Sub-Coulomb α Transfers on ^{12}C and the ^{12}C$(\alpha,\gamma)^{16}$O S Factor

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The ^{12}C$(\alpha,\gamma)^{16}$O reaction is crucial for the understanding of He burning in massive stars, but the low-energy cross section is highly uncertain. To address this problem we have measured at sub-Coulomb energies total cross sections for the ^{12}C$(^6$Li$, d)^{16}$O and ^{12}C$(^7$Li$, t)^{16}$O reactions to the bound 2^+ and 1^- states of ^{16}O. The data are analyzed to obtain the reduced α widths of these states. Together with capture and phase-shift data, these results provide for a more accurate determination of the low-energy 12C$(\alpha,\gamma)^{16}$O S factor: $S_{E1}(0.3\ \text{MeV}) = 101 \pm 17$ keV b and $S_{E2}(0.3\ \text{MeV}) = 42^{+16}_{-23}$ keV b for the $E1$ and $E2$ multipole components of the reaction.

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The ^{12}C$(\alpha,\gamma)^{16}$O reaction is a very important helium-burning process in massive stars. The rate of this reaction greatly affects the resulting ratio of ^{12}C to ^{16}O, the subsequent nucleosynthesis of heavier elements, and the final fate of the star (i.e., black hole or neutron star) [1,2]. The cross section for this reaction at the energies required for stellar helium burning [$E \approx 0.3$ MeV] (we use center-of-mass energies throughout this paper) is far too small for direct measurement using presently available techniques.

The extrapolation of the measured cross sections ($E \geq 1$ MeV) to lower energies is complicated by two states located 45 keV ($J^\pi = 1^-$) and 245 keV ($J^\pi = 2^+$) below the ^{12}C + α threshold. The cross section at astrophysical energies arises largely from the high-energy tails of these states, but the properties of these states are only weakly constrained by cross-section measurements at higher energies. The γ-ray widths of these levels are known [3], but there is considerable uncertainty in their reduced α widths which parametrize the strength of α + ^{12}C clustering at the nuclear surface. The reduced α width of the 1^- state has been inferred in Ref. [4] by fitting a large body of data including the β-delayed α spectrum of ^{16}N. Very little information exists for the 2^+ state. Reviews of the present status of the ^{12}C$(\alpha,\gamma)^{16}$O problem can be found in Refs. [5,6].

It has long been known that the needed reduced α widths can in principle be determined from α transfer reactions. However, the results reported previously for these methods have been subject to rather large uncertainties, attributed mainly to uncertainties in the potential parameters and the influence of compound-nuclear contributions (see Refs. [7,8], and references therein). The Li-induced α transfer reactions at very low energies offer several attractive features, which have not been previously investigated. The slightly negative Q values for the reactions to the subthreshold states [-1.23 and -2.22 MeV for the $(^6$Li$, d)$ and $(^7$Li$, t)$ reactions to the bound 2^+ state] mean that the outgoing deuterons or tritons will also have energies below the Coulomb barrier. Under these conditions, distorted-wave Born approximation (DWBA) calculations are determined mainly by Coulomb potentials, with very little dependence on nuclear potential parameters. The calculated cross sections are thus essentially model independent, except for the absolute normalization, which depends in turn on the reduced α widths of the ^{16}O final state and the Li nucleus which contributed the α particle. The weak binding of the α particles in these final states serves to enhance the direct cross section for sub-Coulomb reactions.

In this Letter, we report the first sub-Coulomb measurements of the ^{12}C$(^6$Li$, d)^{16}$O and ^{12}C$(^7$Li$, t)^{16}$O cross sections to the bound 2^+ and 1^- states of ^{16}O. The data are analyzed to extract the reduced α widths of these states. The reduced widths are then combined with ^{12}C(α,α) and ^{12}C$(\alpha,\gamma)^{16}$O data to determine the low-energy extrapolations of the ^{12}C$(\alpha,\gamma)^{16}$O capture cross section.

