Electric Dipole Moments and the Mass Scale of New *T*-Violating, *P*-Conserving Interactions

M. J. Ramsey-Musolf
Department of Physics, University of Connecticut, Storrs, Connecticut 06269
and Theory Group, Thomas Jefferson National Accelerator Facility, Newport News, Virginia 23606
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We consider the implications of experimental limits on the permanent electric dipole moment (EDM) of the electron and neutron for possible new time-reversal violating (TV) parity-conserving (PC) interactions. We show that the constraints derived from one-loop contributions to the EDM exceed previously reported two-loop limits by more than an order of magnitude and imply a lower bound on the new TVPC mass scale Λ_{TVPC} of 150 TeV for new TVPC strong interactions. These results imply a value of 10^{-15} or smaller for the ratio of low-energy TVPC matrix elements to those of the residual strong interaction.

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The search for physics beyond the standard model is a topic of ongoing interest for both high-energy colliders as well as low-energy experiments involving atoms and nuclei. Although the standard model (SM) is enormously successful in accounting for a plethora of electroweak data of a broad range in energies, there exist strong theoretical reasons for considering the standard model as an effective theory—derived from some broader framework—applicable to physics below the weak scale. Among the questions to be addressed in considering possible frameworks is the mass scale Λ associated with "new physics." In this respect, experiments in atomic parity violation (APV) provide a powerful probe of new physics scenarios which violate parity. Recently, the Boulder Group has used APV to determine the weak charge Q_W of the cesium atom [1]. The reported value for Q_W, which differs from the standard model prediction by 1.5% (2.5σ), implies the existence of new parity-violating (PV) interactions with mass scales Λ_{PV} on the order of 1 TeV or greater [2].

In this Letter, we consider Λ_{TVPC}, the mass scale associated with possible new time-reversal violating (TV), parity-conserving (PC) interactions. Explicit searches for new TVPC effects at low energies have been carried out using studies of detailed balance in nuclear reactions [3] and neutron transmission experiments [4]. These studies imply that $\alpha_T \lesssim$ few $\times 10^{-3}$, where α_T gives the ratio of typical TVPC nuclear matrix elements to those of the residual strong interaction. The corresponding limits on the TVPC mass scale are weak: $\Lambda_{\mathrm{TVPC}} \gtrsim 10$ GeV. As we argue below, significantly more stringent limits can be inferred indirectly from searches for a permanent electric dipole moment of the electron and neutron.

It has been pointed out in a series of recent papers that the lowest-dimension flavor conserving TVPC interactions have dimension seven [5,6]. Such interactions can generate a permanent electric dipole moment (EDM) of an elementary fermion or its many-body bound states in the presence of a PV standard model radiative correction. It was argued in these studies that the most restrictive limit on new dimension seven TVPC interactions is obtained from a two-loop contribution to the EDM, and the expected magnitude of low-energy TVPC observables was inferred. No attempt was made to derive a lower bound on Λ_{TVPC}. In what follows, we show that there exist additional $d = 7$ operators not considered previously which contribute to the EDM at one-loop order and which generate more stringent lower bounds on Λ_{TVPC} than those derived at two-loop order. We also revisit the analysis of Ref. [5] and argue that it is inconsistent with the separation of scales and systematic power counting which underlies low-energy effective field theory (LEEFT). LEEFT is the appropriate framework for analyzing the nonrenormalizable interactions of interest here. The corresponding scale separation, which is preserved when loop integrals are regulated using dimensional regularization (DR), implies a different Λ_{TVPC} dependence for the EDM than obtained in Ref. [5]. Using an explicit calculation, we obtain the correct scaling of the EDM with Λ_{TVPC} and derive lower bounds on Λ_{TVPC} under naturalness assumptions for the coefficients of the $d = 7$ TVPC operators. We find these bounds are significantly stronger than the scale Λ_{PV} obtained from APV. Our results also imply $\alpha_T \sim 10^{-15}$ or smaller, independent of any naturalness assumptions.

