Resonant Two-Body D Decays

Michael Gronau*
Stanford Linear Accelerator Center, Stanford University, Stanford, California 94309
(Received 5 August 1999)

The contribution of a $K^*(1430)$ 0^+ resonance to $D^0 \to K^-\pi^+$ is calculated by applying the soft pion theorem to $D^+ \to K^*\pi^+$, and is found to be about 30% of the measured amplitude and to be larger than the $\Delta I = 3/2$ component of this amplitude. We estimate a 70% contribution to the total amplitude from a higher $K^*(1950)$ resonance. This implies large deviations from factorization in D decay amplitudes, a lifetime difference between D^0 and D^+, and an enhancement of D^0–$\bar{D}^0$ mixing due to SU(3) breaking.

PACS numbers: 13.25.Ft, 11.40.Ha

Hadronic two-body and quasi-two-body weak decays of D mesons, which constitute a sizable fraction of all hadronic D decays [1,2], involve nonperturbative strong interactions. Long distance QCD effects spoil the simplicity of the short distance behavior of weak interactions [3]. Therefore, a simplified approach in which the amplitudes of these processes are given by a factorizable short-distance current-current effective Hamiltonian is not expected to work too well. Various approaches were employed to include long distance effects. The most commonly and very frequently used prescription, motivated by $1/N_c$ arguments [4], is to apply "generalized factorization" [5–8]: The two relevant Wilson coefficients (c_1, c_2), multiplying appropriate four-quark short-distance operators, are replaced by scale-dependent free parameters (a_1, a_2). In this prescription, the magnitudes of isospin amplitudes are calculated from experimentally determined decay constants and form factors, while strong phases (to be determined from experiment) are assigned to these amplitudes to account for final state interactions. In spite of its somewhat *ad hoc* and disputable procedure (evidently final state phases do not occur only in elastic scattering, but are largely due to inelastic processes), this phenomenological treatment seems to work reasonably well in Cabibbo-favored D decays [6,8]. Its failure in the Cabibbo-suppressed $D \to \pi\pi$ and $D \to K\bar{K}$ processes [9] is believed to be associated with inelastic hadronic rescattering.

It was pointed out almost 20 years ago [10] that the observed resonance states in the $K\pi, K\rho/K^*\pi, \pi\pi, \pi\rho$ channels, with masses close to the D mass, may strongly affect final state interactions in D decays [11]. The idea is clear and simple; however, its implementation involves a multichannel rescattering S matrix which cannot be quantified in a model-independent manner [12]. In practice, it is impossible to calculate the effect of s-channel resonance states in two-body D decays without knowing the weak couplings of the D meson to these resonances. If some of these couplings are sufficiently large, the corresponding resonances may have large or even dominating contributions in certain decays. In this case, the apparent success in describing two-body and quasi-two-body decays in terms of generalized factorized amplitudes would be an accident which ought to be further investigated.

Large resonance contributions in D^0 decays could explain the observed $D^+ - D^0$ lifetime difference. Contrary to the D^0, the final states in Cabibbo-favored D^+ decays, made of $\bar{d}s\bar{d}u$, are pure $I = 3/2$ and do not receive such contributions. Also, resonant amplitudes involve large SU(3) breaking in the resonance masses and widths. Consequently, intermediate resonance states are expected to lead to large D^0–$\bar{D}^0$ mixing. We return to these questions in our conclusion.

The purpose of this Letter is to present the first model-independent quantitative study of direct channel resonance contributions to two-body D decays. We will calculate the contribution of $\bar{K}^{*0}(1430)$, a particular excited K meson 0^+ state ($s\bar{d}$ in a P wave), to the Cabibbo-favored $D^0 \to K^-\pi^+$ decay process. In spite of the fact that this resonance peaks at 436 MeV below the D mass, we find its contribution to amount to a sizable fraction, approximately 30%, of the measured $D \to K^-\pi^+$ amplitude. Another 0^+ $K\pi$ resonance, observed around 1900 MeV, is likely to have a larger contribution due its close proximity to the D meson mass. Assuming that its weak coupling to D is approximately equal to that of the resonance at 1430 MeV, we estimate its contribution to be about 70%.

