Connecticut 06520*
[2]*The Niels Bohr Institute, Blegdamsvej 17, DK-2100 Copenhagen Ø, Denmark*
(Received 17 June 1999)

We propose a novel mechanism for disoriented chiral condensate (DCC) formation in a first-order chiral phase transition. In this case the effective potential for the chiral order parameter has a local minimum at $\Phi \sim 0$ in which the chiral field can be "trapped." If the expansion is fast, a bubble of disoriented chiral field can emerge and decouple from the rest of the fireball. The bubble may overshoot the mixed phase and supercool until the barrier disappears, when the potential resembles that at $T = 0$. This situation corresponds to the initial condition realized in a "quench." Thus, the subsequent alignment in the vacuum direction leads to strong amplification of low-momentum modes of the pion field. We propose that these DCCs could accompany the previously suggested baryon rapidity fluctuations.

PACS numbers: 25.75.–q, 11.30.Qc, 11.30.Rd, 24.85.+p

Relativistic heavy-ion collisions might offer the interesting opportunity to study chiral symmetry restoration at nonzero temperature and density, which could possibly lead to the formation of domains of disoriented chiral condensate (DCC) [1–5]. The strongest amplification of the pion field is obtained for the so-called "quenched" initial condition [3]. It is assumed that the heat bath is removed instantaneously after restoration of chiral symmetry.

However, dynamical simulations [4,5] show that the "quench" does not emerge naturally in a heavy-ion collision, if the chiral phase transition is second order or a smooth crossover. In this Letter, we instead propose a new approach to obtain the quenched initial conditions naturally in the presence of a first-order phase transition.

It has been argued [6] that the phase transition for two massless quarks at baryon-chemical potential $\mu = 0$ is second order which then becomes a smooth crossover for small quark masses. On the other hand, a first-order phase transition is predicted for small temperatures and large μ. If, indeed, there is a smooth crossover for $\mu = 0$ and nonzero T, and a first-order transition for small T and nonzero μ, then the first-order phase transition line in the (μ, T) plane must end in a second-order critical point. This point is predicted to be at $T \sim 100$ MeV and $\mu \sim 600$ MeV. However, some lattice QCD results indicate a first-order transition even at vanishing baryon-chemical potential [7].

Such temperatures and baryon-chemical potentials can be reached in the central region of heavy-ion collisions in the forthcoming Pb(40*A* GeV) + Pb experiments at the CERN-SPS [8], and in the fragmentation regions of more energetic collisions at the CERN-SPS, BNL-RHIC, and CERN-LHC ($\sqrt{s} \simeq 20A$, $200A$, $5000A$ GeV) [9]. Furthermore, fluctuations in individual events can also provide rapidity bins with significantly higher μ and lower T than on average [10–13]. In any case, the dynamical scenario for DCC formation described in this Letter applies to the case of a first-order chiral phase transition, and is qualitatively independent of the value of μ. Our calculation described below has been performed at $\mu = 0$, and the parameters of the Lagrangian have been *chosen* such as to yield a first-order phase transition. It can be viewed as a representative example to illustrate the idea; the basic mechanism works equally well also at nonzero μ.

In case of a first-order transition, the thermodynamical potential as a function of the order parameter Φ exhibits a local minimum at $\Phi = 0$ both in the chirally restored phase as well as in the mixed phase [11,12,14]. In high-energy heavy-ion collisions, the expansion rate of the locally comoving three-volume element can become large [15]. This opens the possibility that the system can break up into smaller droplets [12,16], which might not be able to follow an adiabatic expansion. Instead, a bubble can "overshoot" the phase boundary [11] into the low-density broken phase. The chiral field in the bubble is coherent if the bubble radius is on the order of the coherence length or smaller. Close to a first-order phase boundary, the chiral field is light and this coherence length is expected to be large.