Although the origins of possible new TVPC interactions are not known, it has been shown that they cannot arise via tree-level boson exchange in a renormalizable gauge theory [7]. Hence, one expects them to be generated either by loop effects or nonperturbative short-distance dynamics. Consequently, it is convenient to describe its low-energy consequences using effective Lagrangians. Following Ref. [6], we write

$$\mathcal{L}_{\mathrm{new}} = \mathcal{L}_4 + \frac{1}{\Lambda_{\mathrm{TVPC}}}\mathcal{L}_5 + \frac{1}{\Lambda^2_{\mathrm{TVPC}}}\mathcal{L}_6 + \frac{1}{\Lambda^3_{\mathrm{TVPC}}}\mathcal{L}_7 + \dots, \tag{1}$$

where the subscripts denote operator dimension. The TVPV EDM operator appears in $\mathcal{L}_5$:

$$\mathcal{O}_5 = -\frac{i}{2} C_5^f \bar{\psi} \sigma_{\mu\nu} \gamma_5 \psi F^{\mu\nu}. \tag{2}$$

The TVPC operators considered in Refs. [5,6] appear in $\mathcal{L}_7$:

$$\mathcal{O}_7^{ff'} = C_7^{ff'} \bar{\psi}_f \overleftrightarrow{D}_\mu \gamma_5 \psi_f \bar{\psi}_{f'} \gamma^\mu \gamma_5 \psi_{f'}, \tag{3}$$

$$\mathcal{O}_7^{\gamma g} = C_7^{\gamma g} \bar{\psi} \sigma_{\mu\nu} \lambda^a \psi F^{\mu\lambda} G_\lambda^{a\nu}, \tag{4}$$

where f and f' are distinct fermions and $F_{\mu\nu}$ and $G^a_{\mu\nu}$ are the photon and gluon field strength tensors, respectively. Both $\mathcal{O}_7^{ff'}$ and $\mathcal{O}_7^{\gamma g}$ contribute to C_5^f at two-loop order, although only the contribution of the four-fermion interaction has been computed explicitly previously. In addition, we consider the following TVPC operator appearing in $\mathcal{L}_7$:

$$\mathcal{O}_7^{\gamma Z} = C_7^{\gamma Z} \bar{\psi} \sigma_{\mu\nu} \psi F^{\mu\lambda} Z_\lambda^\nu, \tag{5}$$

where $Z_{\mu\nu}$ is the Z-boson field strength tensor. As we show below, $\mathcal{O}_7^{\gamma Z}$ contributes to C_5^f at one-loop order and yields the strongest bound on Λ_{TVPC}.

Implicit in the LEEFT expansion of $\mathcal{L}_{\text{new}}$ in Eq. (1) is a separation of scales. Short-distance effects ($\mu \gtrsim \Lambda_{\text{TVPC}}$) are subsumed into the renormalized operator coefficients C_n. In the present case, these short-distance effects are not calculable, since the full theory for $\mu \gtrsim \Lambda_{\text{TVPC}}$ is unknown. Long distance contributions ($\mu \lesssim \Lambda_{\text{TVPC}}$) arise from matrix elements of the effective operators $\mathcal{O}_n$ taken between states containing only particles having masses and momenta below Λ_{TVPC}. When these matrix elements involve divergent loops containing the $\mathcal{O}_n$, the use of DR and modified minimal subtraction ($\overline{\text{MS}}$) renormalization preserves the LEEFT separation of scales by protecting loop integrals from high-momentum ($p \gtrsim \Lambda_{\text{TVPC}}$) contributions. Preservation of the scale separation is critical to maintaining the power counting (in $1/\Lambda_{\text{TVPC}}$) associated with Eq. (1). Moreover, it implies that renormalization of $\mathcal{O}_5$ from loops containing the $\mathcal{O}_7$ must scale as $(M/\Lambda_{\text{TVPC}})^2$, where M is the mass of one of the particles dynamically relevant for $\mu \lesssim \Lambda_{\text{TVPC}}$.

