Contribution of Nucleon-Nucleon P Waves to nt-nt, dd-pt, and dd-dd Scattering Observables

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(Received 16 June 1999)

The four-body equations of Alt, Grassberger, and Sandhas [Phys. Rev. C **1**, 85 (1970)] are solved for a system of four nucleons, using realistic NN interactions in channels $j \leq 2$. The results of the calculation are compared with data for the reactions $dd \to dd$, $dd \to p^3$H, and n^3H $\to n^3$H. The calculations indicate that the NN P waves have a strong effect on 4N observables including a 10% contribution to the total n^3H $\to n^3$H cross section at the peak of the low energy resonance.

PACS numbers: 21.45.+v, 13.75.Cs, 25.10.+s

In a recent review article on the three-nucleon continuum [1] one finds that ^{1}H$(\vec{d},d)^1$H elastic observables (cross sections, vector, and tensor polarizations) are insensitive to the choice of realistic NN potential. Beyond the persistent A_y discrepancy at low energy, the agreement between calculations and data is excellent in the energy range up to 65 MeV. They also report that the A_y discrepancy at low energy persists even when known static 3N force models are added to any realistic 2N interaction. The same conclusion is reached by others [2], where the NN interaction is modified by the coupling NN-$N\Delta$. No matter what approach one follows to add a 3N force in the 3N system, the outcome is similar: beyond its effect on the 3N binding energy (about 1 MeV more binding), triton wave function related parameters (charge radii, asymptotic normalization constants), and doublet scattering length, all of which correlate almost linearly with the triton binding energy, the 3N force plays almost no role in $d + p$ elastic scattering or breakup for deuteron laboratory energies up to $E_d \simeq 65$ MeV. This low energy behavior, though predicted by a few authors [3] 30 years ago, was confirmed in the last 10 years by the very accurate calculations of the Bochum and Pisa groups. Unlike $d + N$ scattering, the triton is very sensitive to the choice of 2N or $2N + 3N$ interaction. Therefore ^{3}H$(\vec{n},n)^3$H or $d(\vec{d},p)^3$H observables may show greater dependence on force models than ^{1}H$(\vec{d},d)^1$H.

Recent calculations of the four-nucleon system exist for the binding energy of the α particle [4], $p\,^3$He and $n\,^3$H scattering length [5] which may be considered of the same accuracy and sophistication as encountered in the 3N system. At higher energies, but still below three-body breakup threshold, there is the work of the Grenoble group [6] which calculates the cross section for ^{3}H$(n,n)^3$H using Malfliet-Tjon (MT) and Argonne V14 (AV14) potentials in channels $^1S_0$ and $^3S_1$-$^3D_1$. Because their calculation carries no approximations, beyond a limitation on the NN partial waves included or number of channels, the conclusions are very disturbing. They claim that, unlike MT, the realistic interaction (AV14) fails to describe the neutron-triton (n-t) total cross section at the peak of the resonance ($E_n = 3.5$ MeV) and preliminary results on the importance of NN P waves or a 3N force seem to indicate that they have almost no effect on the cross section at this energy; while the combined effect of $^1P_1$, $^3P_0$, and $^3P_1$ interactions decrease the total cross section at the peak, adding $^3P_2$-$^3F_2$ raises it to only about its original value in the absence of NN P waves. Given that low energy n-t elastic scattering is the simplest 4N reaction, such failure, if confirmed, would have significant implications in few-nucleon physics in general and on the nature of 2N force models in particular.

Therefore in the present work we attempt to clarify this issue, in addition to the presentation of the most complete calculation that we know of, for the 4N tensor observables in ^{2}H$(\vec{d},d)^2$H, ^{2}H$(\vec{d},p)^3$H, and ^{3}H$(\vec{n},n)^3$H below four-body breakup threshold in order to identify possible failures that may shed light on the NN interaction one uses, and study the effect of NN P waves. Previous calculations of these observables involve either a combination of perturbation theory coupled with the solution of the 4N equations for the dominant partial waves [7] or the use of a limited number of channels at energies close to the $d + d$ threshold [8].