Suppose the bubble is created in the restored phase, close to the first-order phase boundary. Following [11], we suppose that the chiral field within the bubble is trapped in the $\Phi \sim 0$ local minimum of the potential (chiral symmetry is still restored but the field oscillates in the false direction), while in previous work [12,17] it was assumed that chiral symmetry breaking had already occurred in the bubble.

The preceding expansion will lead to a velocity profile in the bubble. In other words, the bubble will exhibit a Hubble-like expansion with a very large expansion rate [12,15], and supercool. During the period of supercooling the chiral field oscillates coherently within the whole bubble, while its energy dissipates partly due to friction (coupling to the heat bath [18]). The local minimum in the effective potential persists until the droplet reaches the spinodal line where the potential is close to that at $T = 0$ (one single minimum). The moment where the coherent field "leaps" over the barrier depends on the barrier height and the fluctuation. In a quasistatic situation, the field would tunnel to the global minimum. However, in a high-energy heavy-ion collision, the local expansion rate is so large (roughly 10^{20} times larger than that of the Universe at spontaneous chiral symmetry breaking [15]) that it is reasonable to assume that tunneling has no time to occur.

A second possibility for the chiral field to be kicked over the barrier to the global minimum at $\Phi \neq 0$ is via a thermal fluctuation. Estimates within homogeneous nucleation theory [19] show that the time scale for nucleation of critically sized bubbles with $\Phi \neq 0$ is about the same as that needed to reach the spinodal region (≈ 2 fm$/c$ in our calculation below). Finite-size effects delay bubble nucleation even further [20]: If our entire decoupled bubble has the size of the critically sized bubble in homogeneous nucleation theory, it cannot convert to the broken phase via homogeneous nucleation. Instead, it must supercool until it reaches the spinodal instability; cf. also [11]. Thus, the most favorable scenario for amplification of the low-momentum modes of the pion field, i.e., the quenched initial condition, is automatically realized in a natural way if the chiral phase transition is first order.

Similar to previous studies of DCC formation, our scenario also requires a rapid evolution out of equilibrium. However, the required 10%–20% supercooling appears much more moderate than the instantaneous removal of the heat bath at $T \sim T_C$, as is necessary to obtain the quenched initial conditions in a smooth crossover [3].

After the true vacuum is reached, the coherent chiral field will eventually decay into pions due to residual interactions [2,21]. If, at this stage, the heat bath is still very hot and dense, scattering of the DCC pions with particles from the heat bath will randomize the isospin orientation and spread the momentum distribution [22]. However, in case of a first-order transition with supercooling, the DCC decays at a much lower temperature and density of the heat bath. Therefore, it might be more feasible to detect the DCC pions. These pions will be blueshifted, though, according to the velocity of the bubble from which they emerged.

Above, we discussed the mechanism for DCC formation within a decoupled bubble. In principle, however, the same idea can be applied to the entire fireball, thus assuming that it supercools and reaches the spinodal instability as a whole. Nevertheless, we have chosen to describe how our picture works in a smaller droplet.

To illustrate the above idea, we applied the linear σ model coupled to a heat bath [5,16]. The Lagrangian of the linear sigma model with quark degrees of freedom reads

$$\mathcal{L} = \overline{q}[i\gamma^\mu \partial_\mu - g(\sigma + i\gamma_5 \vec{\tau} \cdot \vec{\pi})]q + \tfrac{1}{2}(\partial_\mu \sigma \partial^\mu \sigma + \partial_\mu \vec{\pi} \partial^\mu \vec{\pi}) - U(\sigma, \vec{\pi}), \qquad (1)$$

where the zero temperature potential is

$$U(\sigma, \vec{\pi}) = \frac{\lambda^2}{4}(\sigma^2 + \vec{\pi}^2 - v^2)^2 - H\sigma. \qquad (2)$$

