π States in Josephson Junctions between ^{3}He-B

S.-K. Yip
Physics Division, National Center for Theoretical Sciences, P.O. Box 2-131, Hsinchu, Taiwan 300, Republic of China
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We investigate the dependence of the current-phase relationship on the orientation of the order parameter for a pinhole between two ^{3}He-B reservoirs. We show that, due to the internal spin structure of the superfluid, the energy of the junction may have a relative minimum at phase difference equal to π at low temperatures. The dependence of the supercurrent on the direction of an applied magnetic field can be used to verify the present mechanism for the "π states."

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Superfluid ^{3}He is a remarkable state of matter. It exhibits two superfluid phases A and B in zero magnetic field. In particular, in the B phase, the magnitude of the gap is independent of momentum direction $\hat{p}$ despite the fact that the pairing is triplet. It does so by having $S = 1$ pairs with zero spin projections $[\frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle)]$ along direction $\hat{d}(\hat{p})$, with $\hat{d}$ related to $\hat{p}$ by a rotation: $\hat{d}_i(\hat{p}) = R_{i\mu}(\hat{n},\theta)\hat{p}_\mu$. Here $R(\hat{n},\theta)$ is the rotational matrix with rotational axis $\hat{n}$ and angle θ. In the bulk θ is determined by minimizing the dipole interaction energy and is given by $\theta_L \equiv \cos^{-1}(-\frac{1}{4}) \approx 0.58\pi$, usually referred to as the Leggett angle. For any quantization axis perpendicular to $\hat{d}$, there are only $|\uparrow\uparrow\rangle$ and $|\downarrow\downarrow\rangle$ pairs. In the bulk the amplitudes for these pairs are equal, though the phase angles can be different [1].

Being a superfluid, one naturally expects that a Josephson current can flow across weak links between two reservoirs. This was first studied experimentally by Avenel and Varoquaux [2]. Recently the Berkeley Group have studied this Josephson effect again in much more detail; in particular, the current-phase relationships have been mapped out [3]. In their experiment the weak links consist of a large number of small apertures (diameter ≈ 0.1 μm) made on a thin membrane. A particularly interesting feature is the π states which occur when the temperature T is not too close to T_c. For an ordinary junction between two s-wave superconductors, the current-phase relationship is a slanted sine function, with the current I positive for phase difference χ satisfying $0 < \chi < \pi$. The junction energy E, related to the current I by $I = dE/d\chi$, is maximum at $\chi = \pi$. However, the current-phase relationships found for ^{3}He-B [3] are slanted sines only at T not too far from T_c. At lower temperatures, though I is positive for small χ as usual, it turns negative at a phase difference less than π, and I passes through 0 again at $\chi = \pi$ with $dI/d\chi > 0$. Thus π is a relative minimum instead of maximum in the junction free energy. Explanation of this phenomenon is still controversial. There is a suggestion [4] that this π state is not an intrinsic property of a single junction but rather the collective behavior of many pinholes. Earlier theoretical works [5,6] have also predicted the possible existence of π states, but they rely on finite length or the width of the channels.

Later experiments [7] reveal that there are in fact two possible current-phase relationships. They can be achieved from different cooldowns from the normal state. These two states are distinguishable also by the different magnitudes of the critical current. The authors of Refs. [3,7] suggested that the two different states may be the result of two different relative orientations of $\hat{n}$ on the two sides of the junction; such as parallel or antiparallel [8].

The geometry of the individual apertures in the experiment of [3,7] approaches that of pinholes, i.e., apertures that have dimensions much less than the coherence length (≈ 0.1 μm). Although this criterion is not strictly obeyed for the experiment, pinholes are much easier to study theoretically, since in this case all of the self-consistent fields, including superfluid pairing and Fermi liquid effects, as well as the orientation of the $\hat{n}$ vectors, are the same as those near an impenetrable wall [9]. The current-phase relationship of a single pinhole in ^{3}He-B has been investigated by Kurkijärvi [10]. He implicitly assumed that $\hat{n}$ on both sides of the junction are parallel. Ignoring surface depairing he found that $I(\chi)$ is simply that of an s-wave superfluid/superconductor. We shall reconsider the single pinhole junction, but will allow general relative orientations of $\hat{n}^{l,r}$ on the left and right of the junction. In particular, we shall show that one has a natural mechanism for the π state if $\hat{n}^l$ and $\hat{n}^r$ are not parallel. The basic origin of the π state is due to the internal spin structure of the order parameter. For a given momentum direction $\hat{p}$, quasiparticles of different spin projections actually see different effective phase differences across the junction, thus contributing to $I(\chi)$ with a phase shifted from each other. Provided T is not too close to T_c, the resultant current-phase relationship is anomalous and, in general, can have an energy-phase relation that has a relative minimum at $\chi = \pi$. Thus we have a very unusual situation where the spin structure of the order parameter affects the dynamics of the mass flow.