The starting point involves the solution of Alt, Grassberger, and Sandhas (AGS) equations [9] for the transition operators involving all $(2N) + (2N)$ and $N + (3N)$ channels. For local NN potentials such equations are three-vector variable integral equations which after partial wave decomposition reduce to a set of coupled equations in three continuous scalar variables. Since scattering calculations require a great number of channels for convergence, we follow an approach based on the separable representation of subsystem amplitudes in order to reduce the equations to two or one continuous variable. The integral equations we use are the same as in Ref. [10] and result from the modified AGS equations [11] after one has (a) represented the original NN t matrix by an operator of rank one and (b) represented the resulting 3N t matrix by a finite rank operator and taken as many terms as needed for convergence. Since in the modified AGS equations the $2N + 2N$ subamplitudes are expressed in terms of a convolution integral involving two noninteracting pair propagators, as first proposed in Ref. [12], the sole approximation in this approach involves a rank one representation of the $2N$ t matrix which may be obtained from the well-known method of Ernst, Shakin, and Thaler (EST) [13]. The multiterm representation of the $3N$ t matrix is done using the energy dependent pole expansion (EDPE) method developed in Ref. [14]. This latter representation for the $3N$ t matrix is well under control since one may check the convergence rate of $4N$ observables for increasing rank in the $3N$ t matrix.

This method was first used in Ref. [10] to calculate the binding energy of ^{4}He and later confirmed to be accurate by the exact work of Kamada and Glöckle [15]. More recently [16] the results of our calculations for n-t elastic scattering were shown to agree with the exact results of the Grenoble group [6], for both MT and AV14 potentials taken in $2N$ partial waves with $j \le 1^+$ ($^1S_0, ^3S_1$-$^3D_1$). This benchmark was performed in the energy range $0 \le E_n < 7$ MeV and shows that both works agree within 3% or better. In addition, at $E_d = 3$ MeV, d-n elastic observables calculated with the rank one EST representation of the $2N$ t matrix are in very good agreement with the exact results; the differential cross sections agree within 1%, and the tensor observables T_{20}, T_{21}, and T_{22} within 10% or better over a broad angular range. Only iT_{11} is reduced by 30% at the peak. Therefore, in the energy region below four-body breakup threshold, we expect the rank one approximation to account for the dominant physics.

The four-nucleon calculations we present here make use of the Bonn-B and Argonne V14 potentials in channels $^1S_0, ^3S_1$-$^3D_1, ^1P_1, ^3P_0, ^3P_1$, and $^3P_2$. The first two channels correspond to including all $2N$ partial waves with $j \le 1^+$, while the first five channels to $j \le 1$. The Coulomb repulsion between protons is neglected. Independently of the number of $2N$ partial waves that are included for a given NN interaction, one needs to set upper limits for given subsystem quantum numbers in order to reach a converged $4N$ result. In particular one has to decide the largest $3N$ total angular momentum J to be included and the rank of the corresponding EDPE expansion. Since preliminary work was first presented in Ref. [17], we do not show here how the $4N$ results converge with the rank "r" in the EDPE expansion of the $3N$ t matrix nor with the number of $3N$ subamplitudes for increasing $3N$ total angular momentum J. In general we take all $3N$ subamplitudes up to $7/2^+$ for $3N$ total isospin $I = 1/2$ and up to $5/2^+$ for isospin $I = 3/2$. For a given (J, I) the resulting $3N$ subamplitude is calculated using all underlying $3N$ channels with N-$(2N)$ orbital angular momentum $L \le 3$ and rank r equals 6 for $I = 1/2$ amplitudes and 4 for $I = 3/2$. As expected $I = 3/2$ $3N$ subamplitudes contribute only to isospin $\mathcal{I} = 1$ $4N$ reactions such as ^{3}H$(n,n)^3$H. Finally in all $\mathcal{I} = 1 (\mathcal{I} = 0)$ calculations we solve AGS equations for each $4N$ total angular momentum $\mathcal{J} \le 3$ ($\mathcal{J} \le 6$) in all corresponding $N + (3N)$ and $(2N) + (2N)$ channels with relative orbital angular momentum $\mathcal{L} \le 3$ ($\mathcal{L} \le 5$). The above limits for $3N$ and $4N$ channels are less restrictive than those used in Ref. [6] and for this reason we expect this work to be fully converged on all relevant angular momentum channels and subchannels.