Here, q is the light quark field $q = (u, d)$. The scalar field σ and the pion field $\vec{\pi} = (\pi_1, \pi_2, \pi_3)$ together form a chiral field $\Phi = (\sigma, \vec{\pi})$. This Lagrangian is invariant under chiral $SU_L(2) \otimes SU_R(2)$ transformations if the explicit symmetry breaking term $H\sigma$ is zero. The parameters of the Lagrangian are usually chosen such that the chiral symmetry is spontaneously broken in the vacuum and the expectation values of the meson fields are $\langle\sigma\rangle = f_\pi$ and $\langle\vec{\pi}\rangle = 0$, where $f_\pi = 93$ MeV is the pion decay constant. The constant H is fixed by the partially conserved axial current relation which gives $H = f_\pi m_\pi^2$, where $m_\pi = 138$ MeV is the pion mass. Then one finds $v^2 = f_\pi^2 - m_\pi^2/\lambda^2$. The sigma mass, $m_\sigma^2 = 2\lambda^2 f_\pi^2 + m_\pi^2$, which we set to 600 MeV, yields $\lambda^2 \approx 20$. The coupling to the heat bath, $g = 5.5$, is chosen such as to obtain the potential shown in Fig. 1. It corresponds to a first-order chiral phase transition. Note that this large value for g results in a constituent quark mass at $T = 0$ of $m_q = g f_\pi \sim 512$ MeV. Thus, the nucleon mass is too large. However, this is not relevant for our present considerations. A stronger coupling leads to a stronger first-order phase transition with a more pronounced barrier and even larger constituent quark mass; on the other hand, $g = 3.3$ fits the nucleon mass in the vacuum but results in a smooth crossover [5].

The Euler-Lagrange equations of motion for the fields,

$$\begin{aligned} \partial_\mu \partial^\mu \sigma + \lambda^2[\sigma^2 + \vec{\pi}^2 - v^2]\sigma - H &= -g\rho_s, \\ \partial_\mu \partial^\mu \vec{\pi} + \lambda^2[\sigma^2 + \vec{\pi}^2 - v^2]\vec{\pi} &= -g\vec{\rho}_{ps}, \end{aligned} \qquad (3)$$

are solved self-consistently through the effective quark and antiquark mass, $m_q = g\sqrt{\sigma^2 + \vec{\pi}^2}$, with the continuity equation for the energy-momentum tensor of the heat bath, which is constituted by the quarks:

$$\partial_\mu T^{\mu\nu} + \rho \partial^\nu m_q = 0, \qquad (4)$$

where ρ_s and $\vec{\rho}_{ps}$ are the scalar and the pseudoscalar densities, respectively, and $\rho = \sqrt{\rho_s^2 + \vec{\rho}_{ps}^2}$. To solve Eqs. (3), we employ a second-order leapfrog algorithm, while Eqs. (4) are solved with the relativistic Harten-Lax-Van Leer-Einfeldt algorithm [23], assuming that $T^{\mu\nu}$ is that of an ideal fluid in local thermodynamical equilibrium. For more details please refer to [5].

Let us consider the evolution starting close to the minimum of the potential shown to the left in Fig. 1. The evolution of the fields is shown in Fig. 2. The chiral field is "trapped" for $t \sim 2$ fm$/c$ in the local minimum until the temperature drops to the value corresponding to the potential to the right ($T \sim 100$ MeV). At this point, the bubble is supercooled by about 15% and the barrier to the global minimum has almost disappeared. Here, the field starts to "accelerate" and rolls towards the true vacuum. For this scenario, we find that the pion field is amplified