In the absence of a magnetic field $\hat{n}$ thus prefers to lie along (or opposite) the normal near a surface of the membrane separating the two reservoirs. However, under a magnetic field $\vec{H}$, its orientation can be modified [11]. We shall show that $I(\chi)$ can be changed substantially by

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applying a magnetic field of sufficient magnitude ($\gtrsim$50 G) along a general direction. Moreover, we predict that if one performs cool-downs from the normal state under a magnetic field in general directions, more than two current-phase relationships are possible. These predictions can be used to distinguish among the different hypotheses suggested for the π state.

We shall then consider a single pinhole between two reservoirs l and r with $\hat{z}$ being the direction along the interface normal. The current can be calculated along the same lines as in Refs. [9,10]. Without loss in generality we take the phase of the order parameter to be 0 and χ for the left and right reservoirs, respectively. To obtain the current we need to solve the Andreev [12] equation (suitably generalized to triplet pairing) or the quasiclassical Green's function [13]. For simplicity we shall ignore surface depairing. Under this approximation the problem simplifies enormously by the observation that one has only $\vec{S} \cdot \hat{w} = \pm 1$ pairs along any direction $\hat{w}$ perpendicular to $\hat{d}$. $\hat{n}^{l,r}$ can be considered as constants in the present calculations since the size of the pinhole is much less than that of the coherence length, which is in turn much less than the bending length of the $\hat{n}$ vectors. Thus for the given $\hat{p}$, and thus the quasiparticle path through the pinhole, $\hat{d}$ is piecewise constant and equals either $\hat{d}^l$ or $\hat{d}^r$. By choosing the spin-quantization axis along $\hat{w} \equiv \hat{d}^l(\hat{p}) \times \hat{d}^r(\hat{p})$, the gap matrix is finite only for the $\uparrow\uparrow$ and $\downarrow\downarrow$ components. Explicitly, with the triad $(\hat{u}, \hat{v}, \hat{w})_{\hat{p}}$ as the basis vectors for $\hat{d}(\hat{p})$, the gap matrix has the form

$$\mathbf{\Delta} = \Delta_B \begin{pmatrix} -d_u + id_v & 0 \\ 0 & d_u + id_v \end{pmatrix} = \Delta_B \begin{pmatrix} -e^{-i\phi_p} & 0 \\ 0 & e^{i\phi_p} \end{pmatrix}.$$

where ϕ_p is the azimuthal angle of $\hat{d}$ in the (u, v) plane $[\phi_p = \phi_p^l(\phi_p^r)$ for $z < (>)0]$. The Andreev equation or the quasiclassical equation block diagonalized in spin space, resulting in two matrix equations only in particle-hole space. Each of them can be solved as in the s-wave case. For given $\hat{p}$, an $\uparrow$ quasiparticle effectively sees a phase $\pi - \phi_p^l$ for $z < 0$ and $\pi - \phi_p^r + \chi$ for $z > 0$, i.e., an effective phase difference of $\chi - (\phi_p^r - \phi_p^l)$. Similarly, the effective phase difference for a $\downarrow$ quasiparticle is $\chi + (\phi_p^r - \phi_p^l)$. For future convenience we shall define $\chi_{\hat{p}}^s \equiv \phi_p^r - \phi_p^l$. Obviously $\chi_{\hat{p}}^s$ corresponds to the angle between $\hat{d}^r(\hat{p})$ and $\hat{d}^l(\hat{p})$, thus $\chi_{\hat{p}}^s = \cos^{-1}[\hat{d}^r(\hat{p}) \cdot \hat{d}^l(\hat{p})]$. We see that the contribution of the present quasiparticle path to the current is proportional to the sum

$$\sum_{\sigma=\pm 1} \Delta_B \sin\left(\frac{\chi - \sigma\chi_{\hat{p}}^s}{2}\right) \tanh\left[\frac{\Delta_B}{2T} \cos\left(\frac{\chi - \sigma\chi_{\hat{p}}^s}{2}\right)\right]. \tag{1}$$

