

Squeezed Correlations and Spectra for Mass-Shifted Bosons

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Huge back-to-back correlations are shown to arise for thermal ensembles of bosonic states with medium-modified masses. The effect is experimentally observable in high energy heavy ion collisions.

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Introduction.—The Hanbury Brown–Twiss (HBT) or Goldhaber–Goldhaber–Lee–Pais (GGLP) effect [1,2] is widely used in heavy ion physics to measure the space-time geometry of such reactions. The enhanced correlations of bosons in outgoing states with small relative momentum provide a Fourier transformed picture of the system at freeze-out. The scales measured by the HBT effect coincide with the lengths of homogeneity [3].

In this Letter, we consider the effect of possible mass shifts in the dense medium on two boson correlations in general. Thus far medium modifications of hadron masses have been mainly considered in terms of effects on such observables as dilepton yields and spectra. Hadron mass shifts are caused by interactions in a dense medium and therefore vanish on the freeze-out surface. Thus, a naive first expectation is that in-medium hadron-modifications may have little or no effect on two boson correlations, and so the usual HBT effect has been expected to be concerned with only the geometry and matter flow gradients on the freeze-out surface. However, in this Letter, we show that an interesting quantum mechanical correlation is induced due to the fact that medium modified bosons can be represented in terms of two-mode squeezed states of the asymptotic bosons, which are observables. As a by-product, we solve the finite-size problem for two-mode squeezing, by formulating the theory of intensity interferometry for a finite, inhomogeneous, squeezed, and expanding medium, which is a new result also for quantum optics [4].

In this Letter, we assume the validity of relativistic hydrodynamics up to freeze-out. The local temperature $T(x)$, chemical potential $\mu(x)$, and flow fields $u^\mu(x)$, are given, for example, by those taken from Ref. [5]. In relativistic heavy ion collisions, it has been observed that the one particle spectra can be described by thermal distribution fairly well [6]. We assume that the sudden (nonadiabatic) approximation is a valid abstraction in describing the freeze-out process in relativistic heavy ion collisions quantum mechanically [7] and that there exists an abrupt freeze-out surface, $\Sigma^\mu(x)$. However, the effects of fluctuations of that freeze-out process will also be considered below.

Consider the following model Hamiltonian,

$$H = H_0 - \frac{1}{2} \int d\mathbf{x} d\mathbf{y} \phi(\mathbf{x}) \delta M^2(\mathbf{x} - \mathbf{y}) \phi(\mathbf{y}), \quad (1)$$

$$H_0 = \frac{1}{2} \int d\mathbf{x} (\dot{\phi}^2 + |\nabla\phi|^2 + m_0^2 \phi^2), \quad (2)$$

where H_0 is the asymptotic Hamiltonian, in the rest frame of matter. The scalar field $\phi(\mathbf{x})$ in the Hamiltonian H corresponds to quasiparticles that propagate with a momentum-dependent medium-modified effective mass, which is related to the vacuum mass, m_0 , via

$$m_*^2(|\mathbf{k}|) = m_0^2 - \delta M^2(|\mathbf{k}|).$$

The mass shift is assumed to be limited to long wavelength collective modes: $\delta M^2(|\mathbf{k}|) \ll m_0^2$ if $|\mathbf{k}| > \Lambda_s$.

We are interested in the invariant single-particle and two-particle momentum distributions:

$$N_1(\mathbf{k}_1) = \omega_{\mathbf{k}_1} \frac{d^3N}{d\mathbf{k}_1} = \omega_{\mathbf{k}_1} \langle a_{\mathbf{k}_1}^\dagger a_{\mathbf{k}_1} \rangle, \quad (3)$$

$$N_2(\mathbf{k}_1, \mathbf{k}_2) = \omega_{\mathbf{k}_1} \omega_{\mathbf{k}_2} \langle a_{\mathbf{k}_1}^\dagger a_{\mathbf{k}_2}^\dagger a_{\mathbf{k}_2} a_{\mathbf{k}_1} \rangle, \quad (4)$$