In Fig. 1 we show the new results for the total cross section for n-t elastic scattering obtained with Bonn-B potential. The long dashed line corresponds to including NN partial waves with $j \le 1^+$ while the solid curve to a calculation that carries all NN partial waves up to $^3P_2$ (all $j \le 1$ plus $^3P_2$). For comparison we show the results obtained with AV14 for $j \le 1^+$ (dash-dotted line) which, as mentioned above, coincide with the work of Ref. [6] within 3%. At threshold energies the Bonn-B and AV14 results shown in Fig. 1 deviate from data. This comes from having the wrong triton binding energy (8.28 MeV for Bonn-B and 7.64 MeV for AV14) but this discrepancy is considerably smaller for Bonn-B than for AV14 as the well known correlation [5] between triton binding energy and n-t scattering length (triplet and singlet) predicts. At higher energies the calculation shows that the NN P waves, unlike in Ref. [6], are responsible for a 10% increase in the total n-t cross section at the peak of the resonance, leading to a reasonable description of the data. To save computing time we have added NN P waves to the AV14 calculation at $E_n = 3.5$ MeV alone. The corresponding result is shown by the triangle in Fig. 1. As in Ref. [6] we also find that the total cross section decreases when the NN P waves are restricted to $^1P_1$, $^3P_0$, and $^3P_1$ (see diamond in Fig. 1), but adding $^3P_2$ produces a remarkable increase in the cross section. These results reverse the findings of Ref. [6] and show that NN P waves not only have a strong effect in n-t elastic observables, but also that both AV14 and Bonn-B are able to describe the total n-t cross section at the peak. This may be rationalized in the following manner: as we

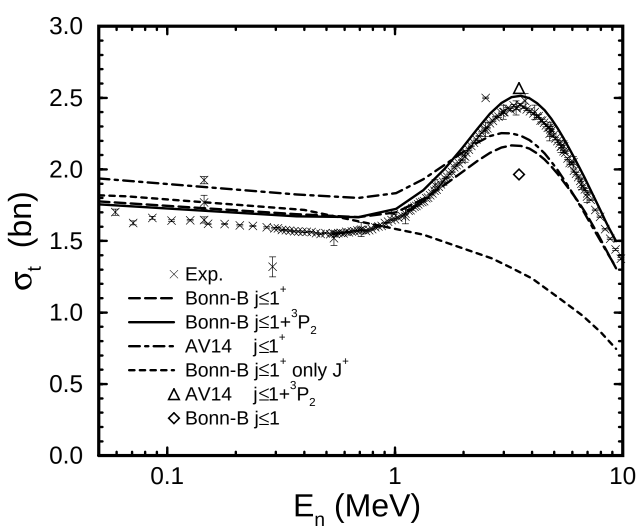

FIG. 1. Total neutron-triton elastic cross section versus energy for both Bonn-B and AV14 potentials. The crosses are experimental points from Ref. [18].

know from $3N$ physics, ${}^1\mathrm{H}(\vec{d},d){}^1\mathrm{H}$ spin observables are strongly dependent on the NN P waves, even at low energy $E_d < 3$ MeV; most of this dependence comes through the odd parity $3N$ amplitudes J^- which, as we know from very early work on the four-nucleon system [19], make up for 50% of the total n-t cross section. This is again shown in Fig. 1 by the dashed line which corresponds to a calculation with Bonn-B potential in channels $j \leq 1^+$ but where the $3N$ subamplitudes included are restricted to J^+. Therefore we claim that the J^- $3N$ subamplitudes, once embedded in the $4N$ system, act as a magnifier for the NN P waves, leading to a 10% increase of the total cross section. Given that the NN P waves have almost no effect in low energy d-N cross sections and less than a 1% contribution to the triton binding, this 10% rise of the total n-t cross section is a remarkable result in low energy few-nucleon physics that may bring new hope to studies of $3N$ force effects in this energy region; static $3N$ forces give rise to NN P wave components whose contribution to $4N$ scattering may also be magnified through J^- $3N$ subamplitudes.