With this, we can immediately see a mechanism for the formation of the Josephson π states if $\chi_{\hat{p}}^s \neq 0$ (see Fig. 1) [14]. It remains to sum over the contributions from all $\hat{p}$. This can easily be done with the final result

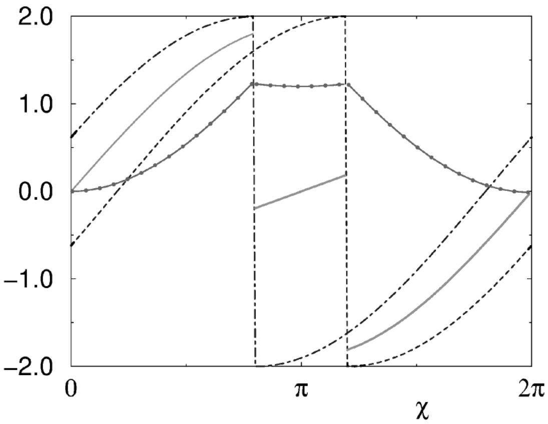

FIG. 1. The basic mechanism for producing a π state. The dashed and dotted-dashed lines are two current-phase relationships shifted from each other by an equal and opposite amount horizontally. They correspond to the two terms in Eq. (1) [shown here at $T = 0$]. The resultant $I(\chi)$, solid line, is anomalous. The corresponding junction energy, being proportional to the integral of I over χ (line decorated with symbols), has a relative minimum at $\chi = \pi$. This mechanism is operative so long as T is not too close to T_c, so that the individual terms in expression (1) are not strictly sinusoidal.

$$I_N = \frac{\pi}{2} A N_f \Delta_B \int \frac{d\Omega_{\hat{p}}}{4\pi} |v_{fz}| \sum_\sigma \sin\left(\frac{\chi_{\hat{p}}^\sigma}{2}\right) \times \tanh\left[\frac{\Delta_B}{2T} \cos\left(\frac{\chi_{\hat{p}}^\sigma}{2}\right)\right], \tag{2}$$

where $\chi_{\hat{p}}^\sigma \equiv \chi - \sigma\chi_{\hat{p}}^s$, A is the area of the pinhole, N_f is the density of states per spin at the fermi energy, and v_f is the Fermi velocity. To complete the calculation we need only to find $\chi_{\hat{p}}^s$ for given $\hat{n}^{l,r}$, with $\hat{d}_i^l(\hat{p}) = R_{i\mu}(\hat{n}^l, \theta_L)\hat{p}_\mu$ and similarly for $l \to r$.

In the absence of any other orientation effects such as magnetic field, $\hat{n}^{l,r}$ are expected to lie along $\pm\hat{z}$ [11]. If $\hat{n}^l = \hat{n}^r$ then obviously $\hat{d}^l \cdot \hat{d}^r = 1$, hence $\chi_{\hat{p}}^s = 0$ for all $\hat{p}$. Our result for the current reduces to that of an s-wave superconductor [10]. For ease of later comparison we plot the current-phase relationship in Fig. 2. This is the configuration with a higher critical current. Now consider

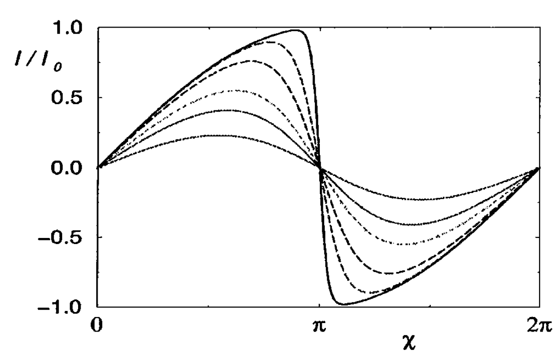

FIG. 2. Current-phase relationships for $\hat{n}^{l,r}$ both along the normal and parallel to each other. The temperatures are, for decreasing critical current, $T/T_c = 0.1, 0.3, 0.5, 0.7, 0.8$, and 0.9. $I_0 \equiv \pi A N_f v_f \Delta_B/2$.