$$\begin{aligned} \langle a_{\mathbf{k}_1}^\dagger a_{\mathbf{k}_2}^\dagger a_{\mathbf{k}_2} a_{\mathbf{k}_1} \rangle &= \langle a_{\mathbf{k}_1}^\dagger a_{\mathbf{k}_1} \rangle \langle a_{\mathbf{k}_2}^\dagger a_{\mathbf{k}_2} \rangle + \langle a_{\mathbf{k}_1}^\dagger a_{\mathbf{k}_2} \rangle \langle a_{\mathbf{k}_2}^\dagger a_{\mathbf{k}_1} \rangle \\ &\quad + \langle a_{\mathbf{k}_1}^\dagger a_{\mathbf{k}_2}^\dagger \rangle \langle a_{\mathbf{k}_2} a_{\mathbf{k}_1} \rangle, \end{aligned} \quad (5)$$

where $a_{\mathbf{k}}$ is the annihilation operator for the asymptotic quantum with four-momentum $k^\mu = (\omega_{\mathbf{k}}, \mathbf{k})$, $\omega_{\mathbf{k}}^2 = m_0^2 + \mathbf{k}^2$ ($\omega_{\mathbf{k}} > 0$), and the expectation value of an operator \hat{O} is given with the density matrix $\hat{\rho}$ as $\langle \hat{O} \rangle = \text{Tr} \hat{\rho} \hat{O}$. Equation (5) has been derived as a generalization of Wick's theorem for *locally* equilibrated (chaotic) systems in Refs. [3,8,9].

In order to simplify notation, we introduce the chaotic and squeezed amplitudes, defined, respectively, as

$$G_c(1, 2) = \sqrt{\omega_{\mathbf{k}_1} \omega_{\mathbf{k}_2}} \langle a_{\mathbf{k}_1}^\dagger a_{\mathbf{k}_2} \rangle, \quad (6)$$

$$G_s(1, 2) = \sqrt{\omega_{\mathbf{k}_1} \omega_{\mathbf{k}_2}} \langle a_{\mathbf{k}_1} a_{\mathbf{k}_2} \rangle. \quad (7)$$

Usually, the chaotic amplitude, $G_c(1, 2) \equiv G(1, 2)$ is dominant, and carries the Bose-Einstein correlations,

while the squeezed amplitude, $G_s(1, 2)$ vanishes:

$$C_2(\mathbf{k}_1, \mathbf{k}_2) = \frac{N_2(\mathbf{k}_1, \mathbf{k}_2)}{N_1(\mathbf{k}_1)N_1(\mathbf{k}_2)} = 1 + \frac{|G(1, 2)|^2}{G(1, 1)G(2, 2)}. \quad (8)$$

The exact value of the intercept, $C_2(\mathbf{k}, \mathbf{k}) = 2$, is a characteristic signature of a chaotic Bose gas without dynamical 2-body correlations outside the domain of Bose-Einstein condensation [10].

For a hydrodynamic ensemble, Eq. (6) reduces to the special form derived by Makhlin and Sinyukov [3],

$$G(1, 2) = \frac{1}{(2\pi)^3} \int d^3 \Sigma_\mu K_{1,2}^\mu e^{iq_{1,2} \cdot x} n(x, K_{1,2}). \quad (9)$$