The measurements were conducted by bombarding ^{12}C targets with ^{6,7}Li beams covering $2.7 \leq E[^6\text{Li}] \leq 7.0$ MeV and $4.75 \leq E[^7\text{Li}] \leq 7.0$ MeV, supplied by the Caltech 3-MV Pelletron accelerator. The targets consisted of 19.2 ± 1.7 μg/cm^2 of ^{12}C (>99.9% enrichment) evaporated onto Cu substrates, and were oriented at 45° with respect to the incident beam. The target thickness was determined by comparing the Cu(d, d) scattering plateau edge for the target with that from a blank Cu target. These determinations were performed with detectors located at $\theta_{\text{lab}} = 135°$ and 145° and for $1.5 \leq E_d \leq 2.5$ MeV using deuteron beams supplied by the Triangle Universities Nuclear Laboratory FN Tandem accelerator.

Total cross sections for populating the bound $J^\pi = 2^+$ and 1^- states were determined by detecting the de-excitation γ rays from these states. High-purity Ge detectors of 85% and 35% relative efficiency were placed at distances (target to front face) of 7 and 9 cm and angles of 31° and 110°, respectively. Note that the

$P_4(\cos\theta)$ Legendre polynomial vanishes at these angles. The branching ratios of these states to the ground state of ^{16}O are essentially 100%, so by applying Gaussian quadrature to the measured yields, the total cross section for populating these states could be determined, without ambiguity from angular-distribution effects. Sample γ-ray spectra are shown in Fig. 1. We do not positively identify the 7.12-MeV γ ray from the ^{12}C(^{7}Li, t)^{16}O reaction, due to background from the ^{12}C(^{7}Li, α)^{15}N reaction. The photopeak efficiencies were determined using a calibrated ^{152}Eu source and the 992-keV resonance in the ^{27}Al(p, γ) reaction.

The systematic error in the measured cross sections due to uncertainties in detector efficiency, target thickness, and beam-current integration is estimated to be $\pm 15\%$. The results for the bound 2^+ and 1^- states are shown in Fig. 2. Cross sections for the 2^+ state have recently been reported for somewhat higher energies [9,10]. We find good agreement for the range of overlap for ^{12}C(^{6}Li, d)^{16}O [10], but some differences in normalization and energy dependence for ^{12}C(^{7}Li, t)^{16}O [9]. We also note that γ-ray angular distributions for the former reaction (which will be presented elsewhere) agree with the findings of Carlson [11].

We have analyzed these data using the finite-range DWBA code FRESCO [12]. The essential ingredients of these calculations are the optical potentials for the incoming and outgoing scattering states, and the wave functions of the α-particle bound states. For sufficiently low energies ($E \lesssim 2.5$ MeV for ^{12}C + ^{6}Li, $E \lesssim 3.3$ MeV for ^{12}C + ^{7}Li) the calculations are insensitive to the assumed optical potentials as both incoming and outgoing channels are well below the Coulomb barrier (the optical potentials can effectively be replaced by a point-Coulomb potential).

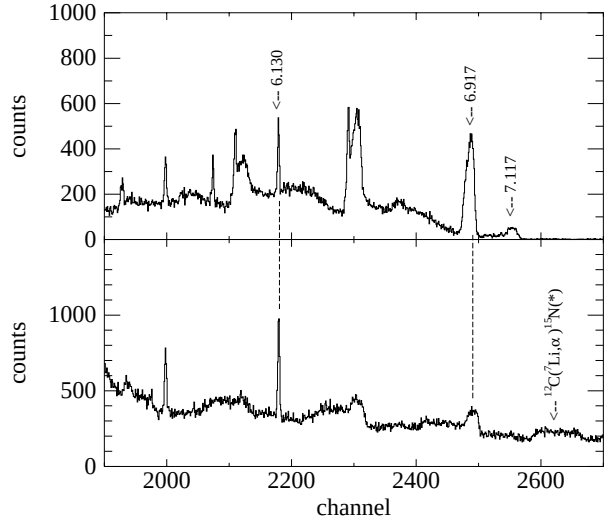

FIG. 1. Sample γ-ray spectra obtained at 31°, for ^{12}C + ^{6}Li at $E = 2.4$ MeV (upper panel), and ^{12}C + ^{7}Li at $E = 3.1$ MeV (lower panel).

It is also found that the calculated cross section at sub-Coulomb energies is sensitive only to the asymptotic part of the α particle bound-state wave functions, while other factors relating to the bound states such as the geometry of the binding potentials, or the numbers of radial nodes, are unimportant.