$\hat{n}^l = -\hat{n}^r$. Parametrizing $\hat{p}$ by its azimuthal and polar angles $(\alpha_{\hat{p}}, \beta_{\hat{p}})$, the corresponding angles of $\hat{d}^{l,r}$ are obviously $(\alpha_{\hat{p}} \pm \theta_L, \beta_{\hat{p}})$, One then easily gets $\hat{d}^l \cdot \hat{d}^r = 1 - 2\sin^2\beta_{\hat{p}}\sin^2\theta_L = 1 - \frac{15}{8}\sin^2\beta_{\hat{p}}$. The resultant $I(\chi)$ is as shown in Fig. 3. This is the configuration with a lower critical current. Except for T very close to T_c where $I(\chi)$ is basically sinusoidal with a slight tilt, π states are evident. These current-phase relationships resemble closely those obtained experimentally [7] for the "low critical current" state.

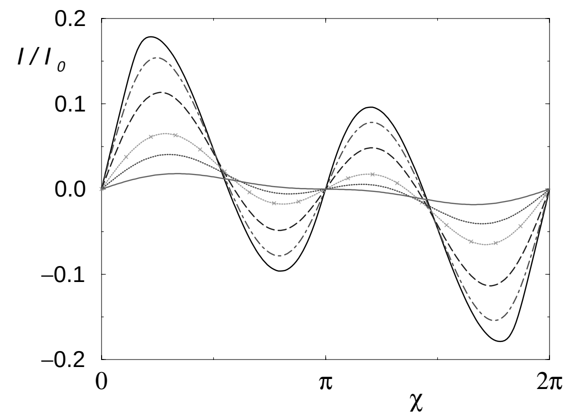

FIG. 3. Current-phase relationships for $\hat{n}^{l,r}$ both along the normal and opposite to each other. The temperatures are, for decreasing critical current, $T/T_c = 0.1$, 0.3, 0.5, 0.7, 0.8, and 0.9.

It is, however, known that the orientation of $\hat{n}$ can be affected by a magnetic field. The relevant terms in surface free energies are proportional to $-(\hat{z} \cdot \hat{n})^2$ and $-(\vec{H}_i R_{i\mu} \hat{z}_\mu)^2$ [11]. The first term prefers $\hat{n} = \pm\hat{z}$. However, for sufficiently large magnetic field ($\gtrsim$50 G) the second term dominates, which tends to orient $\hat{n}$ in a direction such that the rotation $R(\hat{n}, \theta_L)$ rotates $\pm\hat{z}$ to $\hat{H}$. For simplicity in the following we shall consider this case only. Without loss of generality we let $\hat{H}$ be in the y-z plane and denote its angle with the $\hat{z}$ axis by θ_H $(0 < \theta_H < \pi)$. Then the possible orientations of $\hat{n}$ are

$$\left(-\sqrt{\frac{3}{5}}\,\frac{\sin\theta_H}{1+\cos\theta_H}, \pm\frac{\sin\theta_H}{1+\cos\theta_H}\sqrt{\frac{1+4\cos\theta_H}{5}}, \pm\sqrt{\frac{1+4\cos\theta_H}{5}}\right)$$
$$\left(+\sqrt{\frac{3}{5}}\,\frac{\sin\theta_H}{1-\cos\theta_H}, \mp\frac{\sin\theta_H}{1-\cos\theta_H}\sqrt{\frac{1-4\cos\theta_H}{5}}, \pm\sqrt{\frac{1-4\cos\theta_H}{5}}\right).$$

We shall use the letters A, B, C, and D to denote the different orientations of $\hat{n}$. A and B exist only for $\cos\theta_H > -\frac{1}{4}$, whereas C and D exist only for $\cos\theta_H < \frac{1}{4}$. At $\theta_H = 0$ the configurations A and B correspond to $\hat{n} = \pm\hat{z}$, respectively. A and B rotate $\hat{z}$ to $\hat{H}$, whereas C and D rotate $-\hat{z}$ to $\hat{H}$. For the junction we shall denote the order parameter configurations on the two sides by the order pairs AB, etc., where the letters indicate $\hat{n}^{l,r}$, respectively. Thus, if $0 < \theta_H < 0.42\pi$, then the allowed configurations of the junction are AA, AB, BA, and BB, whereas for $0.42\pi < \theta_H < 0.52\pi$ sixteen configurations are allowed. The current-phase relationships of some of these configurations are identical by symmetry considerations alone. A rotation of π around the $\hat{x}$ axis affects the transformations $A \leftrightarrow B$, $C \leftrightarrow D$ and simultaneously interchanges l and r. Thus, e.g., $I_{AC}(\chi) = -I_{DB}(-\chi) = I_{DB}(\chi)$. In our present approximation of no surface pair breaking, $I(\chi)$ depends only on $\hat{d}^l(\hat{p}) \cdot \hat{d}^r(\hat{p})$. Thus we have $I_{AA}(\chi) = I_{BB}(\chi)$ and $I_{AB}(\chi) = I_{BA}(\chi)$, etc. We are thus left with five independent current-phase relationships for AA $(= BB = CC = DD)$ [15] AB $(= BA)$, AC $(= CA = BD = DB)$, AD $(= CB = BC = DA)$, and CD $(= DC)$. It turns out there is also a rather nontrivial relation between AC and AD in that, for any given $\hat{p}$ in AC, there exists another $\hat{p}'$ related by rotation about $\hat{z}$ such that $\hat{d}^A(\hat{p}) \cdot \hat{d}^C(\hat{p}) = \hat{d}^A(\hat{p}') \cdot \hat{d}^D(\hat{p}')$ (Appendix A). Thus $I_{AC}(\chi) = I_{AD}(\chi)$. To summarize, for a given θ_H with $0 < \theta_H < 0.42\pi$ there are two possible $I(\chi)$ (we shall label them AA and AB); for $0.42\pi < \theta_H < 0.58\pi$ there are four possible $I(\chi)$'s. We denote these by AA, AB, AC, and CD. Results for $0.58\pi < \theta_H < \pi$ can be obtained from those of $0 < \theta_H < 0.42\pi$ by $\theta_H \to \pi - \theta_H$.

