

Experimental Demonstration of Generation and Propagation of Acoustic Solitary Waves in an Air-Filled Tube

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Experiments are performed to demonstrate the generation and propagation of acoustic solitary waves in an air-filled tube with a periodic array of Helmholtz resonators connected axially. The purpose is to verify the theoretical findings made so far that nonlinear acoustic waves do not evolve into a shock but into a solitary wave propagating steadily without any change of its smooth profile. To identify the solitary wave, the temporal pressure profile is compared directly with the theoretical profile of the solitary wave. Also checked are the relation between the peak sound pressure of the solitary wave and its half-value width in time, and the relation between the peak sound pressure and the deviation of propagation speed from sound speed. The experimental results show good quantitative agreement with the theory.

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When an intense acoustic wave is propagated in air, it usually evolves into a shock. This is because the propagation speed of the acoustic wave becomes faster where sound pressure is higher [1]. By this nonlinear effect, a wave profile is steepened forward and distorted progressively, whereas the dissipative effect is too weak to counteract it. As a consequence, there emerges a discontinuity in the profile. This is the shock. Once the shock is formed, it is decayed out by “nonlinear damping” even in a lossless case. Hence steady propagation of nonlinear acoustic waves in air has been thought to be impossible. But it has recently been revealed, in theory, that an acoustic solitary wave free from the shock can be propagated steadily in an air-filled tube if a periodic array of Helmholtz resonators is axially connected with the tube [2,3]. Now that theoretical aspects of the acoustic solitary wave have been clarified to some extent, we are in a position to perform experiments to verify them. This Letter reports the results of the experiments. Before doing so, we summarize the theoretical findings obtained so far.

The solitary wave is a compressive pulse localized temporally as well as spatially and it can be propagated steadily without any change of form. The propagation speed is slower than the sound speed a_0 , i.e., subsonic, but is faster than a threshold value determined by $a_0/(1 + \kappa/2)$, where $\kappa (= V/Ad \ll 1)$ is a small parameter that measures the volume of the resonator’s cavity V relative to the volume of the tube per axial spacing between neighboring resonators, A and d being the cross-sectional area of the tube and the axial spacing, respectively. As the speed approaches the upper bound a_0 , the peak sound pressure of the solitary wave, p'_p i.e., the peak height, becomes higher but is bounded by that of the limiting solitary wave, given by $8\gamma\kappa/[3(\gamma + 1)]$ relative to an equilibrium pressure p_0 . As the speed decreases and

approaches the lower bound, the height becomes lower and the profile tends to take the soliton solution of the Korteweg–de Vries equation. In fact, the acoustic solitary waves which are small in height may be identified as the soliton [4,5].

The remarkable change in evolution from the shock to the solitary wave is brought about by weak linear dispersion due to the array of resonators. The spatially periodic structure of the tube yields the dispersion of a Bloch-wave-type, which makes propagation speed dependent on frequencies and renders it slower than sound speed [6]. It is noted, however, that this dispersion does not appear in the form of the higher-order derivative in the evolution equation and, therefore, it cannot always inhibit shock formation. In fact, there are cases in which shocks can still be propagated [2,3]. The Korteweg–de Vries equation is derived only asymptotically in the limit of a high natural frequency of the resonator. Incidentally, for the generation of the acoustic soliton, each Helmholtz resonator in the array may be replaced by a side branch of *any* shape if the side branch is closed otherwise and acoustically compact [7]. But specific profiles of solitary waves will differ depending on specific types of the side branch.

Since the solitary wave is a compressive pulse, both the density and the temperature of air where the wave is located are increased by adiabatic compression while the air is pushed forward to obtain a velocity in the direction of propagation. It is remarked that all of these quantities vary in phase with the pressure. This implies that mass, momentum, and energy can be transferred by propagation of the solitary wave [8]. Such properties should be compared with those of the shock. Whereas the shock is decayed out eventually by nonlinear damping, the solitary wave is free from this damping. Thus it has a great advantage in being capable of transporting the physical quantities steadily over a long distance.