In addition to the total cross section we show in Fig. 2 the differential cross section at $E_n = 3.5$ MeV and 6 MeV, together with the analyzing power A_y at $E_n = 5.54$ MeV and 10.01 MeV. Since A_y comes from p-^{3}He data we have added the two-body Coulomb amplitude to the nuclear amplitude multiplied by the Coulomb phases. Although the differential cross section is in reasonable agreement with data, once the NN P waves are included (solid curves), the A_y is still too low and shows the same type of disagreement with data as one has seen before in ${}^2\mathrm{H}(\vec{n},n){}^2\mathrm{H}$. The effect of the NN P waves on n-t partial waves is centered on ${}^3P_0$ and ${}^3P_2$ phases which rise relative to their values in the absence of NN P waves. The remaining discrepancy in A_y may come from higher partial waves in the NN interaction or missing $3N$ force, but may also be due to uncertainties in the strength of the ${}^3P_j$ partial waves, as discussed in the work of Tornow *et al.* [23]. To investigate this issue we modify the ${}^3P_j$ strength according to their prescription and show the new results in Fig. 2 (dash-dotted lines). The new curves show that $\vec{n}$-t A_y is also sensitive to changes in ${}^3P_j$ partial waves, leading to a marginal improvement relative to experiment, but also an undesirable increase of 5% in the total cross section at this energy. Even if this prescription could explain the A_y discrepancy in the $3N$ continuum, it would leave open the solution of the A_y problem in n-t scattering.

Finally in Figs. 3 and 4 we show the predictions of our calculations for ${}^2\mathrm{H}(\vec{d},d){}^2\mathrm{H}$ and ${}^2\mathrm{H}(\vec{d},p){}^3\mathrm{H}$ tensor observables. All calculations were performed with Bonn-B; the long-dashed line corresponds to $j \leq 1^+$, the dashed line to $j \leq 1 + {}^3P_2$, and the solid line to $j \leq 2$. To compare with data all nuclear amplitudes were multiplied by the appropriate Coulomb phases and in the case of $dd \to dd$ the Coulomb amplitude was also added. Since the number of $3N$ channels grows as additional NN partial waves are included, convergence requirements preclude that the rank r of the EDPE expansion of $3N$ subamplitudes has to increase. Given our limitations in computing power, we are restricted to $r = 8$ for all $3N$ J up to $7/2^+$. Therefore $j \leq 2$ results, unlike the others, may not be fully converged. This shortcoming is more evident in $dd \to dd$ observables where the small size of the tensor analyzing powers results from cancellations between large amplitudes. The results are very interesting because they show the importance of NN P waves in the tensor observables for both reactions, as well as the contribution from higher partial waves; in addition one gets the correct order of magnitude for the observables in both reactions. Nevertheless the results are also disturbing because one finds that higher partial waves, other than ${}^1S_0$ and ${}^3S_1$-${}^3D_1$, change the

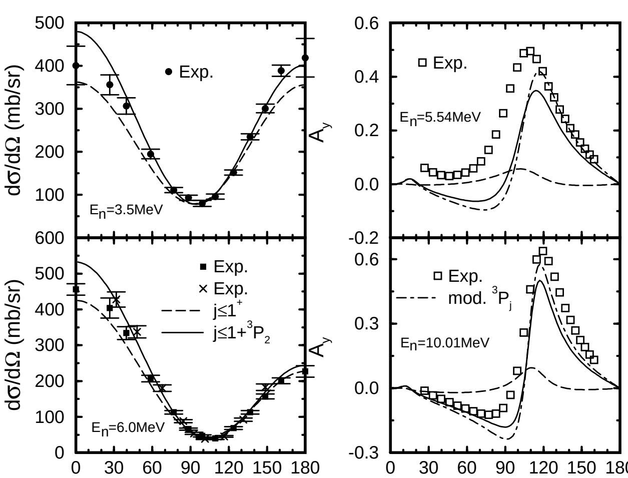

FIG. 2. Angular distributions corresponding to the differential cross sections for ${}^3\mathrm{H}(n,n){}^3\mathrm{H}$ (left) and analyzing powers A_y for ${}^3\mathrm{He}(\vec{p},p){}^3\mathrm{He}$ (right). The experimental points are from Ref. [20] (full circles and squares), Ref. [21], (crosses), and Ref. [22] (open squares).

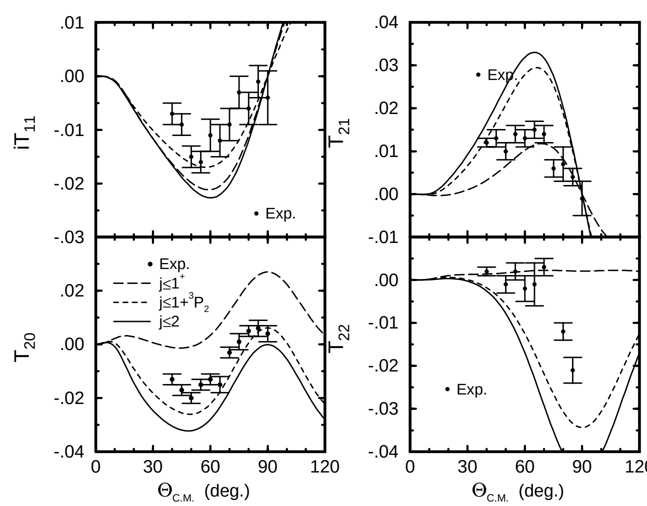

FIG. 3. dd-dd tensor analyzing powers at $E_d = 6.1$ MeV for Bonn-B potential. The experimental points are from Ref. [24].