As an example, we show in Fig. 4 the current-phase relationships for these configurations at $\theta_H = 0.45\pi$, $T = 0.1T_c$. I_{AA} is the same as that between two s-wave superconductors since $\hat{n}^{l,r}$ are parallel. AB has an energy minimum at $\chi = 0$ but also a relative minimum at $\chi = \pi$. AC has a rather conventional shape, except for the phase shift by π, thus having its energy minimum at $\chi = \pi$ rather than 0. CD has a very weak relative minimum at π.

At $\theta_H = \pi/2$ the system possesses an extra symmetry: a rotation of π around the $\hat{z}$ axis induces the transformations $A \leftrightarrow C$ and $B \leftrightarrow D$. Thus at $\theta_H = \pi/2$, I_{AB}

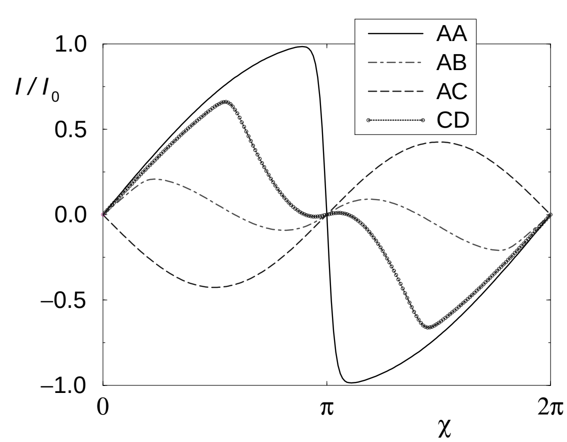

FIG. 4. Current-phase relationships for $\theta_H = 0.45\pi$. $T = 0.1T_c$.

and I_{CD} merge and only three possible $I(\chi)$ remain (not shown).

The above provides a possible test of the hypothesis that the π state is the result of relative $\hat{n}^{l,r}$ orientations. If one performs cool-down from the normal state in a magnetic field (of suitable orientation), in principle, all configurations are reachable. There should be two possible $I(\chi)$ for $0 < \theta_H < 0.42\pi$ but at least four for $0.42\pi < \theta_H < 0.58\pi$ (except $\theta_H = \pi/2$).

Next we consider the evolution of $I(\chi)$ as a function of θ_H for a given configuration. We shall, in particular, discuss the case where θ_H is increased from 0. For AA, $\hat{n}^{l,r}$ remain parallel and thus $I(\chi)$ is independent of θ_H. The result for AB is as shown in Fig. 5. Note, as mentioned, $\theta_H = 0$ corresponds to $\hat{n}$ antiparallel and along $\pm\hat{z}$. As θ_H increases from 0, initially the critical current varies in a nonmonotonic way (Appendix B); $I(\chi)$ then evolves towards the s-wave result, reaching it at $\theta_H \approx 0.58\pi$ where $\hat{n}^{l,r}$ become parallel and equal to $-\hat{x}$.

For completeness we also mention the θ_H dependences of other configurations. $I_{AC}(\chi)$ is θ_H independent under the present approximation (Appendix A). $I_{CD}(\chi)$ can be obtained from I_{AB} by substituting $\theta_H \rightarrow \pi - \theta_H$.