Note that the relative and average pair momentum coordinates, $q_{1,2}^0 = \omega_1 - \omega_2$, $\mathbf{q}_{1,2} = \mathbf{k}_1 - \mathbf{k}_2$, and $\mathbf{K}_{1,2} = \frac{1}{2}(\mathbf{k}_1 + \mathbf{k}_2)$, appear in (9). These variables arise naturally whenever the Wigner operator is used, even in totally nonequilibrium semiclassical limits [11]. The validity of the approximations leading to (9) requires the width of $G(1, 2)$ as a function of the relative momentum, $q = |\mathbf{q}_{1,2}|$, to be small. That width is given by $\sim 1/R$, where R is a characteristic dimension of the system. The semiclassical limit corresponds to $KR \gg 1$, where K is $|\mathbf{K}_{1,2}|$. Note that $\sqrt{\omega_{\mathbf{k}_1} \omega_{\mathbf{k}_2}} \sim K_{1,2}^0$ in this case. For $qR < 1$, the second term in (5) describes the minimal quantum interference associated with the indistinguishability of the bosons. The integration over the freeze-out surface, $\Sigma^\mu(x)$, is implemented with the invariant measure $d^3 \Sigma_\mu K_{1,2}^\mu$ that reduces to $K^0 d^3 x$ in the case of a constant freeze-out time.

Results for a homogeneous system.—The terms neglected in (8) involving $G_s(1, 2)$ become nonnegligible when mass shift becomes nonvanishing, i.e., $\delta M^2(|\mathbf{k}|) \neq 0$. Given such a mass shift, the dispersion relation is modified to $\Omega_{\mathbf{k}}^2 = \omega_{\mathbf{k}}^2 - \delta M^2(|\mathbf{k}|)$, where $\Omega_{\mathbf{k}}$ is the frequency of the in-medium mode with momentum \mathbf{k} . The annihilation operator for the in-medium quasiparticle with momentum \mathbf{k} , $b_{\mathbf{k}}$, and that of the asymptotic field, $a_{\mathbf{k}}$, are related by a Bogoliubov transformation [12]:

$$a_{\mathbf{k}_1} = c_{\mathbf{k}_1} b_{\mathbf{k}_1} + s_{-\mathbf{k}_1}^* b_{-\mathbf{k}_1} \equiv C_1 + S_{-1}, \quad (10)$$

where $c_{\mathbf{k}} = \cosh[r_{\mathbf{k}}]$, $s_{\mathbf{k}} = \sinh[r_{\mathbf{k}}]$, and $r_{\mathbf{k}}$ is given by

$$r_{\mathbf{k}} = \frac{1}{2} \log(\omega_{\mathbf{k}}/\Omega_{\mathbf{k}}). \quad (11)$$

We introduce the shorthand, C_1 and S_{-1}^\dagger , to simplify later notation. As is well known, the Bogoliubov transformation is equivalent to a squeezing operation, and so we call $r_{\mathbf{k}}$ the mode dependent squeezing parameter. While it is the a quanta that are observed, it is the b quanta that are thermalized in medium. Thus, we consider the thermal average for a globally thermalized gas of the b quanta, that is homogeneous in volume V :

$$\hat{\rho} = \frac{1}{Z} \exp\left(-\frac{1}{T} \frac{V}{(2\pi)^3} \int d\mathbf{k} \Omega_{\mathbf{k}} b_{\mathbf{k}}^\dagger b_{\mathbf{k}}\right). \quad (12)$$

When this thermal average is applied,

$$G_c(1, 2) = \sqrt{\omega_{\mathbf{k}_1} \omega_{\mathbf{k}_2}} [\langle C_1^\dagger C_2 \rangle + \langle S_{-1} S_{-2}^\dagger \rangle], \quad (13)$$

$$G_s(1, 2) = \sqrt{\omega_{\mathbf{k}_1} \omega_{\mathbf{k}_2}} [\langle S_{-1}^\dagger C_2 \rangle + \langle C_1 S_{-2}^\dagger \rangle]. \quad (14)$$