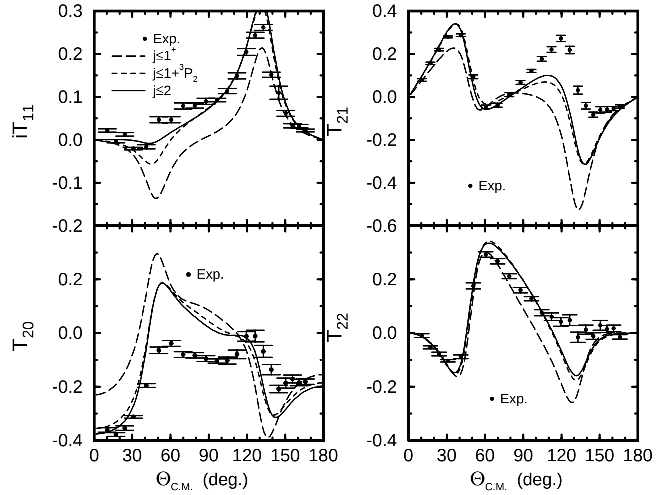

FIG. 4. Same as in Fig. 3 for dd-p^3H.

magnitude of the calculated observables but do not change the shape as much as required by the data; some observables are reasonably well described by the calculation, while others remain poorly reproduced. To attribute such deviations from data to the use, in this work, of the rank one EST representation of the $2N$ t matrix is, in our view, too simple a way out. Higher order terms may modify the representation of the interaction and consequently introduce changes in the calculation, but we do not expect them to alter substantially the present shape of the calculated observables. In Ref. [17] we tried different potentials (Paris, AV14, Bonn-A), limited to $j < 1^+$ partial waves, to test the sensitivity of the observables to different off-shell behavior of the NN interaction, but only a 10% change on the magnitude was observed without any modification of the shape. From experience with p-d versus n-d scattering observables [25], proper treatment of the Coulomb force may also introduce small changes in the magnitude of the tensor observables but not large corrections to their shape.

In the present work we have calculated the total n-t cross section and reversed the conclusion of Ref. [6] *vis-à-vis* the contribution of *NN P* waves at the peak of the total cross section. The difference between the results of Ref. [6] and this work may be due to channel restrictions in Ref. [6] not included in this work, or to a more subtle reason not yet understood. Unlike in Ref. [6] we find the contribution of *NN P* waves to be about 10% of the n-t cross section at $E_n = 3.5$ MeV for both Bonn-B and AV14 potentials. In spite of this success that is also extensive to the differential cross sections for $^3\mathrm{H}(\vec{n},n)^3\mathrm{H}$, we fail to reproduce A_y much like what one finds in $^2\mathrm{H}(\vec{n},n)^2\mathrm{H}$. Modification in the ^3P_j strength according to the prescription by Tornow *et al.* [23] improves the agreement with data as in $\vec{n}$-d A_y. The same equations are applied to study $^2\mathrm{H}(\vec{d},d)^2\mathrm{H}$ and $^2\mathrm{H}(\vec{d},p)^3\mathrm{H}$ tensor observables at 6.1 MeV. Although some observables are well described by the calculation others show large disagreement relative to the data. If these findings are confirmed by others, one may be confronted with a situation where either existing $3N$ force models are able to correct for the present discrepancies and thus show their importance in low energy few-nucleon scattering, or one faces a serious challenge that may urge the development of new $2N + 3N$ force models.

We thank J. Carbonell for very useful discussions, J. Haidenbauer for providing the parameters of the rank one EST expansion, and the Triangle Universities Nuclear Laboratory for the hospitality and initial support without which this work would not have started. This work was funded in part through FCT Grant No. PCERN/C/FIS/1157/97.

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