Lastly, we comment on the magnitude of the supercurrent. At zero magnetic field the theory here expects π states only for $\hat{n}$ antiparallel (and along the normal to the interface). We therefore must identify the result of Refs. [3,7] as being due to this configuration. The value of I_0 defined in the caption of Fig. 2 corresponds to a mass current of $\sim 2 \times 10^{-7}$ g/sec. Thus the critical current of the "low critical current state" at, e.g., $T = 0.5T_c$ is expected to be about 2.2×10^{-8} g/sec according to Fig. 3, which is in reasonable agreement with the experimental value ($\sim 1.5 \times 10^{-8}$ g/sec) [3]. Anyway, the prediction of strong $\hat{H}$ dependence of $I(\chi)$ here can serve as an important test of the hypothesis that the π states observed are due to the internal spin structure of the superfluid.

Appendix A.—In this appendix we discuss $\hat{d}^r(\hat{p}) \cdot \hat{d}^l(\hat{p})$ for configurations AC and AD. Obviously, this dot product is given by $\hat{p}_i \mathcal{R}_{i\mu} \hat{p}_\mu$, where $\mathcal{R}$ is the rotational matrix formed by $[R(\hat{n}^r, \theta_L)]^{-1} R(\hat{n}^l, \theta_L)$. $\mathcal{R}$ is thus the combined action of $R(\hat{n}^l, \theta_L)$ and then the inverse of $R(\hat{n}^r, \theta_L)$. We observe that since $R(\hat{n}^l, \theta_L)$ rotates $\hat{z}$ to $\hat{H}$ whereas $R(\hat{n}^r, \theta_L)$ rotates $-\hat{z}$ to $\hat{H}$, $\mathcal{R}$ rotates $\hat{z}$ to $-\hat{z}$. From the expressions for $R(\hat{n}^{r,l}, \theta_L)$ one can easily evaluate the rotational angle Θ associated with $\mathcal{R}$ by the formula $\mathrm{Tr}\mathcal{R} = (1 + 2\cos\Theta)$. After some straightforward algebra, one can obtain $\Theta = \pi$. Thus $\mathcal{R}$ must correspond to a rotation of π around an axis in the x-y plane. $\mathcal{R}$ for AC and AD differ only by the direction of this axis. Thus for any given $\hat{p}$ for AC, there exists another $\hat{p}'$ related to $\hat{p}$ by a rotation around $\hat{z}$ such that $\hat{d}^A(\hat{p}) \cdot \hat{d}^C(\hat{p}) = \hat{d}^A(\hat{p}') \cdot \hat{d}^D(\hat{p}')$ and thus their $I(\chi)$ are identical. Also, θ_H affects only the direction of the rotational axis for $\mathcal{R}$. Thus $I(\chi)$ for these configurations are independent of $\hat{H}$.

Appendix B.—For the configuration AB, using considerations along the same lines as in Appendix A, we see that $\mathcal{R}$ now leaves $\hat{z}$ invariant and thus $\mathcal{R}$ must correspond to a rotation around $\hat{z}$ itself. Θ can be evaluated to be $\cos^{-1}[\frac{1}{2}(\frac{1-2\cos\theta_H}{1+\cos\theta_H})^2 - 1]$. The quantity in the brackets and thus Θ is nonmonotonic in θ_H. This nonmonotonic behavior of Θ results in the nonmonotonic dependence of the critical current on θ_H, as shown in Fig. 5.

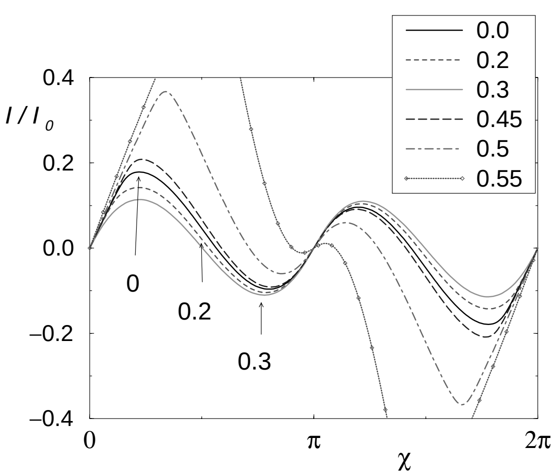

FIG. 5. Current-phase relationships for the AB configuration as a function of θ_H/π given in the legend. $T = 0.1T_c$.

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