An important step in our analysis is the evaluation of the weak interaction matrix element between a D meson and the 1430 MeV resonance state. For this purpose, we apply the soft pion theorem which relates this amplitude to the measured $I = 3/2\, D^+ \to \bar{K}^{*0}\pi^+$ amplitude [13]. It is crucial in our argument that the final state $\bar{K}^{*0}\pi^+$ is "exotic," in which case the amplitude does not involve a pole term ("surface term") and varies smoothly and only slightly in the soft pion limit.

The 1430 0^+ K^* resonance contribution to $D^0 \to K^-\pi^+$ is given by a Breit-Wigner form

$$A(1430, K^-\pi^+) = \frac{h_1 g}{m^2(D^0) - m^2 + im\Gamma}, \qquad (1)$$

where $h_1 \equiv \langle \bar{K}^{*0}|H_W|D^0\rangle$, $m(D^0) = 1864.6 \pm 0.5$ MeV, $m \equiv m(K^{*0}) = 1429 \pm 6$ MeV, $\Gamma \equiv \Gamma(K^{*0}) = 287 \pm 23$ MeV [14]. The strong $K^{*0}K\pi$ coupling g is obtained from the K^{*0} width [14]:

$$g^2 = \frac{8\pi m^2 \Gamma f}{p_\pi}, \; f \equiv B(\bar{K}^{*0} \to K^- \pi^+) = 0.62 \pm 0.07, \quad p_\pi = 621 \text{ MeV}. \tag{2}$$

The hadronic weak matrix element h_1 is related to the measured $I = 3/2$ amplitude $h_2 \equiv \langle \bar{K}^{*0}\pi^+(q_\pi)|H_W|D^+\rangle$ through the soft pion theorem [15],

$$\lim_{q_\pi \to 0} \langle \bar{K}^{*0}\pi^+(q_\pi)|H_W|D^+\rangle = \frac{-i}{f_\pi} \langle \bar{K}^{*0}|[Q_5^-, H_W]|D^+\rangle, \tag{3}$$

where $f_\pi = 130$ MeV and Q_5^- is the axial charge. Note that the amplitude on the left-hand side involves no pole term since $\bar{K}^{*0}\pi^+$ is an $I = 3/2$ state. [On the other hand, the $I = 1/2\, D \to K^*\pi$ amplitude contains such a pole term from an intermediate $0^-(1460)$ $K\pi$ resonance [14], and, consequently, does not vary smoothly in the soft pion limit.] The $(V - A)(V - A)$ structure of the $\Delta I = 1$ weak Hamiltonian implies

$$[Q_5^-, H_W] = -[Q^-, H_W], \tag{4}$$

and the isospin-lowering operator Q^- obeys $Q^-|D^+\rangle = |D^0\rangle, \langle \bar{K}^{*0}|Q^- = 0$. Neglecting the small variation in the $D^+ \to \bar{K}^{*0}\pi^+$ amplitude as one moves the pion four momentum from its physical value to zero, one finds

$$|h_1| \approx f_\pi |h_2|. \tag{5}$$

The amplitude h_2 is obtained from the measured width $\Gamma(D^+ \to K^{*0}\pi^+)$ [14,16]:

$$h_2^2 = \frac{8\pi m^2(D^+)\Gamma(D^+ \to K^{*0}\pi^+)}{q_\pi}, \; m(D^+) = 1869 \pm 0.5 \text{ MeV}, \; q_\pi = 368 \text{ MeV},$$
$$\Gamma(D^+ \to K^{*0}\pi^+) = \frac{0.023 \pm 0.003}{\tau(D^+)f}, \; \tau(D^+) = 1.051 \pm 0.013 \text{ ps}. \tag{6}$$

Combining (1), (2), (5), and (6), one finds

$$|A(1430, K^-\pi^+)| = (7.85 \pm 0.65) \times 10^{-7} \text{ GeV}. \tag{7}$$

The error contains only experimental errors. The uncertainty due to taking the soft pion limit $q_\pi \to 0$ in the smoothly varying amplitude is assumed to be smaller and is neglected. It would be interesting to study this correction, which could slightly increase or decrease the amplitude.