We observe that if the parity symmetry broken by the SM is not restored for scales $\mu \gtrsim \Lambda_{\text{TVPC}}$, then the coefficient C_5 must exist at tree level in the LEEFT. Since both the SM PV interaction and the fundamental, but not calculable, interactions which generate the $d = 7$ TVPC interactions exist at such scales, there exists no reason for them not to conspire in generating a nonvanishing C_5. In this case, power counting implies that loops containing the $\mathcal{O}_7$ will generate subdominant contributions to C_5 (see below), so that the EDM limits cannot be used to constrain $d = 7$ TVPC operators. At best, one may employ dimensional arguments involving C_5 to derived lower bounds on Λ_{TVPC}. For example, taking $C_5 = 4\pi\kappa^2 e$ and using the present limits on the electron EDM, one obtains $\Lambda_{\text{TVPC}} \gtrsim 10^{14}\kappa^2$ GeV. This bound is considerably stronger than that obtained by the authors of Ref. [5], who presume, incorrectly, to be able to calculate short-distance effects via loops.

A more interesting scenario occurs when parity symmetry is restored above the weak scale but below Λ_{TVPC} (e.g., in a left-right symmetric scenario). In this case, $C_5 = 0$ at tree level in the LEEFT and becomes nonvanishing only through PV radiative corrections to the $d = 7$ (and higher) TVPC interactions. A conservative lower bound on Λ_{TVPC} can be obtained by considering the SM PV radiative corrections. The leading order contributions to the fermion EDM arising from the TVPC operators in $\mathcal{L}_7$ arise from the diagrams of Figs. 1 and 2. For simplicity, we consider only the effects of $\mathcal{O}_7^{\gamma Z}$ (Fig. 1) and $\mathcal{O}_7^{ff'}$ (Fig. 2). The conclusions obtained from the two-loop gluon-Z graphs will be similar. Following Refs. [5,6], we also restrict our attention to neutral current PV corrections. The diagrams diverge quadratically. Following the standard practice of LEEFT, we regulate the integrals using DR and subtract the pole terms in the $\overline{\text{MS}}$ scheme with the appropriate counterterm in C_5^f. In the case of Fig. 1, the PV effect arises from the axial vector Z-fermion coupling. In the leading-log approximation, the resulting finite contribution to the EDM from $\mathcal{O}_7^{\gamma Z}$ is

$$C_5^f \sim e C_7^{\gamma Z} \left(\frac{M_Z}{\Lambda_{\text{TVPC}}}\right)^2 \left(\frac{1}{s_W c_W}\right) g_A^f \left(\frac{1}{96\pi^2}\right) \ln\frac{M_Z^2}{\mu^2}, \tag{6}$$

where we have dropped terms quadratic in the fermion mass, where g_A^f is the axial vector Zff coupling, and where $s_W = \sin\theta_W$ is the sine of the Weinberg angle.

In the case of the two-loop contribution generated by $\mathcal{O}_7^{ff'}$, the dominant terms arise from the graphs appearing in Fig. 2. All other two-loop contributions containing this operator are suppressed by powers of m_f/M_Z where m_f is a (light) fermion mass. Turning first to the graphs of Fig. 2a, we note that the closed fermion loop containing three insertions is identical to the triangle graph appearing in the Adler-Bell-Jackiw anomaly. Here, the vector current insertions are associated with the neutral gauge bosons and the axial vector insertion arises from $\mathcal{O}_7^{ff'}$. Denoting its nominally linearly divergent amplitude

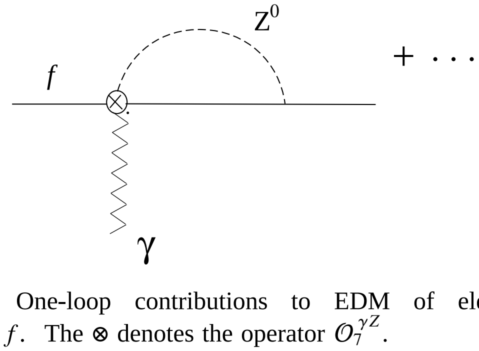

FIG. 1. One-loop contributions to EDM of elementary fermion f. The $\otimes$ denotes the operator $\mathcal{O}_7^{\gamma Z}$.

