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Observation of Quantum Accelerator Modes

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We have realized a quantum accelerator mode using a system consisting of ultracold cesium atoms falling through a pulsed standing wave of off resonant light. We present a picture of this system based on diffraction and show that the effect arises from the application of blazed matter wave diffraction gratings. The implications of our results for quantum chaos and the prospect of constructing a large angular separation matter wave beam splitter are discussed.

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The concept of accelerator modes has been known for many years in the fields of chaos [1] and plasma physics [2]. They occur when a particle is periodically kicked in such a way that with each kick the maximum momentum is transferred. As in the case of a surfer on an ocean wave, the momentum which is imparted depends on the particle's position and velocity in the potential. How does this translate into quantum mechanics? In this Letter we utilize the quantum delta kicked rotor realized with atoms in a regularly pulsed sinusoidal potential [3] to demonstrate a quantum accelerator mode. In this situation the accelerator mode can be understood as the consecutive application of blazed diffraction gratings for matter waves. As in light optics where an enhancement of diffraction occurs for orders which match the geometrical paths, so the matter wave diffraction is enhanced in the directions of the classical paths.

For our experiments we use laser cooled cesium atoms in a far off resonant standing light wave aligned parallel to the Earth's gravitational field. This produces a sinusoidal potential [4] with a period of half the optical wavelength, which is switched on periodically for short times to realize deltalike kicks. After a number of kicks the momentum distribution of the atomic ensemble is observed.

The signature of an accelerator mode is a linear increase in momentum with the number of kicks. The experimental momentum distributions for different numbers of kicks are shown in Fig. 1. Clearly the atoms in the accelerator mode move out linearly in momentum space. It is important to note that the probability of staying in this mode is very high and from the experiment is found to be 99.3 ± 0.2% per light pulse. The data show that the atoms were accelerated to a maximum velocity corresponding to ~100 photon recoils. Furthermore, we expect

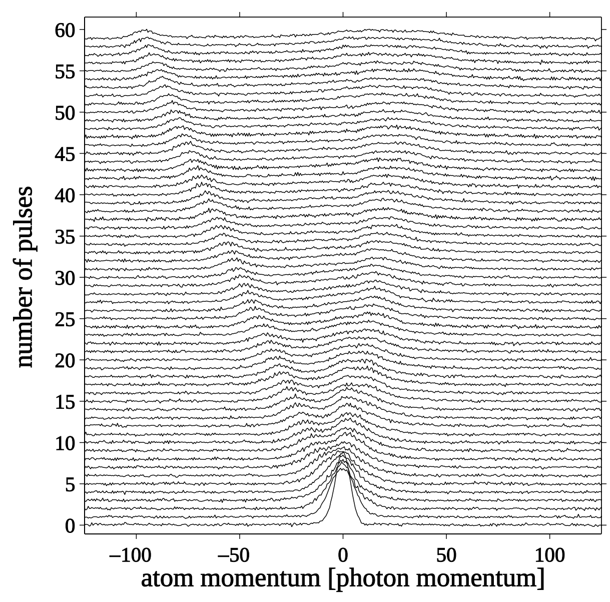

FIG. 1. The experimental momentum distributions for a 60 μs pulse separation as a function of the number of light pulses. About 20% of the initial momentum distribution, which has a full width at half maximum of nine photon momenta, is in an accelerator mode and gains about two photon kicks per pulse. The probability for an atom to stay in the accelerator mode after a light pulse is 99.3 ± 0.2%.

that the momentum is imparted coherently since this is the result of diffraction.

The momentum distributions measured after 30 kicks for different times between the kicks are shown in Fig. 2. Part of the initial distribution is accelerated for certain kick separations. Additionally, the direction of the acceleration reverses as the time between the pulses changes about a characteristic value.