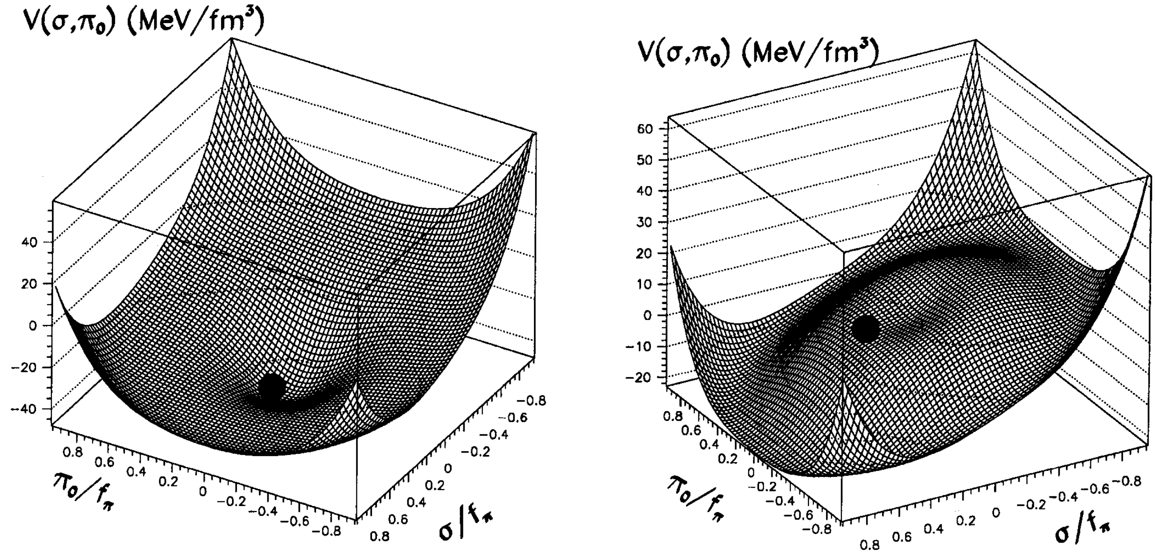

FIG. 1. Finite temperature effective potential (i.e., the grand canonical potential) within the linear σ model at $T = 130$ MeV (left) and $T = 100$ MeV (right). The black dot depicts the average chiral field at the corresponding time.

by more than a factor of 100, corresponding to about 18 π_0's and 3 σ's for an initial radius of 4 fm; see Fig. 3. For the simple quench scenario, where the fields evolve in the zero temperature potential U and without coupling to any heat bath, we get similar results: 21 π_0's and 4 σ's. We computed these numbers as described in [5,24].

In summary, we discussed a novel mechanism for DCC formation in a first-order chiral phase transition. We study a bubble of disoriented chiral field that decouples from the rest of the system before reaching the phase boundary. The bubble supercools, and as the effective potential approaches the $T = 0$ form the local minimum at $\Phi = 0$ disappears. This leads to the quenched initial condition. The subsequent alignment in the vacuum direction leads to very strong amplification of low-momentum modes of the pion field.

If the chiral phase transition is first order, as is particularly likely the case for finite baryon density [6], and if the system breaks up into smaller droplets or bubbles, rapidity fluctuations (e.g., of baryon number) can occur, as proposed in [11–13,16]. If a DCC is formed, in coincidence "pion spikes" of given bubble isospin could appear in the same p_T and rapidity range.

We thank the Yale Relativistic Heavy Ion Group for kind hospitality and support from Grant No. DE-FG02-91ER-40609. Also, we gratefully acknowledge fruitful discussions with A. D. Jackson, I. Mishustin, D. H. Rischke, J. Schaffner, and U. A. Wiedemann. We