In order to compare the calculated $K^*(1430)$ resonance contribution to the measured $I = 1/2$ term in $D^0 \to K^-\pi^+$, one expresses all three $D \to \overline{K}\pi$ amplitudes in terms of isospin amplitudes. Using a somewhat different normalization than elsewhere [2,17], we write

$$A(D^0 \to K^-\pi^+) = A_{1/2} + A_{3/2},$$
$$\sqrt{2}A(D^0 \to \bar{K}^0\pi^0) = -A_{1/2} + 2A_{3/2},$$
$$A(D^+ \to \bar{K}^0\pi^+) = 3A_{3/2}. \tag{8}$$

Consequently,

$$|A_{1/2}|^2 = \frac{2}{3}[|A(D^0 \to K^-\pi^+)|^2 + |A(D^0 \to \bar{K}^0\pi^0)|^2 - \frac{1}{3}|A(D^+ \to \bar{K}^0\pi^+)|^2],$$
$$|A_{3/2}|^2 = \frac{1}{9}|A(D^+ \to \bar{K}^0\pi^+)|^2,$$
$$\cos\delta_I = \frac{|A(D^0 \to K^-\pi^+)|^2 - 2|A(D^0 \to \bar{K}^0\pi^0)|^2 + \frac{1}{3}|A(D^+ \to \bar{K}^0\pi^+)|^2}{6|A_{1/2}A_{3/2}|}, \tag{9}$$

where δ_I is the relative phase between isospin amplitudes. One then finds from the experimental rates [14,16] the values [17]

$$|A_{1/2}| = (24.5 \pm 1.2) \times 10^{-7} \text{ GeV},$$
$$|A_{3/2}| = (4.51 \pm 0.22) \times 10^{-7} \text{ GeV},$$
$$\delta_I = (90 \pm 7)^\circ. \tag{10}$$

This and (7) imply

$$\frac{|A(1430, K^-\pi^+)|}{|A_{1/2}|} = 0.32 \pm 0.03. \tag{11}$$

That is, the 1430 MeV $K\pi$ resonance contribution is about 30% of the dominant $I = 1/2$ amplitude in $D \to K\pi$. Its contribution to $D^0 \to K^-\pi^+$ is larger than the $I = 3/2$ component of this amplitude. Note that $A(D^0 \to K^-\pi^+) \approx A_{1/2}$, since $|A_{3/2}|^2 \ll |A_{1/2}|^2$ and $\delta_I \approx 90^\circ$.

In view of this sizable result, which is rather striking for a resonance peaking 436 MeV below the D mass, one raises the question of possibly larger contributions to $D \to K\pi$ from resonances lying closer to the D. One such resonance state, around 1900 MeV [denoted $K^*(1950)$ in [14]], was observed in $K\pi$ scattering [18], with a mass $m' = 1945 \pm 22$ MeV and a width $\Gamma' = 201 \pm 86$ MeV. Somewhat different values, $m' = 1820 \pm 40$ MeV, $\Gamma' = 250 \pm 100$ MeV, were obtained in a K-matrix analysis [19]. Since this resonance lies right at the D mass, its contribution to $D^0 \to K^-\pi^+$ is likely to be larger than that of $K^*(1430)$. In order to calculate this contribution, one must know the matrix element $\langle K^*(1950)|H_W|D\rangle$, for instance by relating it to $\langle K^*(1430)|H_W|D\rangle$. The higher resonance is most likely a radial $n = 2$ excitation of the state at 1430 MeV which is an $n = 1$ P-wave $s\bar{d}$ state. In both amplitudes the local H_W connects a $c\bar{u}$ S-wave state to an $s\bar{d}$ P-wave state which is more spread out. The radially excited $n = 2$ state is slightly less localized than the $n = 1$ state. Consequently, one expects $\langle K^*(1950)|H_W|D\rangle$ to be slightly smaller than $\langle K^*(1430)|H_W|D\rangle$.