The narrowing of the momentum distribution at 67 μs has been observed previously in the context of quantum chaos experiments with the delta kicked rotor [5]. It was called a quantum resonance and happens at half multiples of a specific time which we will call the Talbot time. In these experiments no distinct accelerator mode was observed because the standing light wave was oriented perpendicular to the direction of the Earth's gravitational acceleration. This is different from our experiment in which the light wave is parallel to the direction of Earth's acceleration. The matter wave interference, in the near field, localizes the atom distribution in the accelerator mode to less than the periodicity of the kicking potential. Gravity pushes this localized atom distribution for each light pulse to a position where the classical momentum kick from the periodic potential is maximal.

We now develop a diffractive picture to explain this phenomenon which we will use later in our numerical analysis. The periodic potential is of the form

$$U(x,t) = \frac{U_{\max}}{2}\,[1 + \cos(Gx)] \sum_{N_p} \delta(t - N_p T) \qquad (1)$$

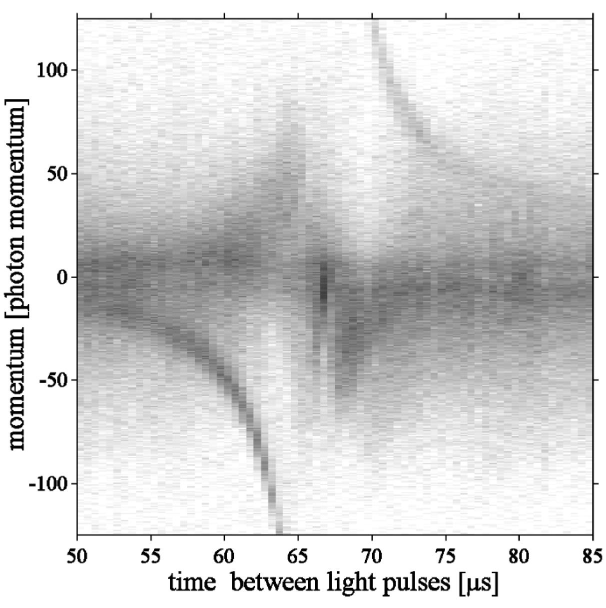

FIG. 2. The measured momentum distribution after 30 kicks for different times between the pulses. At 67 μs the momentum distribution becomes small, as has been observed in the context of the delta kicked rotor [5]. Below and above this time, a subensemble of the initial atom distribution matches the condition for an accelerator mode.

with $G = 2k$ where k is the light wave vector. The delta function expresses the periodic switching of the light, with T the time between pulses and N_p giving the pulse number. Wave mechanics describes the interaction of a de Broglie wave with a periodic potential as diffraction from a phase grating because the potential spatially changes the phase of an atomic wave as $\phi(x) = -U(x)\Delta t/\hbar$, where Δt is the interaction time. Since the pulse length is assumed to be very short the movement of the particle during the interaction is negligible, a situation known as the Raman-Nath regime. Thus the standing light wave acts as a thin phase diffraction grating.

The diffraction amplitude for the nth order is given by $i^n J_n(\phi_d)$, where J_n is the nth order Bessel function of the first kind and $\phi_d = U_{\max}\Delta t/(2\hbar)$. For the free propagation one is in the near field regime, where all the diffracted orders interfere. A characteristic length scale for the near field is the Talbot length $L_{\mathrm{Talbot}} = \frac{2d^2}{\lambda_{\mathrm{dB}}}$, where d is the grating period, equal to half the wavelength of the light, and λ_{dB} is the wavelength of the diffracted de Broglie wave. At this distance from a diffractive object one can observe a one to one image of the wave function which exists directly behind the diffracting object. This arises from the fact that the individual diffraction orders accumulate a phase difference of multiples of 2π. Similarly, π out of phase images occur at odd multiples of half the Talbot length. These reimages are known as Fourier images and are discussed in detail in [6]. The corresponding Talbot time is given by $T_{\mathrm{Talbot}} = L_{\mathrm{Talbot}}/v$, where v is the atomic velocity. Thus,

$$T_{\mathrm{Talbot}} = \pi\,\frac{4m}{\hbar G^2}\,, \qquad (2)$$

which for cesium is 133 μs.