If this thermal b gas freezes out suddenly at some time at temperature T , the observed single a -particle distribution takes the following form:

$$N_1(\mathbf{k}) = \frac{V}{(2\pi)^3} \omega_{\mathbf{k}} n_1(\mathbf{k}), \quad (15)$$

$$n_1(\mathbf{k}) = |c_{\mathbf{k}}|^2 n_{\mathbf{k}} + |s_{-\mathbf{k}}|^2 (n_{-\mathbf{k}} + 1), \quad (16)$$

$$n_{\mathbf{k}} = \frac{1}{\exp(\Omega_{\mathbf{k}}/T) - 1}. \quad (17)$$

This spectrum includes a squeezed vacuum contribution in addition to the mass modified thermal spectrum. Its shape, however, remains essentially that of the conventional thermal distribution. Although the squeezed vacuum contribution results in a slowly decaying power-law tail of the single-particle spectra, this power-law tail arises for typical values of mass shifts and temperatures at high transverse momentum only, e.g., above $|\mathbf{k}| > 1$ GeV, where the local hydrodynamic description breaks down and correlated perturbative QCD (minijets) starts to play a dominant role. We emphasize that the spectra and correlation functions should be described by the same set of model parameters, and any signal found in the correlation function could be cross checked against its contribution to $N_1(\mathbf{k})$.

In the homogeneous limiting case, $G_c(1, 2) \propto V \delta_{1,2}$, while $G_s(1, 2) \propto V \delta_{1,-2}$. The resulting two-particle correlation function is therefore unity except for the parallel and antiparallel cases:

$$C_2(\mathbf{k}, \mathbf{k}) = 2, \quad (18)$$

$$C_2(\mathbf{k}, -\mathbf{k}) = 1 + \frac{|c_{\mathbf{k}}^* s_{\mathbf{k}} n_{\mathbf{k}} + c_{-\mathbf{k}}^* s_{-\mathbf{k}} (n_{-\mathbf{k}} + 1)|^2}{n_1(\mathbf{k}) n_1(-\mathbf{k})}. \quad (19)$$

The *dynamical* correlation due to the two-mode squeezing associated with mass shifts is therefore *back-to-back* as first pointed out in [12]. The HBT correlation intercept remains 2 for identical momenta. Evaluating Eq. (19) for $T = 140$ MeV, $|\mathbf{k}| = 0, 300$ or 500 MeV for ϕ mesons, as a function of the medium modified m_ϕ^* , one finds back-to-back correlations (BBC) as big as 100–1000 for reasonable values of m_ϕ^* .

It follows from Eq. (19) that the BBC are unbounded from above, $1 \leq C_2(\mathbf{k}, -\mathbf{k}) < \infty$. As $|\mathbf{k}| \rightarrow \infty$, $C_2(\mathbf{k}, -\mathbf{k}) \simeq 1 + 1/|s_{-\mathbf{k}}|^2 \simeq 1 + 1/n_1(\mathbf{k}) \rightarrow \infty$. Hence at large values of \mathbf{k} , particle production is *dominated* by that of back-to-back correlated pairs for any nonvanishing value of in-medium mass shifts. This huge enhancement renders our effect measurable. This prediction has to be contrasted to the results of Ref. [7], which predicted large values for the HBT correlations, $C(\mathbf{k}, \mathbf{k})$ as a consequence

of an incorrect Bogoliubov transformation. Reference [9] discussed a different kind of back-to-back correlation, the particle-antiparticle correlation (PAC), which is based on a classical current formalism, and showed $C_2^{(\text{PAC})}(\mathbf{k}, -\mathbf{k}) \leq 3$. However, their upper limit was shown to be reachable for massless quanta at $|\mathbf{k}| = 0$ only. For massive final-state bosons and for any realistic finite duration of particle emission, the effects found in Ref. [9] are suppressed by at least 2–3 orders of magnitude, resulting in an unmeasurably small effect. In contrast, we consider here an entirely quantum effect, which cannot be obtained in classical current formalism. The *unlimited strength* of the back-to-back correlations of mass-shifted mesons in our case results in a *measurable effect* even if the decay of the medium is not completely sudden and orders of magnitude suppressions reduce its strength.