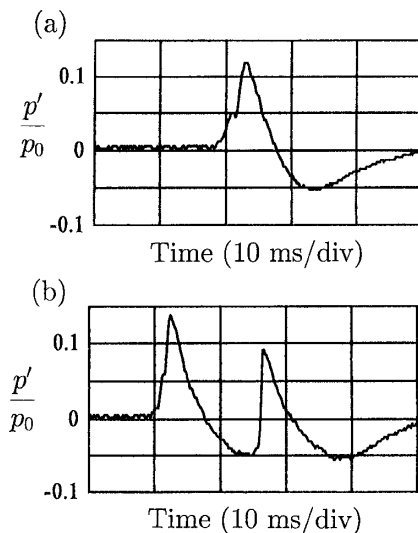


FIG. 1. Temporal profiles of the pressure disturbance p' (in excess) relative to the atmospheric pressure p_0 measured, respectively, at the locations 0.4 m (a) and 4.8 m (b) distant from the valve in the tube without the array of Helmholtz resonators, where the abscissa denotes the time and the scale is 10 ms per division.

Having summarized the theoretical findings, we now describe the experiments. We use a tube of stainless steel, whose length is 7.4 m long and whose inner diameter is 80 mm wide. Let the same Helmholtz resonators be connected along the tube with axial spacing 50 mm on both sides of the tube in a stagger. Each resonator has a cavity of volume $V (= 4.94 \times 10^{-5} \text{ m}^3)$ and a throat of length $L (= 35.6 \text{ mm})$ and of diameter $2r (= 7.11 \text{ mm})$. Thus the parameter κ takes a value of 0.197. A lossless natural angular frequency ω_0 is calculated by $\sqrt{\pi r^2 a_0^2 / LV}$, and it corresponds to 257 Hz. By taking account of the end corrections on both openings of the throat [2,9], L is lengthened effectively to $L + 2 \times 0.82r$ so that the natural frequency is lowered to 238 Hz.

One end of the tube is connected to a high-pressure chamber through a valve that is normally closed, while the other end of the tube is closed by a flat plate. By opening the valve to release the pressurized air stored in the tube, pressure disturbances are produced in the air under the atmospheric pressure and the room temperature 15 °C. As long as the initial pressure in the high-pressure chamber before opening the valve is fixed, almost the same pressure disturbances can be produced so that an experiment can be repeated with high accuracy. A pressure in the tube can be measured by microphones at 18 locations along the tube, each separated by axial spacing 0.4 m from the end of the tube on the valve side.

Although the solitary wave assumes the absence of dissipation, linear damping is unavoidable in the experiments, which is due mainly to wall friction when a frequency is well off the natural frequency of the resonator [6]. Suppose a harmonic wave having an angular frequency ω

is propagating unidirectionally in a tube of diameter D . Its amplitude then decays as $\exp(-qx)$ over a propagation distance x with the rate q given by $C\sqrt{2\nu/\omega}(\omega/a_0)/D$, where $C = 1 + (\gamma - 1)/\sqrt{\text{Pr}}$, with γ and Pr being, respectively, the ratio of specific heats and the Prandtl number, and ν is the kinematic viscosity of air. Taking the natural frequency ω_0 as ω , for example, the decay in amplitude is calculated to be 8% after propagation over the tube length 7.4 m, where $a_0 = 340 \text{ m/s}$, $\gamma = 1.4$, $\text{Pr} = 0.72$, and $\nu = 1.45 \times 10^{-5} \text{ m}^2/\text{s}$ for air at 15 °C. But because a typical angular frequency of the solitary wave ω is much lower than ω_0 (as will be seen in Fig. 4 below), q becomes smaller by the factor $\sqrt{\omega/\omega_0}$ and the decay is estimated to be about 3% over the whole length of the tube.

