Pion Cloud and the Q^2 Dependence of $\gamma^* N \leftrightarrow \Delta$ Transition Form Factors

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Recent experiments indicate that the behavior exhibited by the ratios E_{1^+}/M_{1^+} and S_{1^+}/M_{1^+} of the $\gamma^* N \leftrightarrow \Delta$ transition remains small and *negative* for $Q^2 \leq 4.0$ GeV^2. It implies that perturbative QCD is still not applicable at these momentum transfers. We show that these data can be explained in a dynamical model for electromagnetic production of pions, together with a simple scaling assumption for the bare $\gamma^* N\Delta$ form factors. Within our model we find that the bare Δ is almost spherical and the electric $E2$ and Coulomb $C2$ quadrupole excitations of the physical Δ are nearly saturated by the pion cloud contribution in $Q^2 \leq 4.0$ GeV^2.

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It has been well recognized that the study of the excitations of the hadrons can shed light on the nonperturbative aspects of QCD. One case which has recently been under intensive study is the electromagnetic excitation of the $\Delta(1232)$ resonance. At low four-momentum transfer squared Q^2, the interest is motivated by the possibility of observing a D state in the Δ [1–3]. The existence of a D state in the Δ has the consequence that the Δ is deformed and the photon can excite a nucleon through electric $E2$ and Coulomb $C2$ quadrupole transitions. In a symmetric SU(6) quark model, the electromagnetic excitation of the Δ could proceed only via $M1$ transition. In pion electroproduction, $E2$ and $C2$ excitations would give rise to nonvanishing $E_{1+}^{(3/2)}$ and $S_{1+}^{(3/2)}$ multipole amplitudes. Recent experiments give nonvanishing ratio $R_{EM} = E_{1+}^{(3/2)}/M_{1+}^{(3/2)} \sim -0.03$ [1] at $Q^2 = 0$ which has been widely taken as an indication of the Delta deformation.

At sufficiently large Q^2, the perturbative QCD (pQCD) is expected to work. It predicts that only helicity-conserving amplitudes contribute at high Q^2 [4], leading to $R_{EM} = E_{1+}^{(3/2)}/M_{1+}^{(3/2)} \rightarrow 1$ and $R_{SM} = S_{1+}^{(3/2)}/M_{1+}^{(3/2)} \rightarrow$ const. This behavior in the perturbative domain is very different from that in the nonperturbative one. It is an intriguing question to find the region of Q^2 which signals the onset of the pQCD.

In a recent measurement [5], the electromagnetic excitation of the Δ was studied at $Q^2 = 2.8$ and 4.0 GeV^2 via the reaction $p(e, e'p)\pi^0$. The extracted ratios R_{EM} and R_{SM} remain small and *negative*. This disagrees with the previous analysis [6] of the earlier DESY data [7] which gave small but *positive* R_{EM} and R_{SM} at $Q^2 = 3.2$ GeV^2, though both analyses indicate that pQCD is still not applicable in this region of Q^2. In this Letter, we want to show that the recent data of Ref. [5] can be understood from the dominance of the pion cloud contribution at low Q^2, in both $E_{1+}^{(3/2)}$ and $S_{1+}^{(3/2)}$, as predicted by a dynamical model [8,9] for electromagnetic production of pion, together with a simple scaling assumption for the bare $\gamma^* N\Delta$ form factors.

The main feature of the dynamical approach to the pion photo- and electroproduction [8,9] is that the unitarity is built in by explicitly including the final state πN interaction in the theory, namely, t matrix is expressed as

$$t_{\gamma\pi}(E) = v_{\gamma\pi} + v_{\gamma\pi} g_0(E) t_{\pi N}(E), \qquad (1)$$

where $v_{\gamma\pi}$ is the transition potential operator for $\gamma^* N \rightarrow \pi N$, and $t_{\pi N}$ and g_0 denote the πN t matrix and free propagator, respectively, with E the total energy in the CM frame.