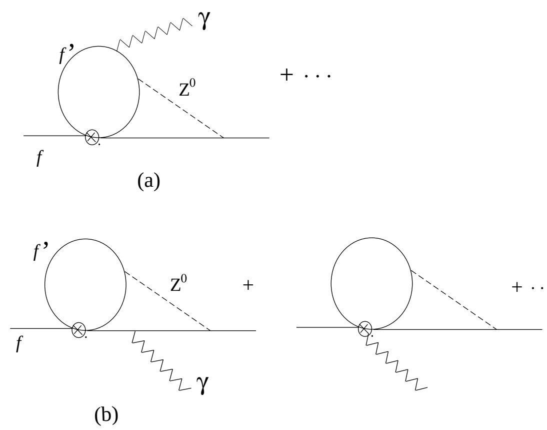

FIG. 2. Two-loop contributions containing $\mathcal{O}_7^{ff'}$ (denoted by ⊗) to the EDM of elementary fermion f. (See text.)

$T^{\mu\lambda\alpha}$, we choose the loop momentum routing to satisfy $q^\mu T_{\mu\lambda\alpha} = 0 = k^\lambda T_{\mu\lambda\alpha}$, where q_μ and k_λ are the photon and Z-boson momenta, respectively. The result is finite. We have verified that our result produces the textbook result for $(q + k)^\alpha T_{\mu\lambda\alpha}$ for $k^2 = q^2 = 0$ [8].

The remaining integration for the two-loop amplitude of Fig. 2a is straightforward. Since the amplitude contains no infrared singularities, we follow Ref. [5] and neglect the $m_{f'}$ dependence of $T^{\mu\lambda\alpha}$. As with the amplitude for Fig. 1, the two-loop amplitude diverges quadratically, and we follow the same subtraction procedure as in the one-loop case. The corresponding, leading-log finite contribution to the EDM is

$$C_5^f \sim -eC_7^{ff'}\left(\frac{M_Z}{\Lambda_{\text{TVPC}}}\right)^2 Q_{f'} g_V^{f'} g_A^f \left(\frac{G_F M_Z^2}{\sqrt{2}}\right) \times \left(\frac{1}{8\pi^2}\right)^2 \ln\frac{M_Z^2}{\mu^2}, \qquad (7)$$

where $g_V^{f'}$ (g_A^f) is the vector (axial vector) coupling of the Z to fermion f' (f) and $Q_{f'}$ is the electric charge of fermion f', with f' denoting the species of fermion in the closed loop.

In the case of Fig. 2b, the closed fermion loop contains the axial vector Z-fermion insertion, while the external fermion couples to the Z through the vector current. The closed fermion loop subgraph diverges quadratically and must be renormalized by the appropriate $\overline{\text{MS}}$ counterterm before the second loop integration is carried out. These graphs receive contributions from both a photon insertion on the line for fermion f (the external fermion) as well as from the EM seagull vertex generated by the covariant derivative in $\mathcal{O}_7^{ff'}$. To leading-log order, the sum of the amplitudes for Fig. 2b gives

$$C_5^f \sim -eC_7^{ff'}\left(\frac{5}{12}\right)\left(\frac{M_Z}{\Lambda_{\text{TVPC}}}\right)^2 Q_f g_V^f g_A^{f'} \left(\frac{G_F M_Z^2}{\sqrt{2}}\right) \times \left(\frac{1}{8\pi^2}\right)^2 \left(\ln\frac{M_Z^2}{\mu^2}\right)^2, \qquad (8)$$

where as before f' is the fermion in the closed loop. The appearance of the $\ln^2$ arises from the presence of two subdivergences in the graphs of Fig. 2b, whereas the closed fermion loop in Fig. 2a is finite. For $\ln(M_Z^2/\mu^2) \sim 1$, the contribution in Eq. (8) will be of the same order as that in Eq. (7). As we argue below, however, we expect $\ln(M_Z^2/\mu^2) \sim 10$, so that the graphs in Fig. 2b generally give a somewhat larger contribution than those of Fig. 2a.

