Comment on "Evidence for an Anisotropic State of Two-Dimensional Electrons in High Landau Levels"

In a recent Letter Lilly *et al.* [1] have shown that a highly anisotropic state can arise in certain 2D electron systems. (This effect has also now been seen by another group [2].) Most of the samples studied are square. In these samples, the resistances in the two perpendicular directions are found to have a ratio R_{yy}/R_{xx} that may be as much as 60 or larger at low temperature and at certain magnetic fields. Hall bar measurements were also performed which can be thought of, at least conceptually, as being measurements on rectangular samples (with side lengths L_x and L_y). In such measurements R_{yy} is measured in a sample with $L_y \gg L_x$ and R_{xx} in a sample where $L_x \gg L_y$. In the Hall bar experiments it is found that the anisotropy ratio is much smaller—with $R_{yy}/R_{xx} \approx 5$. In this Comment we resolve this discrepancy by noting that the anisotropy of the underlying sheet resistivities is correctly represented by Hall bar resistance measurements but shows up exponentially enhanced in the resistance measurements on square samples due to simple geometric effects. We note, however, that the origin of this underlying resistivity anisotropy remains unknown, and is not addressed here.

Consider a rectangular geometry (L_x by L_y), and assume that the system is described by a uniform anisotropic sheet resistivity tensor ρ. In the so-called "principle" basis, ρ is antisymmetric. Lack of any large observed anisotropy [3] between the resistances $R_{(x+y)(x+y)}$ and $R_{(x-y)(x-y)}$ allows us to assume that the principle basis axes are aligned with the edges of the sample. To calculate the resistance we need to solve $\mathbf{E} = \rho\,\mathbf{j}$, with $\nabla \cdot \mathbf{j} = 0$, and $\nabla \times \mathbf{E} = 0$ with the boundary condition that a net current J is injected at the current source and extracted at the drain. By solving for $\mathbf{j}$ using the Fourier series methods, the voltage between two contacts is obtained by integrating the current density along the edge $\int \mathbf{j} \cdot d\mathbf{l}$ and multiplying by the longitudinal resistivity in that direction. As in the experiment, we place the source and drain at the center of the faces of length L_y and the voltage contacts at the corners. We obtain

$$R_{xx} = \frac{4}{\pi}\sqrt{\rho_{yy}\rho_{xx}} \sum_{n=\mathrm{odd}^+}\left[n \sinh\left(\sqrt{\frac{\rho_{yy}}{\rho_{xx}}}\,\frac{\pi n}{2}\,\frac{L_y}{L_x}\right)\right]^{-1} \approx (L_x/L_y)\rho_{xx} - C\sqrt{\rho_{xx}\rho_{yy}} \quad \text{for } L_x^2\rho_{xx} > L_y^2\rho_{yy}$$

$$\approx \frac{8}{\pi}\sqrt{\rho_{yy}\rho_{xx}}\, e^{-[\pi L_y/2L_x]\sqrt{\rho_{yy}/\rho_{xx}}} \quad \text{for } L_x^2\rho_{xx} < L_y^2\rho_{yy}\,,$$

where $C = 2\ln 2/\pi \approx 0.44$. The case of $L_x \gg L_y$ corresponds to the Hall bar case, whereupon the measured resistance R_{xx} is the sheet resistivity ρ_{xx} times a geometric factor L_x/L_y. However, in the experimental case of a square geometry ($L_x = L_y$), an anisotropy $\rho_{yy} > \rho_{xx}$ in the sheet resistivity results in an exponential decrease in the measured resistance. This decrease is due to the nature of the current paths in a square sample. Most of the current flows straight across the sample in the $\hat{\mathbf{x}}$ direction from the current source in the center of one face to the drain in the center of the opposite face in something close to the shortest route possible. Only a very small amount of current—exponentially small in the parameter $(L_y/L_x)\sqrt{\rho_{yy}/\rho_{xx}}$—extends to the edge of the sample between the voltage contacts a distance away in the $\hat{\mathbf{y}}$ direction. However, it is precisely this small current that determines the voltage drop measured between these two contacts. When the ratio ρ_{yy}/ρ_{xx} is increased, the current flows more directly in the $\hat{\mathbf{x}}$ direction, and the current between the voltage contacts, and hence the measured voltage, decreases exponentially.

Analogously, R_{yy} is given by switching x and y everywhere they occur in the above equations. For a square sample with $\rho_{yy} > \rho_{xx}$, we then have

$$R_{yy}/R_{xx} \approx (\pi/8)\,[(\rho_{yy}/\rho_{xx})^{1/2} - C]\,e^{(\pi/2)\sqrt{\rho_{yy}/\rho_{xx}}}.$$

Thus an underlying anisotropy ratio of $\rho_{yy}/\rho_{xx} \approx 7$ is sufficient to yield a measured anisotropy ratio of 60. It is interesting that the Hall bar measurements yield a ratio $\rho_{yy}/\rho_{xx} \approx 5$ which is slightly lower. Several factors might explain this discrepancy: (i) The two Hall bar measurements and the square measurements are made on physically different samples (although all are from the same wafer) and thus may not all have the same resistivity tensors. (ii) Lack of perfect knowledge of where the contacts are placed could slightly change the effective aspect ratio of the sample. (iii) Misalignment of the crystal axes from the principle axes of the resistivity tensor would change these results (although an exponential enhancement would remain, in general). (iv) Some of the conduction may be from edge state or ballistic transport which cannot be described in terms of a simple local sheet resistivity. (v) Large scale inhomogeneities can give spurious results in the Hall bar measurements if the width of the Hall bar is smaller than this disorder length scale.

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[1] M. P. Lilly *et al.*, Phys. Rev. Lett. **82**, 394 (1999).
[2] R. R. Du *et al.*, Solid State Commun. **109**, 389 (1999).
[3] M. P. Lilly *et al.* (unpublished).