In the (3,3) channel where Δ excitation plays an important role, the transition potential $v_{\gamma\pi}$ consists of two terms

$$v_{\gamma\pi}(E) = v_{\gamma\pi}^B + v_{\gamma\pi}^\Delta(E), \qquad (2)$$

where $v_{\gamma\pi}^B$ is the background transition potential which includes Born terms and vector mesons exchange contributions, as described in Ref. [10]. The second term of Eq. (2) corresponds to the contribution of bare Δ, namely, $\gamma^* N \rightarrow \Delta \rightarrow \pi N$.

In accordance with Ref. [11], we decompose Eq. (1) in the following way, as shown in Fig. 1,

$$t_{\gamma\pi} = t_{\gamma\pi}^B + t_{\gamma\pi}^\Delta, \qquad (3)$$

where

$$t_{\gamma\pi}^B(E) = v_{\gamma\pi}^B + v_{\gamma\pi}^B g_0(E) t_{\pi N}(E), \qquad (4)$$

$$t_{\gamma\pi}^\Delta(E) = v_{\gamma\pi}^\Delta + v_{\gamma\pi}^\Delta g_0(E) t_{\pi N}(E). \qquad (5)$$

The advantage of such a decomposition (3) is that all the

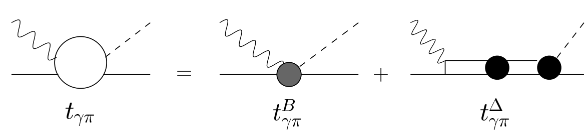

FIG. 1. Graphical representation of the pion electroproduction t matrix.

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processes which start with the electromagnetic excitation of the bare Δ are summed up in $t^{\Delta}_{\gamma\pi}$. The solid blob for both the intermediate Δ states and the $\pi N\Delta$ vertex means that they are dressed [2,9]. Equation (3) provides us with a prescription to extract information concerning bare Δ excitation.

For physical multipole amplitude in channel α, multipole decomposition of $t_{\gamma\pi}$ gives [8]

$$t^{(\alpha)}_{\gamma\pi}(q_E, k_E; E + i\varepsilon) = e^{i\delta_\alpha}\cos\delta_\alpha\left[v^{(\alpha)}_{\gamma\pi}(q_E, k_E) + P\int_0^\infty dq' \frac{q'^2 R^{(\alpha)}_{\pi N}(q_E, q'; E) v^{(\alpha)}_{\gamma\pi}(q', k_E)}{E - E_{\pi N}(q')}\right], \tag{6}$$

where δ_α, $R^{(\alpha)}_{\pi N}$, $E_{\pi N}(q)$, and P denote the πN phase shift, reaction matrix in channel α, total CM energy of momentum q, and principal value integral, respectively; $k_E = |\mathbf{k}|$ is the photon momentum, and q_E the pion on-shell momentum. Equation (6) manifestly satisfies the Watson theorem and shows that $\gamma\pi$ multipoles depend on the half-off-shell behavior of the pion-nucleon interaction. To make principal value integration associated with $v^B_{\gamma\pi}$ convergent, we introduce an off-shell dipole form factor, which characterizes the finite range aspects of the potential. The cutoff parameter Λ is determined by requiring that it provides the best fit to the $M^{(3/2)}_{1+}$, which turns out to be $\Lambda = 440$ MeV.

Note that due to the off-shell rescattering effects in the principal value integral of Eq. (6), gauge invariance is violated. In the present model for the pion electroproduction we restore gauge invariance by the following substitution:

$$J^B_\mu \to J^B_\mu - k_\mu \frac{k \cdot J^B}{k^2}, \tag{7}$$

where J^B_μ is the electromagnetic current corresponding to the background contribution $v^B_{\gamma\pi}$.