We emphasize that the Λ_{TVPC} dependence appearing in Eqs. (6)–(8) differs substantially from that obtained in Refs. [5,6]. The reason is that the regulator used in the calculation of Ref. [5] does not preserve the LEEFT separation of scales and effectively mixes all orders in the $1/\Lambda_{\text{TVPC}}$ expansion. The two-loop integral containing $\mathcal{O}_7^{ff'}$ was regulated by assuming this interaction arises from the exchange of a hypothetical axial vector boson of mass Λ_{TVPC} having a nonrenormalizable coupling to fermion f. The presence of the axial vector propagator renders the loop integral finite. In effect, this propagator functions as a form factor $(p^2/\Lambda_{\text{TVPC}}^2 - 1)^{-1}$ which contains an infinite power series in $(p/\Lambda_{\text{TVPC}})^2$ with predetermined (model-dependent) coefficients. To be consistent, the effects of an infinite tower of higher-dimension operators in $\mathcal{L}_{\text{new}}$ must also be included in tandem with this form factor, though as a practical matter this was not done in the calculation of Ref. [5]. Furthermore, each operator in the tower will generate an equally important contribution to the EDM, and truncation at $d = 7$ will be unjustified.

This loss of power counting incurred by form factors can be seen in the following example. Consider the tower of operators

$$\mathcal{O}_{7+2n} = C_{7+2n}^{ff'} \bar{\psi}_f \overleftrightarrow{D}_\mu \gamma_5 \psi_f (\partial^2)^n \bar{\psi}_{f'} \gamma^\mu \gamma_5 \psi_{f'}, \qquad (9)$$

where $n = 0, 1, \ldots$. Inserting these operators into the loops of Fig. 2 generates divergent contributions to C_5. Following the spirit of Ref. [5], we may regulate the integrals by including the form factors $(p^2/\Lambda_{\text{TVPC}}^2 - 1)^{-(n+1)}$. Doing so is equivalent to repeating the calculation of Ref. [5] with additional factors of $(p^2/\Lambda_{\text{TVPC}}^2)^n (p^2/\Lambda_{\text{TVPC}}^2 - 1)^{-n} = 1 + \ldots$ in the loop integrals. The first term of order unity generates the same leading-log contribution as given in Ref. [5], while the remaining terms $(+\cdots)$ generate finite contributions for $\Lambda_{\text{TVPC}} \to \infty$. At leading-log order, then, the contribution from the entire tower of operators in Eq. (9) is proportional to $\sum_{n=0}^\infty C_{7+2n}^{ff'}$. A similar conclusion follows if a different form factor is used to cut the integrals off at $p \sim \Lambda_{\text{TVPC}}$; each of the $C_{7+2n}^{ff'}$ will contribute with a similar weight. Consequently, no information about the $d = 7$ interactions alone can be extracted from EDM limits.

In effect, the use of a form factor as in Ref. [5] allows contributions from intermediate states having momenta $p \sim \Lambda_{\text{TVPC}}$, thereby blurring the separation of scales implicit in the low-energy expansion of Eq. (1). Consequently, the renormalization of $\mathcal{O}_5$ due to any $d \geq 7$

operator is dominated by these high-momentum intermediate states—a feature reflected by the absence of the factors $(M_Z/\Lambda_{\mathrm{TVPC}})^2$ in the expressions of Ref. [5]. One therefore has no systematic power counting to justify truncation at $d = 7$ in the expansion of Eq. (1). In contrast, the use of DR and $\overline{\mathrm{MS}}$ subtraction as above avoids these high-mass contributions and maintains the power counting in $\Lambda_{\mathrm{TVPC}}^{-1}$ appropriate to the LEEFT separation of scales.

From the one- and two-loop results of Eqs. (6)–(8) and the experimental limits on EDM's, one may derive conservative lower bounds on Λ_{TVPC}. In the case of $\mathcal{O}_7^{ff'}$ contributions, one must specify the fermion species f' involved in the closed fermion loop. Since the result in Eq. (7) is proportional to $g_V^{f'}$, contributions involving closed, charged lepton loops are suppressed by $g_V^{\ell^-} = -1 + 4\sin^2\theta_W \approx 0.1$ with respect to quark loop contributions. Consequently, we consider only the latter. In this case, the constants relevant to the electron and neutron EDM's are C_7^{eu}, C_7^{ed}, and C_7^{ud}. Moreover, since $|g_V^u|$ and $|g_V^d|$ differ by less than a factor of 2, and since the contributions from $\mathcal{O}_7^{ff'}$ to the EDM go as $1/\Lambda_{\mathrm{TVPC}}^3$, the lower bounds on Λ_{TVPC} from u-quark and d-quark loops differ negligibly. For the results in Eq. (8), the additional factor of $\ln(M_Z^2/\mu^2)$ renders the contribution from Fig. 2b comparable in magnitude to that of Fig. 2a when μ is chosen as discussed below.