The DWBA calculations are performed using α-particle bound-state wave functions which are normalized to unity over all space. The experimental and calculated cross sections are then related by

$$\sigma_{\text{exp}} = S_1 S_2 \sigma_{\text{DWBA}}\,, \tag{1}$$

where S_1 and S_2 are the α spectroscopic factors for the Li ion which contributed the α particle and the ^{16}O final state, respectively. The asymptotic part of the α-particle wave functions depends on the spectroscopic factors S_i and the model wave functions $u_i(r)/r$, and can be conveniently parametrized by an asymptotic normalization constant C_i, such that

$$S_i^{1/2} u_i(r) = C_i W(r)\,, \tag{2}$$

where $W(r)$ is the exponentially decaying Whittaker function, and r is any radius beyond the range of the nuclear force. The R-matrix reduced widths γ_i^2 are directly related to C_i via

$$C_i^2 = \frac{2\mu a}{\hbar^2 W^2(a)} \left(\frac{\gamma_i^2}{1 + \gamma_i^2 \frac{dS}{dE}} \right), \tag{3}$$

where μ is the reduced mass, a is the channel radius, and $S(E)$ is the shift function. This relation holds only for the R-matrix boundary condition choice $B = S(E_i)$, where E_i

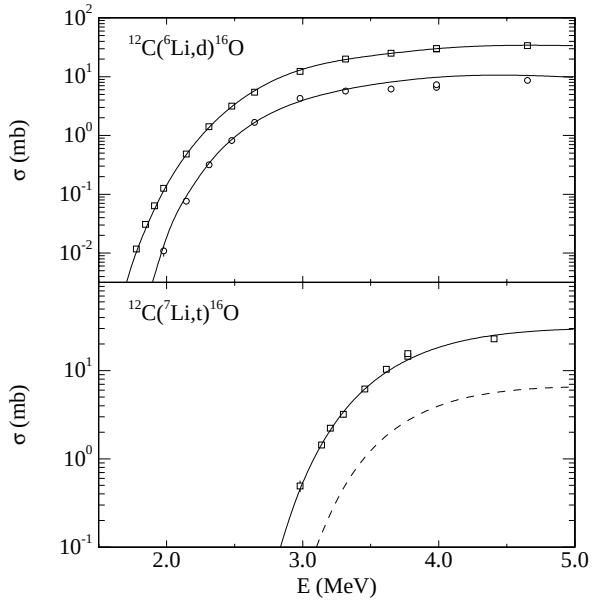

FIG. 2. Total cross sections measured using ^{6}Li (upper panel) and ^{7}Li (lower panel) beams for the 6.92-MeV 2^+ state of ^{16}O (□) and the 7.12-MeV 1^- state (○, ^{6}Li beam only). The solid curves are DWBA calculations normalized to the data; the dashed curve is described in the text.

is the bound-state energy [see Eq. (16) of Barker [13] and Ref. [14]].

The DWBA results are shown in Fig. 2, normalized to the experimental data with $E \leq 2.5$ MeV for $^{12}C + {}^{6}Li$ and $E \leq 3.3$ MeV for $^{12}C + {}^{7}Li$. The calculations are seen to predict the energy dependence of the measured cross sections very well, particularly at sub-Coulomb energies, where they are expected to be most reliable. The normalization of the theoretical cross sections to the experimental ones then determines the product of spectroscopic factors (or normalization constants). The dashed curve shows the yield for $^{12}C(^{7}Li, t)^{16}O$ to the bound 1^- state predicted from the measured $(^{6}Li, d)$ cross section.

The absolute cross section also depends on the reduced α width (or asymptotic normalization constant) of the Li ion which contributed the α particle. This information appears to be reasonably well determined; we utilize $C^2(^{6}Li) = 5.3 \pm 0.5$ fm^{-1} [15,16] and $C^2(^{7}Li) = 12.6 \pm 1.9$ fm^{-1} [17–19].