Assuming about equal weak amplitudes for the two resonance states, one estimates from (1), (2) [18,19]

$$\frac{|A(1950, K^-\pi^+)|}{|A(1430, K^-\pi^+)|} \approx \sqrt{\frac{\{[m^2(D^0) - m^2]^2 + m^2\Gamma^2\}m'^2\Gamma' f' p_\pi}{\{[m^2(D^0) - m'^2]^2 + m'^2\Gamma'^2\}m^2\Gamma f p'_\pi}} = 2.1\text{–}2.4\,,$$

$$f' \equiv B[K^*(1950) \to K^-\pi^+] = 0.35, \quad p'_\pi = 904 \text{ MeV}, \tag{12}$$

depending somewhat on m' and Γ'. Namely, in the absence of a radial suppression of its weak coupling to D, the resonance around 1900 MeV contributes about 70% of the $I = 1/2\, D \to K\pi$ amplitude. In reality, the contribution may be somewhat (but not very much) smaller.

The combined contribution of the two resonances, at 1430 MeV and in the range 1820–1945 MeV, is considerably larger than the $I = 3/2$ amplitude in $D \to K\pi$. These contributions dominate the $I = 1/2$ amplitude if the two resonances interfere constructively. This is the case if the mass of the second resonance is lower than m_D, as claimed in [19]. This explains the $I = 1/2$ dominance observed in these decays. In view of its important role in D decays, it would be helpful to determine the mass of the higher resonance more precisely.

The above calculations show that direct channel resonances have very large contributions in certain two-body D decays. In a four-quark operator language (or in a diagram language) these contributions are manifestations of annihilation (or W-exchange) amplitudes. A possible phenomenological way of incorporating them in D decays is by employing a diagrammatic language [20], decomposing the $D \to K\pi$ amplitudes, for instance, into a color-favored "tree" amplitude T, a "color-suppressed" amplitude C, and an "exchange" amplitude E. In a more general context this description is based on flavor SU(3) [21]. Here we assume only isospin symmetry. The three amplitudes T, C, and E are an overcomplete set. Only two combinations are required to describe the two isospin amplitudes:

$$\begin{aligned} 3A_{1/2} &= 2T - C + 3E, \\ 3A_{3/2} &= T + C. \end{aligned} \tag{13}$$

The amplitude T may be chosen to be real, C obtains a complex phase from rescattering, while E is given by the sum of two Breit-Wigner forms, representing the two resonances in $D \to K\pi$.

Clearly this scheme, which is more appropriate for the case of large resonance contributions, deviates substantially from the generalized factorization framework [6–8]. In the latter prescription one combines the real amplitudes

$$\begin{aligned} T &= \frac{G_F}{\sqrt{2}}|V_{ud}V_{cs}|a_1 f_\pi(m_D^2 - m_K^2)F^{DK}(m_\pi^2)\,, \\ C &= \frac{G_F}{\sqrt{2}}|V_{ud}V_{cs}|a_2 f_K(m_D^2 - m_\pi^2)F^{D\pi}(m_K^2)\,, \\ E &= 0\,, \end{aligned} \tag{14}$$

into isospin amplitudes (13) which are assigned arbitrary phases. A large nonzero E term, which is required in order to describe resonating amplitudes, modifies the values obtained from the experimental data for a_1 and a_2 relative to their values in the generalized factorization prescription. Although the numerical changes may not be very large, which is the reason for the *apparent success* of the generalized factorization approach, the difference between the physical interpretations of the two descriptions, with and without the E term, is evident.