We evaluate $t^B_{\gamma\pi}$ with $t_{\pi N}$ matrix elements obtained in a meson-exchange model [12]. From the structure of $t^{\Delta}_{\gamma\pi}$ as depicted in Fig. 1, we can describe its energy dependence of the corresponding multipole amplitudes A^{Δ} with a Breit-Wigner form, as was done in the isobar model of Ref. [10],

$$A^{\Delta}(W, Q^2) = \bar{\mathcal{A}}^{\Delta}(Q^2)\frac{f_{\gamma\Delta}\Gamma_\Delta M_\Delta f_{\pi\Delta}}{M^2_\Delta - W^2 - iM_\Delta\Gamma_\Delta}e^{i\phi}, \tag{8}$$

where $f_{\pi\Delta}(W)$ is the usual Breit-Wigner factor describing the decay of the Δ resonance with total width $\Gamma_\Delta(W)$ and physical mass $M_\Delta = 1232$ MeV. The W dependence of the $\gamma N\Delta$ vertex is given in $f_{\gamma\Delta}(W)$ with normalization $f_{\gamma\Delta}(M_\Delta) = 1$. The expressions for $f_{\gamma\Delta}$, $f_{\pi\Delta}$, and Γ_Δ are taken from Ref. [10]. The phase $\phi(W, Q^2)$ in Eq. (8) is to adjust the phase of A^{Δ} to be equal to the corresponding pion-nucleon scattering phase δ_{33}. At the resonance $\phi(M_\Delta, Q^2) = 0$, and it does not affect the Q^2 dependence of the electromagnetic vertex.

The main parameters in the bare $\gamma^* N\Delta$ vertex are the $\bar{\mathcal{A}}^{\Delta}$'s in Eq. (8). For the magnetic dipole $\bar{\mathcal{M}}^{\Delta}$ and electric quadrupole $\bar{\mathcal{E}}^{\Delta}$ transitions they are related to the conventional electromagnetic helicity amplitudes $A^{\Delta}_{1/2}$ and $A^{\Delta}_{3/2}$ by

$$\bar{\mathcal{M}}^{\Delta} = -\frac{1}{2}(A^{\Delta}_{1/2} + \sqrt{3}A^{\Delta}_{3/2}),$$
$$\bar{\mathcal{E}}^{\Delta} = \frac{1}{2}\left(-A^{\Delta}_{1/2} + \frac{1}{\sqrt{3}}A^{\Delta}_{3/2}\right). \tag{9}$$

For the real photons, they are equal to the standard $M1$ and $E2$ amplitudes of the $\gamma N \to \Delta$ transition as defined by the Particle Data Groups.

In the present work, we parametrize the Q^2 dependence of the dominant $\bar{\mathcal{M}}^{\Delta}$ amplitude by

$$\bar{\mathcal{M}}^{\Delta}(Q^2) = \bar{\mathcal{M}}(0)\frac{|\mathbf{k}|}{k_\Delta}(1 + \beta Q^2)e^{-\gamma Q^2}G_D(Q^2), \tag{10}$$

where G_D is the nucleon dipole form factor. The parameters β and γ will be determined later. For the small $\bar{\mathcal{E}}^{\Delta}$ and $\bar{S}^{\Delta}$ amplitudes, following Refs. [10,13], we assume that they have the same Q^2 dependence as $\bar{\mathcal{M}}^{\Delta}$. This is motivated by the scaling law which has been observed for the nucleon form factors. It is plausible if a bare Δ is pictured as simply flipping one of the quark spins in the nucleon. It is also known that $\bar{\mathcal{E}}^{\Delta}(0) = \bar{S}^{\Delta}(0)$ [13].

To proceed, we first consider $M^{(3/2)}_{1+}$ and $E^{(3/2)}_{1+}$ multipoles at $Q^2 = 0$. By combining the contributions of $t^B_{\gamma\pi}$ and $t^{\Delta}_{\gamma\pi}$ and using the bare amplitudes $\bar{\mathcal{M}}^{\Delta}(0)$ and $\bar{\mathcal{E}}^{\Delta}(0)$ of Eq. (8) as free parameters, results of our best fit to the multipoles obtained in the recent analyses of Mainz [14] and VPI group [15] are shown in Fig. 2 by solid curves. The dashed curves denote the contribution from $t^B_{\gamma\pi}$ only. The dotted curves represented the K-matrix approximation to $t^B_{\gamma\pi}$, namely, without the principal value integral term included.