We have investigated the importance of compound-nuclear contributions using the Hauser-Feshbach code STATIS [20]. The predicted absolute cross sections depend somewhat on the assumed optical potentials, but the calculated energy dependences and ratios between different ^{16}O final states are nearly independent of potential assumptions. When the calculations are normalized to the 3^- state data of Refs. [9,10], or the 8.87-MeV 2^- state data at higher energies [21], the predicted cross sections for the 2^+ and 1^- states are at most 5% of the measured values. It is interesting to note that the DWBA and Hauser-Feshbach calculations predict that the ratio of direct to compound-nuclear cross sections is largest at the lowest energies measured.

For the bound ^{16}O states, we find $C^2(2^+) = (1.24 \pm 0.24) \times 10^{10}$ fm^{-1} and $C^2(1^-) = (4.33 \pm 0.84) \times 10^{28}$ fm^{-1} from the $^{12}C(^{6}Li, d)^{16}O$ measurements, and $C^2(2^+) = (1.33 \pm 0.29) \times 10^{10}$ fm^{-1} from $(^{7}Li, t)$. The quoted uncertainties include contributions from cross-section data, the Li normalization constants, and possible effects from compound-nuclear contributions. The values for the 2^+ state are seen to be in good agreement; the average value (taking into account common uncertainties) is $C^2(2^+) = (1.29 \pm 0.23) \times 10^{10}$ fm^{-1}.

In order to determine the effect of these results on the $^{12}C(\alpha, \gamma)^{16}O$ reaction, we have incorporated the reduced α widths determined here in R-matrix fits to $^{12}C(\alpha, \gamma)^{16}O$ S factors and $^{12}C(\alpha, \alpha)$ phase shifts. The reduced α width for any desired channel radius can be deduced from the reported C_i using Eq. (3). The $E1$ and $E2$ cross-section data were fitted separately, in conjunction with phase-shift data from Ref. [22]. All fits were carried out with the R-matrix boundary-condition constant chosen to make the level shift vanish at the energy of the subthreshold state. We report results for a channel radius of $a = 6.5$ fm; other channel radii in the range $5.5 \leq a \leq 8$ fm yield nearly identical results. The best fits were determined via χ^2 minimization. Following Ref. [4], we determined the acceptable ranges for the low-energy S factors by allowing fits with $\chi^2 \leq \chi^2_{\min}(1 + 9/\nu)$, where ν is the number of degrees of freedom, and also taking into account the uncertainties in the fixed parameters.

The $l = 1$ fitting was performed using a 3-level fit, with capture-cross-section data from Refs. [23–26]. The parameters associated with the bound 1^- level are fixed by the known excitation energy, γ-ray width [3], and reduced α width (determined above from α transfer). We find $S_{E1}(0.3 \text{ MeV}) = 101 \pm 17$ keV b, with the best fit shown in Fig. 3 and the parameters given in Table I. This result is in reasonable agreement with the finding $S_{E1}(0.3 \text{ MeV}) = 79 \pm 21$ keV b [4] based on ^{16}N β-delayed α decay. We find an additional χ^2 minimum with destructive interference near $E = 0.3$ MeV which yields $S_{E1}(0.3 \text{ MeV}) = 10$ keV b, but this solution increases χ^2 by 48, which is ruled out by the adopted χ^2 criterion.

The $l = 2$ fitting was carried out utilizing capture-cross-section data from Refs. [24,26] and the R-matrix formalism of Barker and Kajino [27]. Four levels were included, corresponding to the bound state, the narrow ($\Gamma_\alpha = 0.625 \pm 0.100$ keV [3]) level at $E = 2.68$ MeV, the 4.36-MeV level, and a background state. The known excitation energy and partial widths of the 2.68-MeV level are fixed by including in the data set three pseudo capture-cross-section points around the resonance peak. Likewise, the γ-ray width of the 4.36-MeV state is fixed by including one additional pseudo data point at the peak of the resonance. The pseudo data points were assumed to have 10% uncertainties.