A fit of D decays to $\bar{K}\pi, \bar{K}\eta, \bar{K}\eta'$ in terms of diagrammatic amplitudes, assuming flavor SU(3) by which T, C, E can be separated, was carried out recently by Rosner [22]. He finds (in units of 10^{-6} GeV) $|T| \simeq 2.7, |C| \simeq 2.0, |E| \simeq 1.6$. A large phase ($-114°$) is found in E/T. The large magnitude of E, comparable to the other two amplitudes, and its sizable phase relative to T, are evidence for the important role of resonances in these decays.

To demonstrate the insensitivity of the naive factorization prescription to large nonfactorizable resonant contributions, we note the following: Extracting a_1 and a_2 from the above values of $|T|$ and $|C|$, using in (14) the values $F^{DK}(m_\pi^2) = 0.77$ [23], $F^{D\pi}(m_\pi^2) = 0.70, f_K =$ 160 MeV, gives $|a_1| = 1.06, |a_2| = 0.64$. These values do not differ by too much from $a_1 = c_1(m_c) = 1.26, a_2 = c_2(m_c) = -0.51$, obtained in the traditional way which disregards resonance contributions [6–8].

While intermediate resonances were shown here to be important in D^0 decays, they do not contribute to Cabibbo-favored D^+ decays, where the final states consisting of $\bar{d}s\bar{d}u$ are pure $I = 3/2$. This can be a qualitative explanation for the measured longer D^+ lifetime. A calculation of the D^+/D^0 lifetime ratio, including resonance contributions in D^0 decay, is a challenging task.

To conclude, we comment on the possible effect of direct channel resonances on $D^0-\bar{D}^0$ mixing. Reasonably small SU(3) breaking in D decays to two pseudoscalar mesons was shown to enhance the mixing by several orders of magnitudes relative to the short distance box diagram contribution [24]. The actual enhancement was argued to be much smaller when summing over all decay modes, if a large energy gap existed between the charmed quark mass and $\Lambda_{\rm QCD}$ [25]. Resonance states close to the D mass violate this assumption. Moreover, resonant contributions lead to particularly large SU(3) breaking between SU(3)-related D decay rates. For instance, mass and width differences between $K\pi$ and $\pi\pi$ resonances show up as large rate differences (when Cabibbo-Kobayashi-Maskawa factors are included), since direct channel resonance amplitudes peak strongly when the resonance mass approaches the D mass. This raises the possibility that SU(3) breaking in resonance amplitudes enhances $D^0-\bar{D}^0$ mixing beyond predictions based on the contributions of a few two-body decays [24]. Such effects were discussed recently in [26], where it was noted that, in the lack of information about weak Hamiltonian matrix elements between a D meson and the resonances, some crude assumptions must be made. The authors assume vacuum saturation for these matrix elements, implying that P-wave 0^+ resonances (for which the wave functions vanish at the origin) do not contribute to $D^0-\bar{D}^0$ mixing. Our model-independent calculation finds a large matrix element for the 0^+ $K\pi$ resonance at 1430, which indicates that the mixing can indeed be larger than estimated in [24]. This interesting possibility deserves further study.

I thank Yuval Grossman, Alex Kagan, David Leith, Alexey Petrov, Dan Pirjol, Jon Rosner, Michael Scadron, and Michael Sokoloff for useful discussions. I am grateful to the SLAC Theory Group for its very kind hospitality. This work was supported in part by the United States–Israel Binational Science Foundation under Research Grant No. 94-00253/3, and by the Department of Energy under Contract No. DE-AC03-76SF00515.

*Permanent address: Physics Department, Technion–Israel Institute of Technology, 32000 Haifa, Israel.