For standing light waves switched on at multiples of this time, the wave function at the time of each grating is the same as the wave function directly after the previous grating. Thus the action of the periodically switched standing light waves is the same as one "thick" standing light wave. No momentum spread with increasing number of pulses is expected since one enters the regime of Bragg scattering where only certain initial momenta are diffracted. For odd multiples of half the Talbot time, where the reimage is π out of phase, each consecutive phase grating compensates the phase modulation and thus leads to no diffraction at all. These two cases are known as quantum resonances and were first observed by Moore *et al.* [5] for the delta kicked rotor.

In order to understand the experimental observations one has to look more closely at the atomic wave field near the half Talbot time. For times longer or shorter than this an intensity modulated wave field is formed due to the specific phase evolution of the diffracted orders. The positions of the maxima depend on the initial velocity v_i in the direction of the grating through the relative phases of the different diffraction orders n established with the first light pulse (diffraction grating). These phases are given by $\phi_n = \frac{\hbar G^2}{2m} T n^2 + v_i G T n$. The first phase term comes from the recoil frequency shift while the second is a result of the Doppler shift due to the initial velocity v_i.

The gravitational acceleration leads to an additional shift of the phase which can be understood as an extra Doppler shift due to the change in velocity of the atom. It is given by $\phi_n^{\mathrm{grav}} = (g N_p T) G T n$. If the time between the pulses is chosen to be 60 μs as in the experiment shown in Fig. 1, an initial velocity corresponding to one photon recoil will have its near field intensity maxima at the positions of the steepest gradient of the potential. The stability of the accelerator mode implies that between the light pulses the phase differences accumulated between the orders which constitute the accelerator mode are multiples of 2π. Without an acceleration of the atoms relative to the standing light wave (such as gravity provides) it would be impossible to fulfill this condition for a large number of pulses.

The near field of the accelerator mode at the time of a given light pulse is shown by the solid lines in Fig. 3. The position of the periodic potential is given by the broken lines. Clearly for separations between the light pulses of 60 and 76 μs the near field is localized on opposite sides of the periodic "kicking" potential. The localization arises because the accelerator mode consists of several different diffraction orders, and the position is determined by the free evolution. The slightly different efficiencies of the accelerator mode above and below the resonance time are a result of the different amounts of localization of the atomic wave field.

From light optics one knows that if a geometrical path overlaps with a diffraction order, diffraction enhancement is observed. Similarly in wave mechanics a diffraction order is enhanced if it overlaps with the classical path. Because of the properties of the diffraction at a phase grating [7] the order diffracted with the highest probability is given roughly by $n_{\max} \approx (\phi_d - 1)$, corresponding to a most likely momentum of $p_{\mathrm{quantum}} = n_{\max} \hbar G$. The wave function is localized at the position of steepest potential gradient so that the classical change in momentum is $p_{\mathrm{class}} = \phi_d \hbar G$. Since a highly probable diffraction order closely coincides with the classical path for all ϕ_d, enhancement should work for every potential height. Since the wave function is not perfectly localized at the steepest gradient the classical force leads to a range of classical paths and thus the enhancement is shared over many orders. As the phase modulation ϕ_d increases, the change in the potential gradient over the localization of the wave function becomes greater. It was confirmed numerically and experimentally that an efficient accelerator mode is present in the range $0.3\pi < \phi_d < 1.5\pi$.

A numerical simulation with this picture at hand is straightforward. The interaction can be described by a unitary transformation $U_{mn}^{\mathrm{int}} = i^{n-m} J_{n-m}(\phi_d) |m\rangle\langle n|$,