Suppression by finite emission times.—To describe a more gradual freeze-out, the probability distribution $F(t_i)$ of the decay times t_i is introduced. [The sudden approximation is recovered in the $F(t_i) = \delta(t_i - t_0)$ limiting case.] The time evolution of $a_{\mathbf{k}}(t)$ will be $a_{\mathbf{k}}(t) = a_{\mathbf{k}}(t_i) \exp[-i\omega_{\mathbf{k}}(t - t_i)]$, which leads to

$$C_2(\mathbf{k}, -\mathbf{k}) - 1 = \frac{|c_k^* s_{\mathbf{k}} n_{\mathbf{k}} + c_{-\mathbf{k}}^* s_{-\mathbf{k}} (n_{-\mathbf{k}} + 1)|^2}{n_1(\mathbf{k}) n_1(-\mathbf{k})} \times |\tilde{F}(\omega_{\mathbf{k}} + \omega_{-\mathbf{k}})|^2, \quad (20)$$

where $\tilde{F}(\omega) = \int dt F(t) \exp(-i\omega t)$. For a typical exponential decay, $F(t) = \theta(t - t_0) \Gamma \exp[-\Gamma(t - t_0)]$, the suppression factor is

$$|\tilde{F}(\omega_{\mathbf{k}} + \omega_{-\mathbf{k}})|^2 = 1/[1 + (\omega_{\mathbf{k}} + \omega_{-\mathbf{k}})^2/\Gamma^2].$$

In the adiabatic limit, $\Gamma \rightarrow 0$, this factor suppresses completely the BBC, while in the sudden approximation, $\Gamma \rightarrow \infty$, the full BBC are preserved. For $\delta t = \hbar/\Gamma = 2 \text{ fm}/c$, and for BBC of ϕ mesons with $\omega \sim 1\text{--}2 \text{ GeV}$, one finds that the BBC are suppressed by a factor of $\sim 10^{-3}$. As shown in Fig. 1, the BBC survive this large suppression with a measurable strength, as large as 2–3, the scale of HBT correlations. This emphasizes the enormous strength of squeezed BBC for mass-shifted bosons.

Results for inhomogeneous systems.—Following Ref. [3], we divide the inhomogeneous fluid into independent cells labeled i and assume

$$\hat{\rho} = \prod_i \hat{\rho}_i = \prod_i \frac{1}{Z_i} \exp[-(H_i - \mu_i \tilde{N}_i)/T_i], \quad (21)$$

where $\hat{\rho}_i$ is the local thermal density matrix of cell i at the decoupling time t_i , and \tilde{N}_i is the local number operator of b quanta. The hydrodynamic limit applies only if the cells are small compared to the scale of change of the temperature, flow, and chemical potential fields, however, they are big enough so that one can apply quantization and statistics cell-by-cell. The validity of hydrodynamics in heavy ion collisions is not obvious, but must be checked

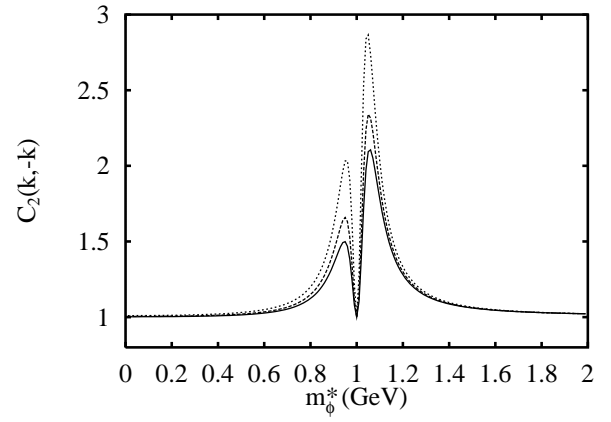


FIG. 1. Dependence of the back-to-back correlations (BBC) on the medium modified ϕ meson mass, m_ϕ^* , for $T = 140 \text{ MeV}$ and $\mu = 0$, where solid, dashed, and dotted lines stand for $|\mathbf{k}| = 0, 300, \text{ and } 500 \text{ MeV}$, respectively. The magnitude of the BBC is large in spite of the finite time suppression factor $1/[1 + (2\delta t \omega_{\mathbf{k}})^2] \sim 0.001$, for $\delta t = 2 \text{ fm}/c$.