We first observe that the shock is really formed and propagated when the array of resonators is not connected. Figure 1(a) shows the temporal profile of the excess pressure p' in reference to p_0 measured at the location 0.4 m distant from the valve. The profile is smooth and no shock is involved. The peak excess pressure is $1.2 \times 10^4 \text{ Pa}$. Because the valve's diameter is much smaller than the tube's, the air spreads out spherically into the tube so that there appears an expansive tail behind the main compressive pulse. The pressure profile measured at the location 4.8 m distant from the valve is shown in Fig. 1(b). Two peaks can be seen: the first (left) pulse is the one traveling down the tube and the second (right) pulse is the one reflected back by the closed end. Comparing the pulse in (a) with the first and second pulses in (b), the first pulse is seen to steepen forward (leftward) and the reflected pulse has been fully transformed into a triangular pulse with shock ahead. The reflected pulse has traveled the distance 10 m in total, with the distance after reflection added. The shock formation point is located at 8.8 m from the valve, which is estimated by solving a lossless nonlinear wave equation for a simple wave [1].

We now demonstrate the evolution when the array of resonators is connected. At first, the experiment is performed under the same condition as that in the tube without the array. Figures 2(a) and 2(b) show the temporal profiles of the excess pressure measured at the locations 0.4 and 4.8 m from the valve, respectively. The peak in Fig. 2(a) is slightly smaller than the one in Fig. 1(a), and the profile seems to have spread a little. This difference results from the dispersive effects due to several resonators located between the valve and the measuring point. Figure 2(b) shows clearly that no steepening takes place and no shock is formed. The reflected pulse becomes smoother and broader in width and it looks like a solitary wave. We now check whether or not this is so.

The analytical expression of the solitary wave is given for the excess pressure p' in reference to p_0 as follows [3]:

$$\frac{p'}{p_0} = \frac{\gamma\kappa}{\gamma + 1} f(\xi), \quad (1)$$

with

$$\zeta = \omega_0 \left[t - \left(1 + \frac{\kappa s}{2} \right) \frac{x}{a_0} \right] + \text{const}, \tag{2}$$

where x and t are the axial coordinates along the tube and the time, respectively, and s is a parameter in $0 < s < 1$. The function $f(\zeta)$ is expressed inversely in terms of ζ as follows:

$$4 \tan^{-1} \sqrt{\frac{f_+ - f}{f - f_-}} - \frac{2s}{\sqrt{-f_+ f_-}} \log \left| \frac{[\sqrt{-f_-(f_+ - f)} - \sqrt{f_+(f - f_-)}]^2}{(f_+ - f_-)f} \right| = |\zeta|, \tag{3}$$

where f_{\pm} are given by

$$f_{\pm} = -2 \left(s - \frac{2}{3} \right) \pm \sqrt{-\frac{4}{3}s + \frac{16}{9}}, \tag{4}$$

with the signs vertically ordered. The profile of $f(\zeta)$ takes a pulse-form with a single peak of height f_+ (>0) at $\zeta = 0$ and symmetric with respect to $\zeta = 0$, decaying out monotonously and exponentially as $|\zeta| \rightarrow \infty$ [3]. Since f is always positive, the solitary wave consists of a compressive phase only and the propagation speed is given by $a_0/(1 + \kappa s/2)$ from (2). When the limit as $s \rightarrow 0$ is taken to approach the upper bound of the propagation speed, f tends to the limiting solitary wave given by

$$f = \frac{8}{3} \cos^2 \left(\frac{\zeta}{4} \right), \tag{5}$$

in $|\zeta| \leq 2\pi$ and $f \equiv 0$ in $|\zeta| > 2\pi$, while when the limit as $s \rightarrow 1$ is taken to approach the lower bound, f tends to the soliton solution asymptotically given by [4,7]

$$f = \alpha \operatorname{sech}^2 \sqrt{\frac{\alpha}{12}} \zeta, \tag{6}$$

where $s = 1 - \alpha/3$ ($0 < \alpha \ll 1$).

