Manipulation of Motional Quantum States of Neutral Atoms

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Cold Cs atoms are cooled into the ground state ($n = 0$) of a far detuned 1D optical lattice. From this state, we prepare and detect several quantum states of motion. First, we produce squeezed states of $n = 0$ (and $n = 1$) and record the time evolution of the velocity distribution of these states. A velocity distribution compressed by a factor of 4 compared to that of the ground state is obtained. Second, we prepare a linear superposition of $n = 0$ and $n = 1$ and observe its time evolution.

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Production of pure quantum states of motion of trapped particles provides not only a clear illustration of quantum mechanics, but it also constitutes the ultimate control of particle motion. For instance, the ground state of a particle in a harmonic potential is a minimum uncertainty state and this control has a variety of applications in atom optics, atom lithography and interferometry, quantum logic, and quantum communications. Several theoretical papers have proposed controllable systems enabling multiparticle entanglement [1]. On the experimental side, quantum motional states of a single trapped ion have been observed [2], and a quantum logic gate with two trapped ions has recently been demonstrated [3]. Cold neutral atoms in optical lattices may offer a long decoherence time because of their very weak coupling to the environment and the possibility of parallel processing [4,5].

In this Letter, we present the production of various quantum states of motion of neutral Cs atoms in a far detuned 1D optical lattice. Squeezed states, coherent states, and linear superpositions of $n = 0$ and $n = 1$ states are observed. Velocity measurement is performed using a ballistic expansion (BE) absorption imaging technique. As the cold atom sample contains $\sim 10^5$ atoms, a single image provides the full velocity distribution of the quantum state $|\psi(v)|^2$ and a set of images gives its time evolution. Such velocity distributions were not accessible in single trapped ion experiments, where the particular quantum states were inferred from the analysis of Rabi oscillations between vibrational states [2]. To avoid thermal excitation, the particles are first cooled to the ground state of the lattice potential [6,7] and from this state the $n = 1$ Fock state has previously been produced [8]. Here we first present the production of squeezed states of motion by a very simple technique. The width of the velocity distribution of the squeezed state oscillates between a minimum value much smaller than the standard quantum limit and a maximum value corresponding to the antisqueezing phase, in agreement with calculation. Finally, we display the time evolution of a linear superposition of $n = 0$ and $n = 1$ states as well as that of a coherent state.

In our experiments, Cs atoms are confined in a dipole trap which consists of two 5 W YAG laser beams having 180(20) μm waists, parallel linear polarizations, and crossing each other in a vertical plane [8]. After loading from a magneto-optical trap, about 1.5×10^5 atoms are confined at a temperature of about 20 μK. The interference of the two beams leads to a modulation of the potential along the vertical axis z with a period of 665 nm and with an amplitude of about 0.2 mK sufficient to trap the atoms in each microwell independently. Each microwell supports about 30 bound states and the harmonic approximation is valid up to $n = 10$. The oscillation frequency along z, measured using the $\Delta n = \pm 1$ sidebands in a two photon Raman spectrum [8], is $\omega_{\text{osc}}/2\pi = 85(5)$ kHz at the center of the trap. The Lamb Dicke factor is $\eta = kz_0 = 0.16$, where k is the wave number of the D2 line. $z_0 = \sqrt{\hbar/2m\omega_{\text{osc}}} = 21$ nm is the rms size of the ground state and $p_0/m = \sqrt{\hbar\omega_{\text{osc}}/2m} = 11$ mm/s its rms velocity width. The trapped cloud has a nearly Gaussian shape with a vertical rms size $z_{\text{trap}} = 53(5)\mu$m so that about 160 horizontal microwells are occupied, each containing about 1000 atoms. In the following, we restrict our attention to the motion along z. First we perform sideband cooling of motion along z as described in [8] and put, in 10 ms, 95% of the atoms in the ground state. The cooling process uses stimulated Raman transitions between the $|F = 3\rangle$ and $|F = 4\rangle$ hyperfine ground states. The Raman beams are vertical and counterpropagating in order to allow transitions between different vibrational states. Measurement of the velocity distribution using BE is performed as follows: the potential is suddenly switched off, the atoms expand freely for a time called T_{BE}, and a 2D absorption image of the atom cloud is taken in 50 μs with a horizontal beam [8].