The numerical values obtained for $\bar{\mathcal{M}}^{\Delta}$ and $\bar{\mathcal{E}}^{\Delta}$ and the helicity amplitudes, at $Q^2 = 0$, are given in Table I along with the corresponding "dressed" values. At the resonance position $t^B_{\gamma\pi}$ vanishes within K-matrix approximation and only the principal value integral term survives. The latter corresponds to the contribution where Δ is excited by the pion produced via $v^B_{\gamma\pi}$. Consequently, the addition of this contribution to $t^{\Delta}_{\gamma\pi}$ can be considered as a dressing of the $\gamma N\Delta$ vertex. The dressed helicity amplitudes obtained in this way are in very good agreement with the results of Ref. [10] and with PDG values.

One notices that the bare values for the helicity amplitudes determined above, which amount to only about 60% of the corresponding dressed values, are close to the predictions of the constituent quark model (CQM), as was pointed out by Sato and Lee [16]. The large reduction of

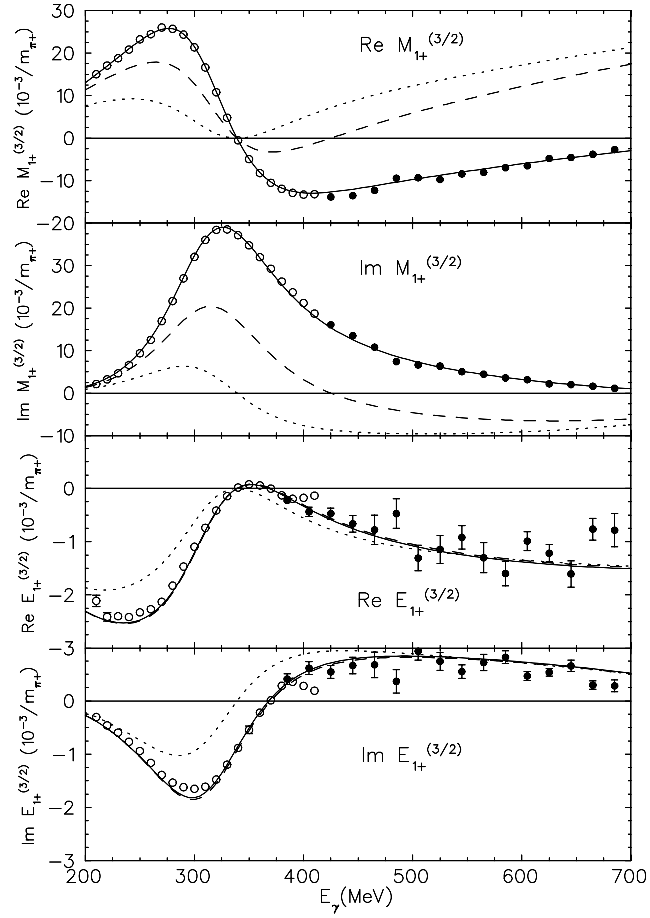

FIG. 2. Real and imaginary parts of the $M_{1+}^{(3/2)}$ and $E_{1+}^{(3/2)}$ multipoles. Dotted and dashed curves are the results for the $t_{\gamma\pi}^B$ obtained without and with principal value integral contribution in Eq. (6), respectively. Solid curves are the full results with bare Δ excitation. For the E_{1+} dashed and solid curves are practically the same due to the small value of the bare $\bar{\mathcal{E}}^\Delta$. The open and full circles are the results from the Mainz dispersion relation analysis 14 and from the VPI analysis 15.

the helicity amplitudes from the dressed to the bare ones result from the fact that the principal value integral part of Eq. (6), which represents the effects of the off-shell pion rescattering, contributes approximately half of the M_{1+} as indicated by the dashed curves in Fig. 2.