Besides the three parameters for each level, the reduced α width of the ^{16}O ground state γ_{10} must be specified. This parameter essentially scales the magnitude of the external contribution to the capture matrix element [27]. The

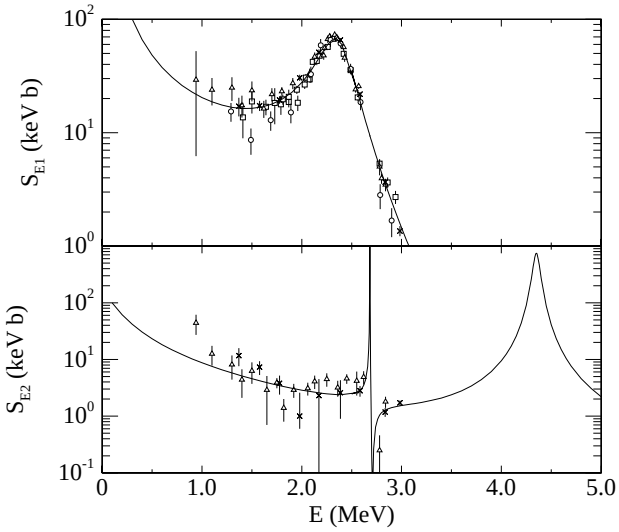

FIG. 3. The best R-matrix fits (solid curves) to the $E1$ and $E2$ S-factor data (□ [23], △ [24], ○ [25], × [26]).

TABLE I. Best-fit R-matrix parameters for $a = 6.5$ fm. Parameters in parentheses are fixed.

	$l = 1$	$l = 2$
E_{1l} (MeV)	(-0.0451)	(-0.2448)
γ_{1l} (MeV$^{1/2}$)	(0.0912)	(0.1450)
$\gamma_{1l\gamma}$ (MeV^{-1})	(8.763×10^{-6})	(1.771×10^{-6})
E_{2l} (MeV)	2.851	2.683
γ_{2l} (MeV$^{1/2}$)	0.3254	0.0123
$\gamma_{2l\gamma}$ (MeV^{-l})	-2.343×10^{-6}	-1.693×10^{-7}
E_{3l} (MeV)	33.30	4.362
γ_{3l} (MeV$^{1/2}$)	1.932	0.0673
$\gamma_{3l\gamma}$ (MeV^{-l})	-4.955×10^{-6}	-1.275×10^{-6}
E_{4l} (MeV)	⋯	11.036
γ_{4l} (MeV$^{1/2}$)	⋯	0.9949
$\gamma_{4l\gamma}$ (MeV^{-l})	⋯	5.613×10^{-7}
γ_{10} (MeV$^{1/2}$)	⋯	-0.0001
$S_{El}(0.3)$ (keV b)	101 ± 17	42^{+16}_{-23}
$\chi^2_{\delta l}$	23 (26 points)	80 (48 points)
$\chi^2_{\gamma l}$	144 (71 points)	54 (27 points)

parameters E_{12} and γ_{12} are fixed from the known excitation energy and the value of $C^2(2^+)$ determined above, respectively; $\gamma_{12\gamma}$ is fixed by the known γ-ray width [3] and the value of γ_{10}. The fit thus has ten free parameters, of which four are tightly constrained by the pseudo data points. It was found essential to systematically investigate all possible combinations of parameter signs in the fitting process. We find $S_{E2}(0.3\ \text{MeV}) = 44^{+16}_{-23}$ keV b, with the best fit shown in Fig. 3 and the parameters given in Table I. This result represents considerable improvement upon the previously available result, $S_{E2}(0.3\ \text{MeV}) < 140$ keV b [5].

In summary, we determined the reduced α widths of the bound 2^+ and 1^- states of ^{16}O from sub-Coulomb measurements of $^{12}\text{C}(^6\text{Li}, d)^{16}\text{O}$ and $^{12}\text{C}(^7\text{Li}, t)^{16}\text{O}$. These reduced widths were then included in the fitting of radiative capture and elastic-scattering data to determine the low-energy extrapolations of the $E1$ and $E2$ S factors. In the case of the $E2$ S factor, the result is considerably more accurate than previously available. Allowing 16 keV b for the cascade S factor [5], the total S factor is $S(0.3\ \text{MeV}) = 159$ keV b. This value agrees very well with the finding [1] $S(0.3\ \text{MeV}) = 170 \pm 50$ keV b deduced by comparing solar-system abundances to supernova nucleosynthesis calculations. The results reported here will reduce the uncertainties in such calculations, and allow for better study of convective mixing processes [2]. We anticipate further improvements in our understanding of the low-energy S factors to result from high-precision $^{12}\text{C}(\alpha, \alpha)$ measurements which have very recently been completed [28].

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