In the case of the neutron EDM, we use the quark model to relate d_n to the light quark EDM's. Following the procedures of Ref. [9], we obtain

$$d_n = \frac{1}{\Lambda_{\mathrm{TVPC}}} \int d^3x \left(u^2 + \frac{1}{3}\ell^2\right)\left[\frac{4}{3}C_5^d - \frac{1}{3}C_5^u\right], \tag{10}$$

where u and ℓ are the upper and lower component quark model radial wave functions, respectively. Using the wave function normalization condition $\int d^3x(u^2 + \ell^2) = 1$ and expression for the axial vector charge $\int d^3x(u^2 - \frac{1}{3}\ell^2) = \frac{3}{5}g_A$ we obtain a value of $(1/4)[1 + 6g_A/5] \approx 0.63$ for the integral in Eq. (10).

The use of Eqs. (6)–(8) to derive limits on Λ_{TVPC} requires a choice of renormalization scale μ and assumptions regarding the constants C_7. Since the typical momentum of a quark inside a nucleon is $\sim\Lambda_{\mathrm{QCD}}$, we take $\mu = \Lambda_{\mathrm{QCD}}$ for d_n. The precise choice for this scale does not affect the lower bounds on Λ_{TVPC} appreciably, since it enters only logarithmically. Consequently, we use the same choice for the electron EDM, though a smaller scale is likely more appropriate. The lower bounds on Λ_{TVPC} are similarly rather insensitive to the value of the constants C_7 assuming they fall within a natural range. Following common conventions [2], we write $C_7^{ff'} = 4\pi\kappa^2$, where κ specifies the coupling strength of the new TVPC interaction ($\kappa^2 \sim 1$ for new strong interactions). We also take $C_7^{\gamma Z} \sim egC_7^{ff'} = 4\pi\alpha/\sin\theta_W C_7^{ff'}$, since one would expect $C_7^{\gamma Z}$ to be suppressed with respect to $C_7^{ff'}$ by the gauge couplings associated with the γ and Z. With these conventions, we obtain the following lower limits on Λ_{TVPC} from the experimental result $|d_e| < 4 \times 10^{-27} e$ cm [10]: $\Lambda_{\mathrm{TVPC}} \gtrsim 150\kappa^{2/3}$ TeV from the one-loop graph of Fig. 1 and $\Lambda_{\mathrm{TVPC}} \gtrsim 30\kappa^{2/3}$ TeV from the two-loop graphs of Fig. 2. The corresponding bounds from the neutron EDM are somewhat weaker—given that the experimental limit on $|d_n|$ is an order of magnitude larger than the limit on $|d_e|$ [11].

Finally, we note the implications of the EDM results for low-energy measurements of TVPC observables. As argued on dimensional grounds in Ref. [6], the ratio α_T should scale as $C_7(p/\Lambda_{\mathrm{TVPC}})^3$, where p is a typical momentum involved in low-energy hadronic interactions. The experimental EDM limits constrain the ratio $C_7/\Lambda_{\mathrm{TVPC}}^3 \propto \kappa^2/\Lambda_{\mathrm{TVPC}}^3$ as discussed above. Conservatively taking $p = 1$ GeV$/c$ (low-energy hadronic interactions are typically characterized by momentum transfers of 1 GeV$/c$ or less), our results imply that α_T should be of the order of 10^{-15} or smaller, independent of the choice of κ. Presently, direct TVPC measurements—such as compound nucleus studies of detailed balance and neutron transmission experiments—yield limits of about 10^{-3} for α_T [3].

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