in each reaction through the consistency with both single and double inclusive spectra. In each cell, the field can be expanded with creation and annihilation operators, and H_i is diagonalized by a local Bogoliubov transformation. The amplitudes (13) and (14) can be evaluated assuming that the b quanta satisfy locally a generalized Eq. (10):

$$G_c(1, 2) = \frac{1}{(2\pi)^3} \int d^4 \sigma_\mu(x) K_{1,2}^\mu e^{iq_{1,2} \cdot x} \times [|c_{1,2}|^2 n_{1,2} + |s_{-1,-2}|^2 (n_{-1,-2} + 1)], \quad (22)$$

$$G_s(1, 2) = \frac{1}{(2\pi)^3} \int d^4 \sigma_\mu(x) K_{1,2}^\mu e^{2iK_{1,2} \cdot x} \times [s_{-1,2}^* c_{2,-1} n_{-1,2} + c_{1,-2} s_{-2,1}^* (n_{1,-2} + 1)]. \quad (23)$$

Here $d^4 \sigma^\mu(x) = d^3 \Sigma^\mu(x; \tau_f) F(\tau_f) d\tau_f$ is the product of the normal-oriented volume element depending parametrically on τ_f (the freeze-out hypersurface parameter) and the invariant distribution of that parameter $F(\tau_f)$, and $u^\mu(x)$ is the local flow vector at freeze-out. The other variables are defined as follows:

$$\tilde{k}^{(*)\mu}(x) = k^{(*)\mu}(x) - k_\nu^{(*)} u^\mu(x) u^\nu(x), \quad (24)$$

$$\Omega_k(x) = \sqrt{m_0^2 - \tilde{k}^\mu \tilde{k}_\mu - \delta M^2(x, \tilde{k})}, \quad (25)$$

$$\tilde{k} = \sqrt{-\tilde{k}^\mu \tilde{k}_\mu}, \quad (26)$$

$$k_{\pm i}^\mu(x) = u^\mu(x) \omega_{k_i} \mp \tilde{k}_i^\mu(x), \quad (27)$$

$$k_{\pm i}^{*\mu}(x) = u^\mu(x) \Omega_{k_i}(x) \mp \tilde{k}_i^{*\mu}(x), \quad (28)$$

$$n_{i,j}(x) = 1/[\exp\{[K_{i,j}^{*\mu}(x) u_\mu(x) - \mu(x)]/T(x)\} - 1], \quad (29)$$

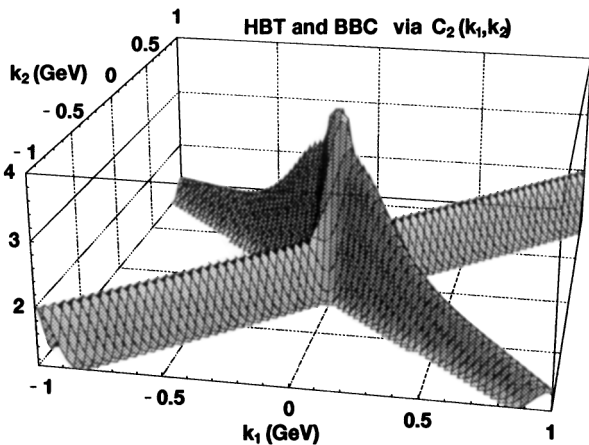


FIG. 2. Schematic illustration of the new kind of correlations for mass-shifted π^0 pairs, assuming $T = 140$ MeV, $G_c \sim \exp[-q_{i2}^2 R_G^2/2]$, $G_s \sim \exp[-2K_{i2}^2 R_G^2]$, with $R_G = 2$ fm. The fall of the BBC for increasing values of $|\mathbf{k}|$ is controlled here by a momentum-dependent effective mass, $m_\pi^* = m_\pi[1 + \exp(-\mathbf{k}^2/\Lambda_s^2)]$, with $\Lambda_s = 325$ MeV in the sudden approximation. Without the Λ_s cutoff, the BBC would increase indefinitely as $|\mathbf{k}| \rightarrow \infty$.