Because the reflected pulse is riding on the tail of the first pulse, as is seen in Fig. 2(b), it is difficult to identify the

base of the solitary wave. In order to measure the height of the reflected pulse, we draw a straight line connecting the minimal points of the pressure on both sides of the reflected pulse and take the line as the base. The peak height is measured from this base and compared with the theoretical value $[\gamma \kappa f_+ / (\gamma + 1)] p_0$ to determine a value of the parameter s . For the profile shown in Fig. 2(b), the peak excess pressure p'_p relative to p_0 is 0.1, and s is calculated to be 0.68. By using this value of s , the profile of the pressure (1) with respect to the time t is drawn in Fig. 3 as a broken line where the origins of t and p'/p_0 are taken so that the peak of the theoretical profile may coincide with that of the experimental one. Both profiles agree very well quantitatively.

Next we examine evolutions in various cases where initial pressures in the high-pressure chamber are varied. Instead of the direct comparison of the pressure profiles, we check the relation between the peak height of the pulse and its half-value width in time. From the solution (1), the half-value width in ζ is defined as the one in which $f_+/2 \leq f \leq f_+$. The solid curve in Fig. 4 shows the half-value width ω^{-1} (in time), normalized by ω_0^{-1} . In passing, ω may be regarded as a typical angular frequency of the solitary wave. The abscissa p'_p/p_0 is limited by the peak height of the limiting solitary wave

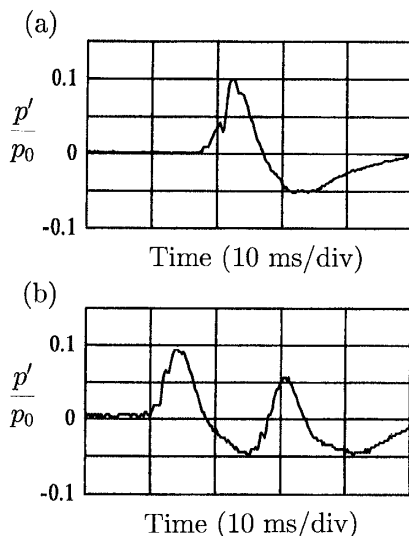


FIG. 2. Temporal profiles of the pressure disturbance p' (in excess) relative to the atmospheric pressure p_0 measured, respectively, at the locations 0.4 m (a) and 4.8 m (b) distant from the valve in the tube with the array of Helmholtz resonators having $\kappa = 0.197$, where the abscissa denotes the time and the scale is 10 ms per division.

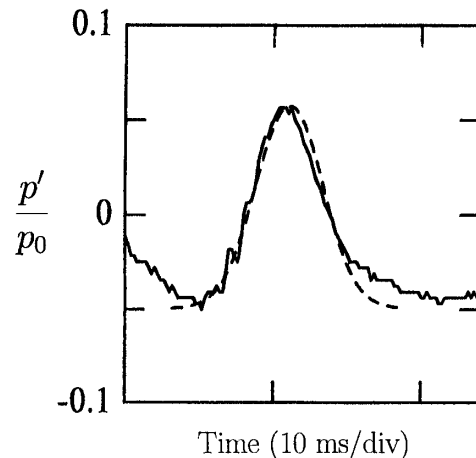


FIG. 3. Comparison of the temporal pressure profile of the reflected pulse measured at the location 4.8 m distant from the valve (solid line) with the theoretical one (dashed line) calculated from the solution (1) for $s = 0.68$, where the abscissa denotes the time, the scale being 10 ms per division, while the ordinate denotes the excess pressure p' relative to the atmospheric pressure p_0 .