For the standard Sach-type form factor $G_M^\Delta(0)$ [17] our bare and dressed values are 1.65 ± 0.02 and 3.06 ± 0.02, respectively. On the other hand, results of CQM calculations lie in the range 1.4–2.2 [18]. From this result we conclude that pion rescattering is the main mechanism responsible for the longstanding discrepancy in the description of the magnetic $\gamma^* N \to \Delta$ transition within CQM. For $E_{1+}^{(3/2)}$, the dominance of background and pion rescattering contributions further leads to a very small bare value for electric transition.

TABLE I. Comparison of the "bare" and "dressed" values for the amplitudes $\bar{\mathcal{A}}^\Delta$, $A_{1/2}^\Delta$, and $A_{3/2}^\Delta$ (in 10^{-3} GeV$^{-1/2}$).

Amplitudes	"Bare"	"Dressed"	PDG
$\bar{\mathcal{M}}^\Delta$	158 ± 2	289 ± 2	293 ± 8
$\bar{\mathcal{E}}^\Delta$	0.4 ± 0.3	-7 ± 0.4	-4.5 ± 4.2
$A_{1/2}^\Delta$	-80 ± 2	-134 ± 2	-140 ± 5
$A_{3/2}^\Delta$	-136 ± 3	-256 ± 2	-258 ± 6

We now turn to the Q^2 evolution of the multipoles in the (3,3) channel. With the parametrization of (10), we fit the recent experimental data [5] as well as old data quoted in Ref. [10] on the Q^2 dependence of $M_{1+}^{(3/2)}$ multipole or equivalently, the G_M^* form factor defined as [10]

$$M_{1+}^{3/2}(M_\Delta, Q^2) = \frac{|\mathbf{k}|}{m_N}\sqrt{\frac{3\alpha}{8\Gamma_{\rm exp} q_\Delta}}\, G_M^*(Q^2)\,, \qquad (11)$$

with $\alpha = 1/137$, $\Gamma_{\rm exp} = 115$ MeV, q_Δ is the pion momentum at the resonance energy, and m_N is the nucleon mass. Our result is shown in Fig. 3. Note that for the G_M^* form factor we use the "Ash" definition [19]. It differs from the definition used in Refs. [5,17] by a factor $[1 + Q^2/(m_N + M_\Delta)^2]^{1/2}$. At $Q^2 = 0$ we have $G_M^*(0) = G_M^\Delta(0)$. The obtained values for the β and γ parameters of Eq. (10) are $\beta = 0.44$ GeV^{-2} and $\gamma = 0.38$ GeV^{-2}. Here the dashed curves correspond to a contribution from the bare Δ, i.e., $t_{\gamma\pi}^\Delta$ of Eq. (5).

With the scaling assumption, i.e., both $\bar{\mathcal{E}}^\Delta$ and $\bar{S}^\Delta$ have the same Q^2 dependence as $\bar{\mathcal{M}}^\Delta$ as given in Eq. (10) the Q^2 dependence for the ratios $R_{EM} = E_{1+}^{(3/2)}/M_{1+}^{(3/2)}$ and $R_{SM} = S_{1+}^{(3/2)}/M_{1+}^{(3/2)}$ can be evaluated. The results are shown in Fig. 4. It is seen that they are in good agreement with the results of the model independent analysis of Ref. [5] up to Q^2 as high as 4.0 GeV^2. Note that since the bare values for the electric and Coulomb excitations are small, the