$$r(i, j, x) = \frac{1}{2} \log\left\{\frac{[K_{i,j}^\mu u_\mu(x)]}{[K_{i,j}^{*\nu}(x) u_\nu(x)]}\right\}, \quad (30)$$

$$c_{i,j} = \cosh[r(i, j, x)], \quad s_{i,j} = \sinh[r(i, j, x)], \quad (31)$$

where $i, j = \pm 1, \pm 2$ and the mean and the relative momenta for the $a(b)$ quanta are defined as $K_{i,j}^{(*)\mu}(x) = [k_i^{(*)\mu}(x) + k_j^{(*)\mu}(x)]/2$ and $q_{i,j}^{(*)\mu} = k_i^{(*)\mu} - k_j^{(*)\mu}$, respectively. We assume, of course, that the mass shift and hence squeezing is nonvanishing over only a finite domain of the freeze-out hypersurface. In terms of Eqs. (22) and (23), the following new expressions describe the particle spectra and the correlation function in the presence of local squeezing:

$$N_1(\mathbf{k}_1) = G_c(1, 1), \quad (32)$$

$$C_2(\mathbf{k}_1, \mathbf{k}_2) = 1 + \frac{|G_c(1, 2)|^2}{G_c(1, 1)G_c(2, 2)} + \frac{|G_s(1, 2)|^2}{G_c(1, 1)G_c(2, 2)}.$$

Figure 2 illustrates the novel character of BBC for two identical bosons caused by in-medium mass-modifications, along with the familiar Bose-Einstein or HBT correlations on the diagonal of the $(\mathbf{k}_1, \mathbf{k}_2)$ plane.

Particle-antiparticle pairs.—As the Bogoliubov transformation always mixes particles with antiparticles, the above considerations hold only for particles that are their own antiparticles, e.g., the ϕ meson and π^0 . However, the extension to particle-antiparticle correlations is straightforward. Let “+” label particles, “−” antiparticles if antiparticle is different from particle, let “0” label both particle and antiparticle if they are identical particles. The nontrivial correlations from mass-modification for pairs of (+ +), (+ −), and (0 0) type read as follows:

$$C_2^{++}(\mathbf{k}_1, \mathbf{k}_2) = 1 + \frac{|G_c(1, 2)|^2}{G_c(1, 1)G_c(2, 2)}, \quad (33)$$

$$C_2^{+-}(\mathbf{k}_1, \mathbf{k}_2) = 1 + \frac{|G_s(1, 2)|^2}{G_c(1, 1)G_c(2, 2)}, \quad (34)$$

$$C_2^{00}(\mathbf{k}_1, \mathbf{k}_2) = 1 + \frac{|G_c(1, 2)|^2}{G_c(1, 1)G_c(2, 2)} + \frac{|G_s(1, 2)|^2}{G_c(1, 1)G_c(2, 2)},$$

where we have assumed for simplicity that mass-modifications of particles and antiparticles are the same.

Summary.—We formulated the theory of particle correlations and spectra for bosons with in-medium mass shifts, which predicts the existence of *unlimited* back-to-back correlations of $\phi\phi$, K^+K^- , $\pi^0\pi^0$, and $\pi^+\pi^-$ pairs that could be searched for at CERN SPS and upcoming RHIC BNL heavy ion experiments [14]. These correlations not only survive orders of magnitude finite time suppressions, but may also be utilized to determine the freeze-out time distribution and in-medium hadron modifications.

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