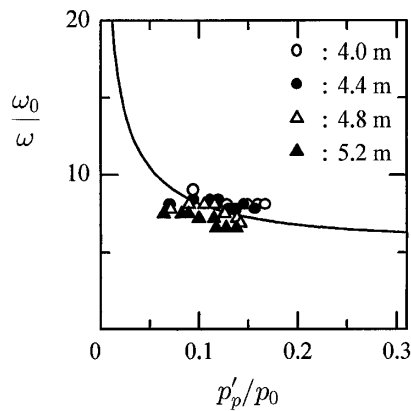


FIG. 4. Relation between the peak excess pressure p'_p relative to the atmospheric pressure p_0 and the half-value width ω^{-1} (in time), normalized by ω_0^{-1} , where the solid curve is the theoretical result derived from the solitary wave solution (1) and the same symbol represents the experimental data measured at the same location indicated in the figure, but under various initial pressures in the high-pressure chamber.

$8\gamma\kappa/[3(\gamma + 1)] (= 0.306)$, at which the half-value width takes 2π . As the peak pressure decreases, on the other hand, the theoretical curve increases to infinity as $\omega_0/\omega \propto \sqrt{p_0/p'_p}$. This is simply the branch derived from the soliton solution (6). The same symbol in Fig. 4 refers to the data measured at the same location indicated but under various initial pressures in the chamber. Although the distances for the solitary waves to be established should differ depending on the initial pressures, the data lie on the theoretical curve. For the pulse shown in Fig. 2(a), the half-value width ω_0/ω takes a value of about 5 at $p'_p/p_0 = 0.1$ and the relation of the solitary wave is not satisfied. However, in the course of propagation, the width of the pulse becomes wider and approaches the theoretical curve.

As another check, we examine the propagation speed of the pulse. Using the reflected pulse propagating toward the valve, we calculate the speed v at a specific location by measuring the ratio of the travel distance to the travel time of the peak between the location and the one that is 0.4 m distant from the valve. Since the speed of the solitary wave is $a_0/(1 + \kappa s/2)$ and is slightly slower than a_0 , the deviation $(a_0 - v)/a_0$ is measured. Figure 5 shows the theoretical curve as a solid line and the experimental data. Note that the propagation speed is always measured to be subsonic and many data scatter above the theoretical curve, meaning v is slower. One cause of the latter may be attributed to the wall friction. It makes the propagation speed slower by $C\sqrt{2\nu/\omega}/D$ relative to a_0 [6], which is estimated to be about 0.007 in the present case. If this is taken into account, the data measured at the locations 4.8 and 5.2 m distant from the valve fall within the range expected with a few exceptions. The scattering of the data measured at the locations 4.0 and 4.4 m may result from

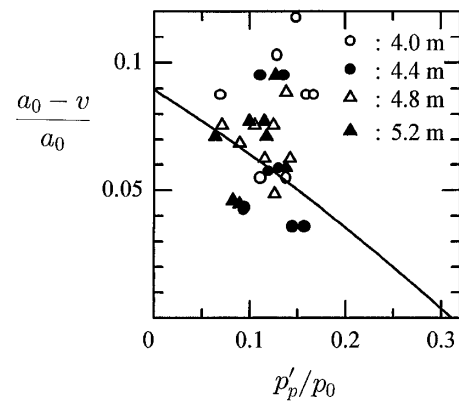


FIG. 5. Relation between the peak excess pressure p'_p relative to the atmospheric pressure p_0 and the deviation of the propagation velocity $v [= a_0/(1 + \kappa s/2)]$ from sound speed a_0 , normalized by a_0 , where the solid curve indicates $\kappa s/(2 + \kappa s)$ given in theory for $\kappa = 0.197$ and $0 < s < 1$, and the same symbol represents the experimental data measured at the same location indicated in the figure, but under various initial pressures in the high-pressure chamber.

the fact that the travel distance and time are too short. This problem would be resolved if a tube was made longer.

In conclusion, it has been clearly demonstrated that the array of Helmholtz resonators can effectively suppress shock formation in the propagation of nonlinear acoustic waves. To verify whether or not the solitary wave is generated, three checks have been made: the direct comparisons of the overall pressure profile, the half-value width, and the propagation speed against the theoretical results. These checks have proved the quantitatively good agreement with the theory.

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