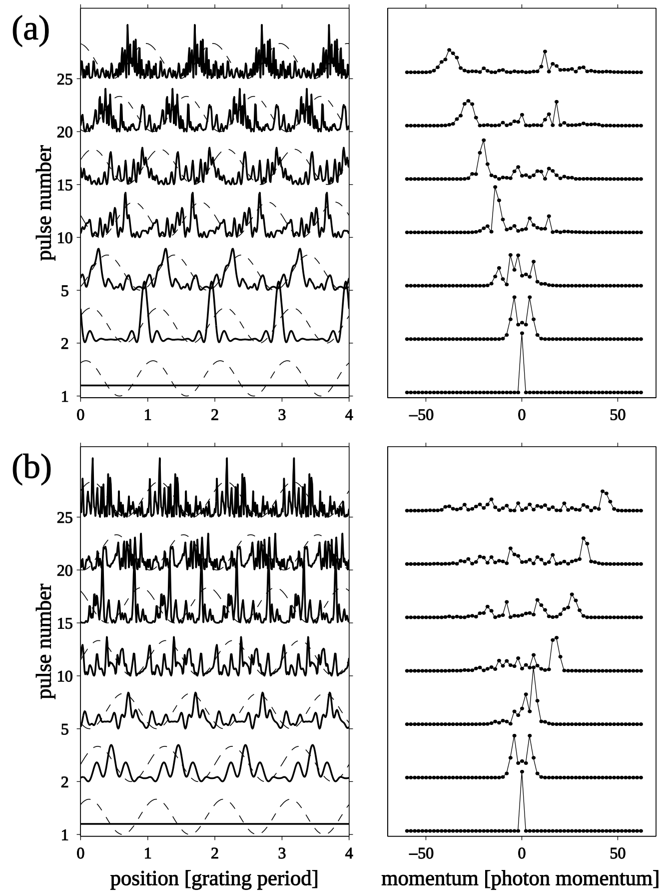

FIG. 3. Near field and momentum distributions of atoms with a starting momentum of one photon recoil for (a) 60 μs and (b) 76 μs pulse separations for different pulse numbers and with $\phi_d = \pi$. The standing light wave potential is indicated by the broken sinusoidal line. In (a) the localization of the wave field is at the position where the potential gradient has its maximum positive value. Thus the classical path overlaps with a negative diffraction order. This is equivalent to an optical blazed grating. In (b) the localization is on the opposite side of the potential so that positive diffraction orders are enhanced.

where m and n are the final and initial diffraction orders, respectively; the free propagation can be described by $U_{mn}^{\mathrm{free}} = e^{i\phi_n} \delta_{mn} |m\rangle\langle n|$. Subsequent applications of the unitary transformations for each light pulse lead to the final momentum distribution. To compare theory with experiment one has to take into account the critical dependence of the near field on the incident momenta. The transformations are applied for each of the momenta initially present and their respective final momentum distributions are added. The numerical results for our experimental parameters are shown in Fig. 4 and reveal the same structure as the experimental results in Fig. 2.

Our experimental demonstration of the accelerator modes was accomplished using cesium atoms dropped out of a magneto-optic trap. Laser diodes close to the cesium $D2$ $6S_{1/2} \rightarrow 6P_{3/2}$ transition were used

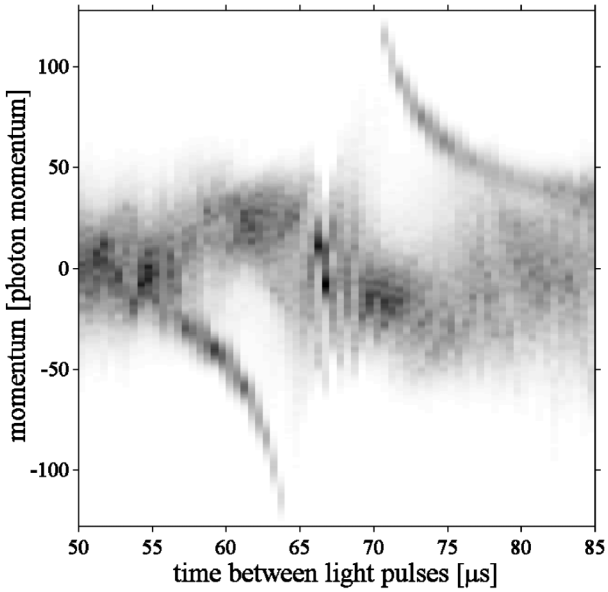

FIG. 4. Theoretical prediction of the momentum distribution after 30 kicks for different times between the pulses and with $\phi_d = \pi$. The theoretical results explain the accelerator mode behavior and the dispersion of the noncontributing atoms. In the numerical calculation 27% of the initial atoms are in the accelerator mode as compared with the experimentally measured efficiency of 20%.