A particle in a harmonic potential is described by the Hamiltonian $H = (p^2 + z^2)/2$ taking units such that $\omega_{\text{osc}} = \hbar = m = 1$. It is equivalent to $H = a^\dagger a + \frac{1}{2}$ with $p = \frac{1}{\sqrt{2}} i(a - a^\dagger)$ and $z = \frac{1}{\sqrt{2}}(a + a^\dagger)$. In analogy with squeezed states of light [9], squeezing of motion amounts to reducing z_{rms} below z_0 or p_{rms} below p_0. The methods proposed so far to generate motional squeezed states use either parametric drive of the potential or sudden drop of the potential [10,11]. The former one was achieved in ion experiments [2] and wave packet compression in the classical regime ($\Delta z > z_0$) was observed in an optical lattice in [12]. Here squeezed states are

generated just by switching off and on the dipole potential (YAG beams). This follows the suggestion of Ref. [13] although its quantum aspect was not realized then. (a) Atoms are cooled to the vibrational ground state, (b) the potential is switched off for a time τ_1, and (c) the potential is switched on again for a time τ_2. Ballistic expansion images are recorded for variable τ_2.

A simple picture of the method is given by the time evolution of the Wigner distribution $W(z, p)$ presented in Fig. 1. In a harmonic potential and in free space the time evolution of $W(z, p)$ is identical to the classical evolution of a phase space distribution. (a) For the ground state (an energy eigenstate), $W(z, p)$ is round and unchanged under time evolution (rotation with angular frequency $\omega_{\rm osc}$). (b) When the potential is turned off, the upper part of the distribution moves to the right because it has a positive momentum, while the lower part moves to the left. The acceleration and displacement of the atoms due to gravity during the time τ_1 are negligible. The distribution becomes elliptic. (c) As the potential is turned on again, the Wigner function rotates at $\omega_{\rm osc}$. For a particular τ_2, the momentum distribution is squeezed while the position distribution is antisqueezed. A quarter period later the situation is reversed. Because the area in phase space is conserved during the time evolution, the product of the lengths of the two principal axes of the ellipse is conserved. In particular, at the squeezed and antisqueezed instants, the principal axes of the ellipse coincide with the z, p axes and we have $\Delta z \Delta p = \hbar/2$. Figure 2(a) shows an absorption image of the trap. Figure 2(b) presents a BE image with $T_{\rm BE} = 14.5$ ms of the $n = 0$ state prepared by Raman sideband cooling [8]. It corresponds to $p_{\rm rms}/m = 11(1)$ mm/s in good agreement with the calculated value (11 mm/s). With such long $T_{\rm BE}$, this picture is essentially the velocity distribution of the atoms. After a squeezing sequence with $\tau_1 = 8\ \mu$s, and $\tau_2 = 0.4\,\mu$s chosen so that atoms are most squeezed in momentum space, the vertical size of the atomic cloud after BE is strongly reduced [Fig. 2(c)]. A fit with a Gaussian gives an rms width z. The velocity width, calculated with a deconvolution with the initial size of the cloud, is 4 times smaller than that of the ground state, $v_{\rm rms} = \sqrt{z^2 - z_{\rm trap}^2}/T_{\rm BE} = 2.7$ mm/s $= \frac{1}{4}(p_0/m)$.

A set of BE absorption images for various τ_2 displays the time evolution of the momentum spread along z (Fig. 3 circles). Several oscillations between squeezed and antisqueezed phases can be seen, as well as a damping which we shall discuss later.

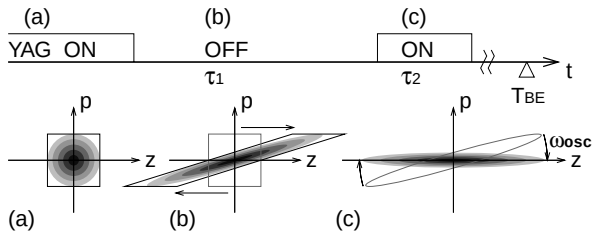

FIG. 1. Experimental time sequence and evolution of the phase space distribution.

We next apply the squeezing sequence to the $|n = 1\rangle$ Fock state produced as in [8]. The momentum distribution of $|n = 1\rangle$ consists in two peaks separated by $2\sqrt{2}\,p_0$. After the squeezing sequence, the momentum distribution keeps the same shape while breathing in time (Fig. 4). For the rather short $T_{\rm BE}$ delay in Fig. 4 (4.8 ms), the two peaks of the modulus of the wave function are not resolved at the maximum squeezing phases ($\tau_2 \sim 0.8$, 6.4, and 12.8 μs) because of the finite initial cloud size. At the antisqueezed phases the two peaks are well resolved ($\tau_2 \sim 3.2$ and 8.8 μs) and their separation is $2.7 \times 2\sqrt{2}\,p_0$.