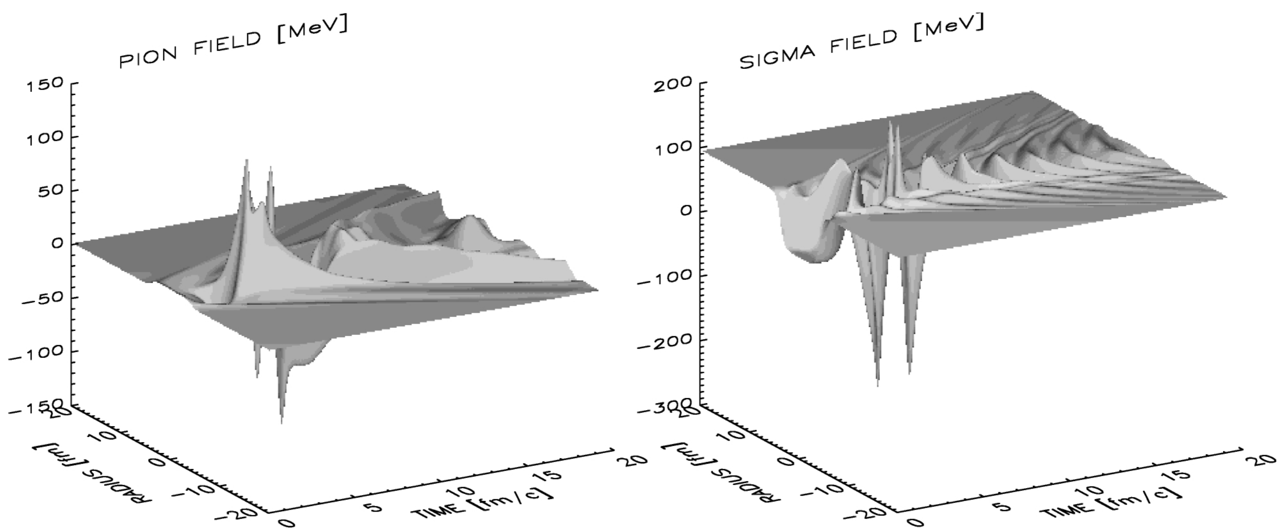

FIG. 2. The π_0 and σ field strengths within the forward light cone for $g = 5.5$ and initial condition $\Phi = (0.1, 0.07, 0, 0)f_\pi$, $\partial\Phi/\partial t = 0$. The initial expansion scalar was chosen as $\partial u \approx 1/(2\ \mathrm{fm}/c)$.

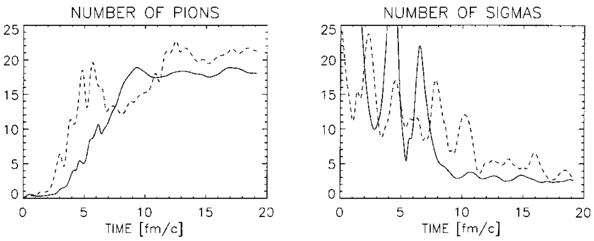

FIG. 3. Number of σ and π_0 mesons produced from the decay of the classical chiral field. The solid lines correspond to the self-consistent calculation with first-order phase transition (initial conditions as in Fig. 2); dotted lines are for the quenched initial conditions $\Phi = (0.45, 0.07, 0, 0) f_\pi$, $\partial\Phi/\partial t = 0$, $V \equiv U$ (this corresponds to roughly the same potential energy at the time when "rolldown" begins).

thank D. H. Rischke for reading the manuscript prior to publication.

[1] A. A. Anselm, Phys. Lett. B **217**, 169 (1989); A. A. Anselm and M. G. Ryskin, Phys. Lett. B **266**, 482 (1991); J. D. Bjorken, Int. J. Mod. Phys. A **7**, 4189 (1992).

[2] J. Blaizot and A. Krzywicki, Phys. Rev. D **46**, 246 (1992).

[3] K. Rajagopal and F. Wilczek, Nucl. Phys. **B399**, 395 (1993); Nucl. Phys. **B404**, 577 (1993); S. Gavin, A. Gocksch, and R. D. Pisarski, Phys. Rev. Lett. **72**, 2143 (1994); S. Gavin and B. Müller, Phys. Lett. B **329**, 486 (1994); M. Asakawa, Z. Huang, and X. Wang, Phys. Rev. Lett. **74**, 3126 (1995); D. Boyanovsky, H. J. de Vega, and R. Holman, Phys. Rev. D **51**, 734 (1995).

[4] J. Randrup, Phys. Rev. Lett. **77**, 1226 (1996); J. Schaffner-Bielich and J. Randrup, Phys. Rev. C **59**, 3329 (1999).