for cooling, trapping, and detection of the atoms. The temperature of the atoms after cooling in optical molasses was 3 μK corresponding to a $\Delta p_{\text{FWHM}} \sim 9$ photon recoils. The pulsed standing light wave was realized parallel to the Earth's gravitational field and was provided by a Ti:sapphire laser, detuned ~5 GHz below the $D1$ transition $(6S_{1/2}, F = 4) \rightarrow (6P_{1/2}, F = 3)$. The switching was performed with an acousto-optic modulator driven with a fast rf amplitude modulator. The ~4 mm diameter Ti:sapphire beams were superimposed with the vertically propagating trap beams to provide a good overlap between the atomic cloud and the kicking standing light wave. The kicking sequence consisted of a series of 500 ns (FWHM), 250 mW pulses with a separation that could be varied between $T = 20$ and 150 μs. After falling 50 cm, the atoms passed through a 2 mm thick sheet of light tuned on resonance to the cooling transition. The amount of absorption in this beam provided a time of flight measurement which allowed us to deduce the atomic momentum distribution in the direction of the kicks with a resolution of two photon recoils. Finally for each set of parameters the momentum distribution measurement was averaged over 15 drops of the atoms.

The quantum accelerator mode has very interesting features in the context of quantum chaos. There the mean energy of the ensemble after a certain number of kicks is used to characterize the system. In the situation where an accelerator mode is present, the quantum mechanical mean energy is not smaller, as is usually the case, but is *larger* than for the equivalent classical system. This is because a quantum accelerator mode may be present even when one is not allowed classically. In the latter case the mode depends on the potential height whereas quantum mechanically it depends mainly on the free evolution. This is the first experiment showing that quantum mechanics is capable of not only inhibiting but also enhancing the momentum spread. This follows intuitively from the fact that the quantum inhibition of the momentum spread, which is usually observed, can be only an interference effect. However, constructive and destructive interference must both exist and the nature of the interference will be determined by the relative positions of the periodic potentials. The diffractive approach we have presented explains the experimental observation of enhanced quantum diffusion when classical accelerator modes are present [8].

The quantum accelerator mode is also a very efficient way of imparting a large amount of momentum coherently to a particle. In our case the data in Fig. 1 shows an efficiency per pulse of 0.993 ± 0.002 with a mean momentum transfer per pulse of 1.6 photon momenta. From Fig. 2 the maximum observed momentum transfer per pulse of 4.5 photon momenta occurs at 64 μs with an efficiency of 0.984 ± 0.005. This leads to an atom accelerated to 100 photon momenta in a total time of 1.9 ms with light on for only 15 μs and a correspondingly small level of spontaneous emission. The efficiency implies that the momentum could be increased simply by applying more laser pulses as long as the Raman-Nath regime is fulfilled.

This process could be used as a beam splitter for an interferometer with a large spatial splitting. However, it is necessary to take into account that the accelerator mode comprises several momentum states (Fig. 3) and also that only a momentum change can be accomplished and not an amplitude splitting. A beam splitter can be formed by applying the scheme we have used in the separated Ramsey interferometer [9]. An internal coherent superposition of two nondegenerate states can be formed via a microwave field so that only one part of the superposition can interact strongly with the light. The momentum of this state can then be changed with an accelerator mode in which the effect of gravity is replaced by acceleration of the standing light wave induced with a Pockels cell. Thus the direction of the acceleration can be readily controlled.

In order to demonstrate this beam splitter clearly, a well defined initial momentum has to be realized and a momentum selective detection is needed to pick out the accelerated atoms. We are currently working towards a Raman-cooling setup which will allow us to prepare and analyze the momentum of the atoms on the recoil scale and build an interferometer based on beam splitting with accelerator modes.

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