Quantitative comparison with theory proceeds as follows. The Hamiltonian is

$$\begin{cases} H_{\rm ON} = \frac{1}{2}(p^2 + z^2) = a^\dagger a + \frac{1}{2}, & [\text{YAG ON: } (a), (c)], \\ H_{\rm OFF} = \frac{1}{2}\,p^2, & [\text{YAG OFF: } (b)]. \end{cases} \tag{1}$$

$U = \exp(-iH_{\rm ON}\tau_2)\exp(-iH_{\rm OFF}\tau_1)$ being the time evolution operator, the time evolved annihilation operator $a' \equiv UaU^\dagger$ obeys

$$a' \equiv UaU^\dagger = \xi z + \zeta p\,, \tag{2}$$

where

$$\begin{cases} \xi = \frac{1}{\sqrt{2}}[\cos\tau_2 + (i - \tau_1)\sin\tau_2], \\ \zeta = \frac{1}{\sqrt{2}}[-\sin\tau_2 + (i - \tau_1)\cos\tau_2]. \end{cases} \tag{3}$$

Starting from the state $|n = 0\rangle$, the state of the atoms after the squeezing sequence $|0\rangle' \equiv U|0\rangle$ obeys

$$a'|0'\rangle = Ua|n = 0\rangle = 0\,. \tag{4}$$

From (2) and (4), the wave function in p representation $\psi_0(p) \equiv \langle p\,|\,0'\rangle$ obeys the differential equation

$$(i\xi\partial_p + \zeta p)\psi_0(p) = 0\,. \tag{5}$$

Solving this equation gives $\psi_0(p) = \gamma e^{-\alpha p^2/2}$ with $\alpha = -i\zeta/\xi$ and the momentum uncertainty

$$\begin{aligned} \Delta p^2 &= \frac{1}{{\rm Re}\,\alpha} = 2|\xi|^2 \\ &= 1 + \frac{\tau_1^2}{2} - \tau_1\sqrt{1 + \frac{\tau_1^2}{4}}\cos(2\tau_2 - \phi), \end{aligned} \tag{6}$$

where $\phi = \tan^{-1}\frac{2}{\tau_1}$ [14].

Equation (6) reproduces qualitatively the experimental results (Fig. 3) but does not explain the damping which we understand as follows. The vertical oscillation frequency depends on the position in the trap and the spread of $\omega_{\rm osc}$ is 10%. This gives different rotation speeds in

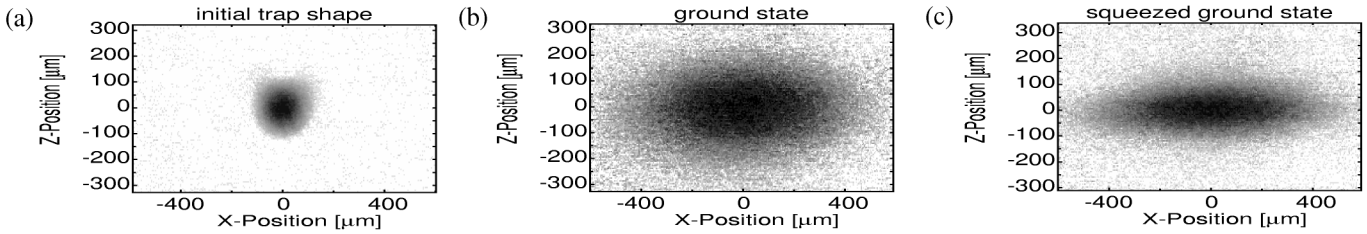

FIG. 2. Absorption image of the Cs dipole trap (a). Images with 14.5 ms ballistic expansion for atoms in the ground state $|n = 0\rangle$ (b), and in a squeezed state obtained with $\tau_1 = 8$ and $\tau_2 = 0.4$ μs (c). From these images, a momentum squeezing factor of 4 is deduced ($p_{\rm rms}/m = 2.7$ mm/s $= \frac{1}{4} p_0/m$). The transverse temperature of the sample is 7 μK ($v_{\rm rms} = 21$ mm/s).

phase space in the 2nd ON time [Fig. 1(c)] and thus provides damping. The calculation including this effect reproduces reasonably well the observed damping in Fig. 3 [15].