[5] I. N. Mishustin and O. Scavenius, Phys. Lett. B **396**, 33 (1997); I. N. Mishustin, J. A. Pedersen, and O. Scavenius, Heavy Ion Phys. **5**, 377 (1997); I. N. Mishustin and O. Scavenius, Phys. Rev. Lett. **83**, 3134 (1999).

[6] M. A. Halasz, A. D. Jackson, R. E. Shrock, M. A. Stephanov, and J. J. Verbaarschot, Phys. Rev. D **58**, 096007 (1998); M. A. Stephanov, Nucl. Phys. **A642**, 90 (1998); M. Stephanov, K. Rajagopal, and E. Shuryak, Phys. Rev. Lett. **81**, 4816 (1998); J. Berges and K. Rajagopal, Nucl. Phys. **B538**, 215 (1999); T. M. Schwarz, S. P. Klevansky, and G. Papp, nucl-th/9903048.

[7] Y. Iwasaki, K. Kanaya, S. Kaya, S. Sakai, and T. Yoshie, Phys. Rev. D **54**, 7010 (1996).

[8] J. Sollfrank, J. Phys. G **23**, 1903 (1997); nucl-th/9707020; J. Cleymans and K. Redlich, Phys. Rev. Lett. **81**, 5284 (1998); L. V. Bravina *et al.*, J. Phys. G **25**, 351 (1999).

[9] A. Dumitru, D. H. Rischke, T. Schönfeld, L. Winckelmann, H. Stöcker, and W. Greiner, Phys. Rev. Lett. **70**, 2860 (1993).

[10] C. Spieles, L. Gerland, H. Stöcker, C. Greiner, C. Kuhn, and J. P. Coffin, Phys. Rev. Lett. **76**, 1776 (1996).

[11] H. Heiselberg and A. D. Jackson, nucl-th/9809013.

[12] I. N. Mishustin, Phys. Rev. Lett. **82**, 4779 (1999).

[13] S. Gavin, nucl-th/9908070.

[14] L. P. Csernai, I. N. Mishustin, and A. Mocsy, Heavy Ion Phys. **3**, 151 (1996); G. E. Brown, M. Buballa, and M. Rho, Nucl. Phys. **A609**, 519 (1996); P. Papazoglou, J. Schaffner, S. Schramm, D. Zschiesche, H. Stöcker, and W. Greiner, Phys. Rev. C **55**, 1499 (1997).

[15] A. Dumitru, hep-ph/9905217.

[16] L. P. Csernai and I. N. Mishustin, Phys. Rev. Lett. **74**, 5005 (1995).

[17] J. I. Kapusta and A. P. Vischer, Z. Phys. C **75**, 507 (1997).

[18] T. S. Biro and C. Greiner, Phys. Rev. Lett. **79**, 3138 (1997); C. Greiner and B. Müller, Phys. Rev. D **55**, 1026 (1997); F. Cooper, S. Habib, Y. Kluger, and E. Mottola, Phys. Rev. D **55**, 6471 (1997); D. H. Rischke, Phys. Rev. C **58**, 2331 (1998).

[19] L. P. Csernai and J. I. Kapusta, Phys. Rev. Lett. **69**, 737 (1992); L. P. Csernai, J. I. Kapusta, G. Kluge, and E. E. Zabrodin, Z. Phys. C **58**, 453 (1993).

[20] E. E. Zabrodin, L. Bravina, H. Stöcker, and W. Greiner, Phys. Rev. C **59**, 894 (1999).

[21] S. Mrowczynski and B. Müller, Phys. Lett. B **363**, 1 (1995); H. Hiro-Oka and H. Minakata, hep-ph/9906301.

[22] M. Bleicher *et al.*, Nucl. Phys. **A638**, 391 (1998).

[23] D. H. Rischke, S. Bernard, and J. A. Maruhn, Nucl. Phys. **A595**, 1346 (1995).

[24] A. Abada and M. Birse, Phys. Rev. D **55**, 6887 (1997); G. Amelino-Camelia, J. D. Bjorken, and S. E. Larsson, Phys. Rev. D **56**, 6942 (1997).