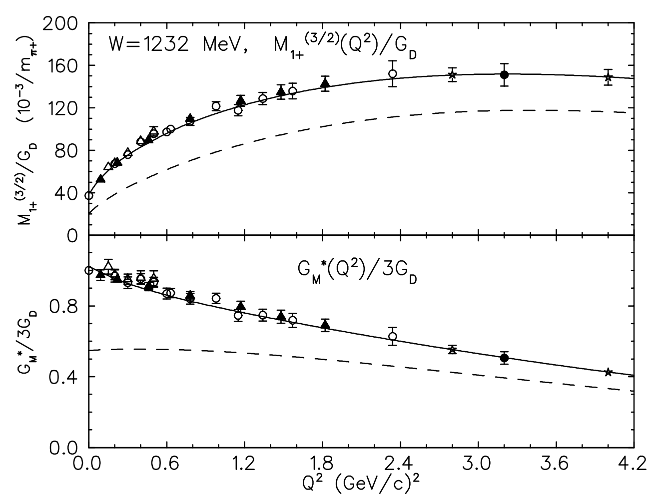

FIG. 3. The Q^2 dependence of Im$M_{1+}^{(3/2)}$ at $W = 1232$ MeV and corresponding G_M^* form factor. The full and dashed curves are the results for the "dressed" and "bare" $\gamma N\Delta$ vertexes, respectively. Experimental data from Refs. [20]. Data at $Q^2 = 2.8$ and 4.0 $(\mathrm{GeV}/c)^2$ from Ref. [5].

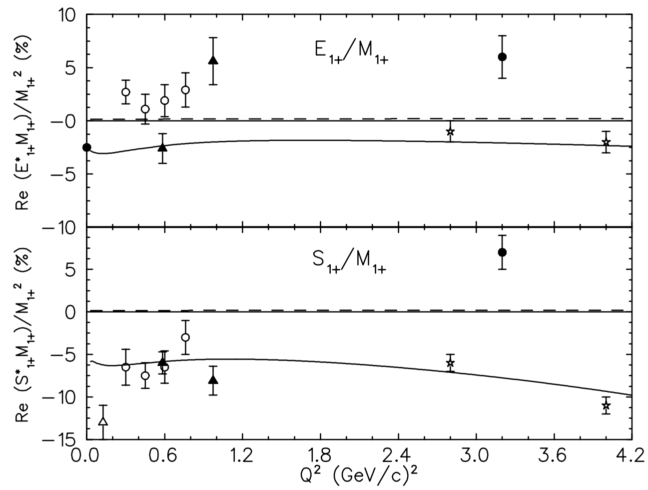

FIG. 4. The Q^2 dependence of the ratios $E_{1+}^{(3/2)}/M_{1+}^{(3/2)}$ and $S_{1+}^{(3/2)}/M_{1+}^{(3/2)}$ at $W = 1232$ MeV. Notations for the curves are the same as in Fig. 3. Experimental data at $Q^2 = 0$ and 3.2 (GeV$/c)^2$ from Refs. [1] and [6], respectively. Data at $Q^2 = 2.8$ and 4.0 (GeV$/c)^2$ from Ref. [5] and others data from Ref. [21].

absolute values and shape of these ratios are determined, to a large extent, by the pion rescattering contribution. The bare Δ excitation contributes mostly to the $M_{1+}^{(3/2)}$ multipole.

In summary, we calculate the Q^2 dependence of the ratios E_{1+}/M_{1+} and S_{1+}/M_{1+} in the $\gamma^* N \to \Delta$ transition, with the use of a dynamical model for electromagnetic production of pions and a simple scaling assumption for the bare $\gamma^* N \to \Delta$ transition form factors. We find that both ratios E_{1+}/M_{1+} and S_{1+}/M_{1+} remain small and *negative* for $Q^2 \leq 4.0\ \mathrm{GeV}^2$. Our results agree well with the recent measurement of Ref. [5], but deviate strongly from the predictions of pQCD. Our results indicate that the bare Δ is almost spherical and hence very difficult to be directly excited via electric $E2$ and Coulomb $C2$ quadrupole excitations. The experimentally observed $E_{1+}^{(3/2)}$ and $S_{1+}^{(3/2)}$ multipoles are, to a very large extent, saturated by the contribution from pion cloud, i.e., pion rescattering effects. It remains an intriguing question, both theoretically and experimentally, to find the region of Q^2 which will signal the onset of pQCD.

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