As a second example of quantum state engineering, we demonstrate now the generation of a coherent superposition of the ground state $|n = 0\rangle$ and the first excited state $|n = 1\rangle$ of motion in the harmonic microtraps. Atoms in $|F = 3, n = 0\rangle$ are transferred into $F = 4$ by a short Raman pulse of duration $\tau = 11$ μs tuned in between the two transitions $|F = 3, n = 0\rangle \rightarrow |F = 4, n = 0\rangle$ and $|F = 3, n = 0\rangle \rightarrow |F = 4, n = 1\rangle$. Nonresonant transfer to $|n = 2\rangle$ is negligible because the $|n = 0\rangle \rightarrow |n = 2\rangle$ coupling is reduced by a factor of $2\sqrt{2}\,\eta^2 = 0.07$ with respect to the $|n = 0\rangle \rightarrow |n = 0\rangle$ coupling. Because atoms transferred into $|F = 4, n = 1\rangle$ are further coupled to $|F = 3, n = 1\rangle$, then to $|F = 4, n = 2\rangle$, and so on, we use a weak pulse which transfers only about half of the atoms into $F = 4$ so that the population in $F = 4$, $n = 2, 3, \ldots$, remains negligible. The ratio between the populations transferred in $n = 1$ and $n = 0$ depends on the detuning of the Raman pulse. It is small if the pulse is resonant with the transition $|F = 3, n = 0\rangle \rightarrow |F = 4, n = 0\rangle$ because η is small, and is increasing as the pulse is tuned nearer the transition $|F = 3, n = 0\rangle \rightarrow |F = 4, n = 1\rangle$. Choosing a detuning which equalizes the two populations, the wave function of atoms in $F = 4$ is then $|\psi\rangle = (|0\rangle + e^{i\phi}|1\rangle)/\sqrt{2}$. After the preparation pulse, the phase difference between these two states evolves in time as $\omega_{\rm osc} t$ leading to a periodic oscillation of the velocity distribution [Fig. 5(b)]. This evolution is obviously nonclassical and contrasts with that of a coherent state which oscillates as a classical particle without deformation.

The velocity distribution of atoms in $F = 4$ at variable time t after the Raman pulse is measured by BE with $T_{\rm BE} = 8$ ms [Fig. 5(c)]. This evolution reproduces reasonably well the expected one [Fig. 5(b)]. We attribute the weaker contrast in the experimental data to the spread of initial positions in the trap. The expected relative phase ϕ at the end of the Raman pulse [Fig. 5(b) lower trace] is $\phi_0 + \omega_{\rm osc} T/2$. Here ϕ_0 is the phase for which the expectation value of the momentum is maximum in the direction of the Raman kick $\hbar(\mathbf{k}_b - \mathbf{k}_r)$. $\mathbf{k}_b$ (respectively,

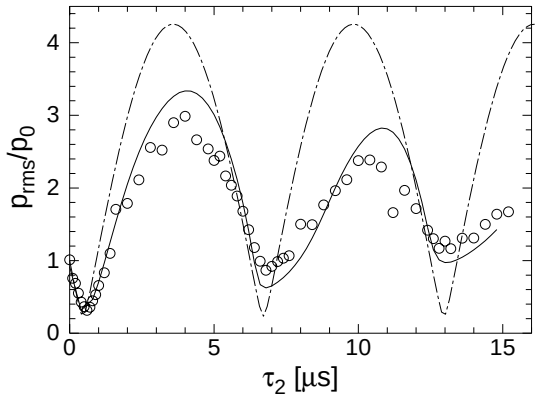

FIG. 3. Time evolution of the momentum spread of $|n = 0\rangle$ squeezed state, in units of the rms width p_0 of $|n = 0\rangle$. Circles: experimental points. Dot-dashed line: Calculated evolution given by (6). Solid line: Calculation taking into account the spread of oscillation frequencies in the lattice wells.

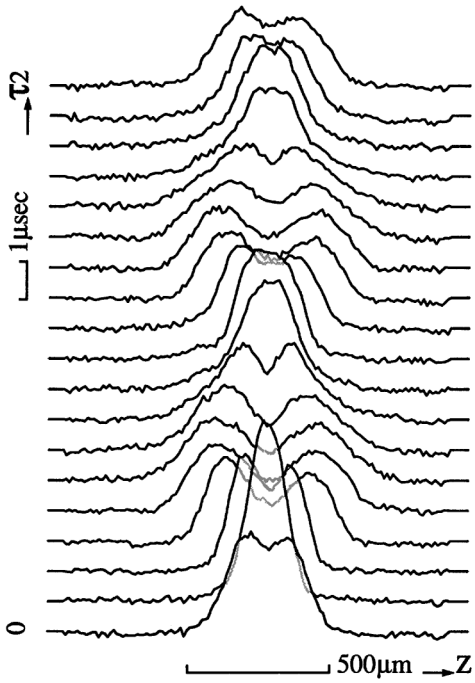

FIG. 4. Time evolution of the squeezed $|n = 1\rangle$ Fock state obtained with $\tau_1 = 4$ μs. Series of cuts along z are taken from the BE absorption images.