[1] S. Stone, in *Heavy Flavors*, edited by A.J. Buras and H. Lindner (World Scientific, Singapore, 1992), p. 334.
[2] M.S. Witherell, in *Lepton and Photon Interactions: Proceedings of the XVIth International Symposium, Ithaca, New York, 1993*, edited by P. Drell and D. Rubin (AIP, New York, 1994), p. 198.
[3] M.K. Gaillard and B.W. Lee, Phys. Rev. Lett. **33**, 108 (1974); G. Altarelli and L. Maiani, Phys. Lett. B **52**, 351 (1974).
[4] M. Fukugita, T. Inami, N. Sakai, and S. Yazaki, Phys. Lett. B **72**, 237 (1977); A.J. Buras, J.M. Gérard, and R. Rückl, Nucl. Phys. B **268**, 16 (1986); B. Yu Blok and M. Shifman, Sov. J. Nucl. Phys. **45**, 135 (1987).
[5] N. Deshpande, M. Gronau, and D. Sutherland, Phys. Lett. B **90**, 431 (1980).
[6] M. Bauer, B. Stech, and M. Wirbel, Zeit. Phys. C **34**, 103 (1987).
[7] M. Neubert, V. Rieckart, Q.P. Xu, and B. Stech, in *Heavy Flavors* (Ref. [1]), p. 315.
[8] H.Y. Cheng, Phys. Lett. B **335**, 428 (1994).
[9] A.N. Kamal and T.N. Pham, Phys. Rev. D **50**, R1832 (1994).
[10] H.J. Lipkin, Phys. Rev. Lett. **44**, 710 (1980); C. Sorensen, Phys. Rev. D **23**, 2618 (1981); A.N. Kamal and E.D. Cooper, Zeit. Phys. C **8**, 67 (1981).
[11] Resonance effects in Cabibbo-suppressed D decays were discussed more recently by F.E. Close and H.J. Lipkin, Phys. Lett. B **372**, 306 (1996).
[12] A.N. Kamal, Int. J. Mod. Phys. A **7**, 3515 (1992), and references therein.
[13] For different applications of the soft pion theorem to *nonresonant* D decays, see R.E. Karlsen and M.D. Scadron, Phys. Rev. D **45**, 4113 (1992).
[14] Particle Data Group, C. Caso *et al.*, Eur. Phys. J. C **3**, 1 (1998).
[15] S.L. Adler and R. Dashen, *Current Algebras and Applications to Particle Physics* (Benjamin, New York, 1968).
[16] CLEO Collaboration, G. Bonvicini *et al.*, Phys. Rev. Lett. **82**, 4586 (1999).
[17] J.L. Rosner, Phys. Rev. D **60**, 0740299 (1999).
[18] LASS Collaboration, D. Aston *et al.*, Nucl. Phys. B **296**, 493 (1988).
[19] A.V. Anisovich and A.V. Sarantsev, Phys. Lett. B **413**, 137 (1997).
[20] L.L. Chau *et al.*, Phys. Rev. D **43**, 2176 (1991), and references therein.
[21] M. Gronau, O.F. Hernández, D. London, and J.L. Rosner, Phys. Rev. D **50**, 4529 (1994).
[22] J.L. Rosner, University of Chicago Report No. EFI-99-20, hep-ph/9905366.
[23] CLEO Collaboration, A. Bean *et al.*, Phys. Lett. B **317**, 647 (1993).
[24] L. Wolfenstein, Phys. Lett. B **164**, 170 (1985); J. Donoghue, E. Golowich, B. Holstein, and J. Trampetic, Phys. Rev. D **33**, 179 (1986); T.A. Kaeding, Phys. Lett. B **357**, 151 (1995).
[25] H. Georgi, Phys. Lett. B **297**, 353 (1992).
[26] E. Golowich and A.A. Petrov, Phys. Lett. B **427**, 172 (1998).