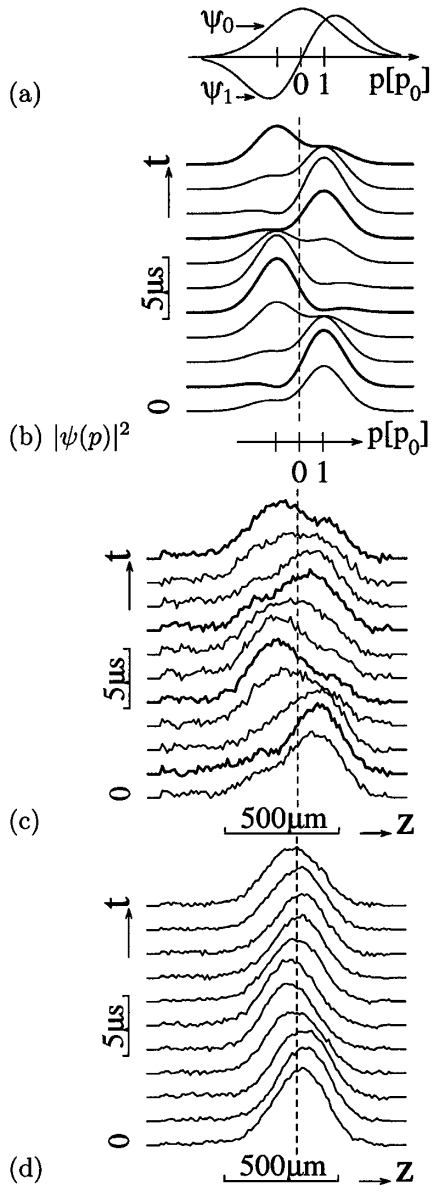

FIG. 5. (a) Wave functions in momentum representation of the states $|n = 0\rangle$ and $|n = 1\rangle$. The momentum unit is the rms ground state width p_0. (b) The calculated evolution of the momentum distribution $|\psi(p)|^2$ of the superposition $|\psi\rangle = (|n = 0\rangle + e^{i\phi}|n = 1\rangle)/\sqrt{2}$. (c) The measured time evolution of the velocity distribution of the superposition state. The time origin is the end of the Raman pulse. (d) Measured time evolution of the coherent state $|\alpha = 0.28\rangle$.

$\mathbf{k}_r$) is the wave vector of the higher (respectively, lower) frequency beam. The initial phase of the measured data [Fig. 5(c)] is in good agreement with this prediction.

If we apply a pulse much shorter than the oscillation period, the atoms have no time to oscillate during the pulse. They behave as free atoms and the wave function transferred in $F = 4$ is displaced in velocity space by $2\hbar k/m = 7.0$ mm/s. If the atoms are initially in the ground state, the produced state is a coherent state. Because of a lack of power in the Raman beams, we rather used a pulse of 11 μs. This pulse is not very short compared to the oscillation period but it still produces a good approximation of a small amplitude coherent state. The pulse is tuned to the $|F = 3, n = 0\rangle \rightarrow |F = 4, n = 0\rangle$ transition. The population transferred in $n = 1$ is small and those of $n = 2, 3, \ldots$, are negligible. The state is then $|0\rangle + \alpha|1\rangle$, close to the coherent state $|\alpha\rangle = e^{-|\alpha|^2/2}[|0\rangle + \alpha|1\rangle + (\alpha^2/\sqrt{2})|2\rangle + \ldots]$. The measured velocity distribution of this state [Fig. 5(d)] oscillates in time while remaining Gaussian as expected. The maximum velocity change is ± 4.5 mm/s $= 0.62 \times 2\hbar k/m$, corresponding to the coherent state $|\alpha = 0.28\rangle$.

In summary we have prepared various quantum superposition states and observed the time evolution of their momentum distribution. Further work will rely on populating a single lattice well so that the decoherence of these states will not be masked by the inhomogeneous broadening of oscillation frequencies. Coherence time of the order of 0.1 s is expected in our system. Extension of this work to two and three dimensions is also straightforward. Finally macroscopic quantum systems such as Bose-Einstein condensates will reveal interesting new effects when coupled to nondissipative optical lattices [16].

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