Bi-state Superfluid ³He Weak Links and the Stability of Josephson π States

A. Marchenkov, R. W. Simmonds, S. Backhaus, A. Loshak, J. C. Davis, and R. E. Packard

Physics Department, University of California, Berkeley, California 94720

We have discovered new features of the current-phase relation, $I(\phi)$, in superfluid ³He weak links. Firstly, we find that at any given temperature there are two distinct $I(\phi)$ functions that characterize the weak link. Secondly, both functions continuously develop an unusual form that ultimately leads to the previously reported π state. The observed form of $I(\phi)$ has recently been predicted for unconventional quantum fluids such as ³He, high-T_c superconductors, and Bose-Einstein condensates. The two distinct states are likely to originate from the textural degree of freedom in superfluid ³He.

At temperatures not too far below the superfluid transition ($T_c = 0.93$ mK), the current-phase relation for ³He aperture-array weak links is given by the dc-Josephson expression, $I(\phi) = I_c \sin\phi$, where I_c is referred to as the link's critical current [1,2]. In this regime experiments have revealed several dynamic phenomena which are analogous to known effects in superconducting weak links: Josephson oscillations [3], plasma mode motion [4], homodyne current spikes [5], and Shapiro steps [6]. The experiments also revealed a new phenomenon: the existence at lower temperatures of a metastable state, characterized by a π phase difference across the link [7]. The appearance of such a state implies that the system's energy has a local minimum at $\phi_{\min} = \pi$, a feature not yet observed experimentally for conventional superconductors.

Several theories have been advanced to explain the physics that gives rise to such minimum [8–10]. To test these theories it is necessary to determine the $I(\phi)$ function of the weak links with better precision. In order to improve on our earlier measurements we have built an elaborate acoustic shield surrounding our experiment in order to decrease noise caused by acoustical disturbances in the environment. Pressure driven fluctuations have been reduced (relative to our earlier work) by at least 1 order of magnitude, and important properties heretofore hidden, particularly near $\phi = \pi$, have now become visible.

We use a double diaphragm cell (which is described more completely elsewhere [3]) to study the dynamics of the weak links. This cell consists of a flat cylindrical container bounded on the top and bottom with metallized flexible plastic membranes of known stiffness. The weak link (a square array of 65×65 holes, each nominally 100 nm diameter separated by 3 μm, in a 50 nm thick SiN wall [11]) is mounted in the lower membrane. The top membrane's position is monitored with a SQUID-based displacement transducer [12]. This cell is immersed into liquid ³He-B (at zero ambient pressure) which also fills the inside region.

By applying a step voltage between the bottom diaphragm and the adjacent rigid electrode we create an initial pressure head across the weak link which eventually relaxes due to dissipation. A sufficiently large impulse sends the system into the Josephson oscillation regime; i.e., the phase is winding through angles greater than 2π and the average pressure head $\langle P \rangle \neq 0$. As the energy dissipates, the frequency of these oscillations falls, and the system eventually reaches a point where $\langle P \rangle = 0$. The phase motion becomes bounded around a (quasi)equilibrium value $\phi_{\min}$. Experimentally, we find that the oscillations occur around two stable (or metastable) points: "pendulum" oscillations [1,4] are characterized by $\phi_{\min} = 0$; at sufficiently low temperatures, "π-state" oscillations [7] are possible with $\phi_{\min} = \pi$. For the first few cycles in both oscillatory states, we observe anharmonic motion, where the ϕ oscillation amplitude is large and the frequency is small. After a few cycles, as the amplitude decreases, the frequency increases, reaching an almost amplitude-independent limit [4].

The top membrane serves as a pressure gauge which records the instantaneous pressure across the array weak link: $P(t) = \lambda x(t)$, where $x(t)$ is the membrane's displacement from the equilibrium position. The calibration constant λ is obtained by measuring the mass-current oscillation frequency, f_J, and using the relation,

$$f_J = \frac{2m_3}{\rho h} P, \qquad (1)$$

where $2m_3$ is the mass of a ³He Cooper pair, ρ is the liquid density, and h is Planck's constant. Equation (1) was experimentally verified in our previous work [3].

Integration of the pressure gives the phase difference through the Josephson-Anderson phase evolution equation,

$$\phi(t) = -\int_0^t \frac{2m_3}{\rho \hbar} P(\tau)\, d\tau. \qquad (2)$$

The velocity of that same membrane gives the mass current,

$$I(t) = \rho A \dot{x}(t), \qquad (3)$$

where A is the membrane's area. Elimination of the common time variable yields the function $I(\phi)$. In this work, in order to extract the $I(\phi)$ function we take several Josephson oscillation cycles and pendulum oscillation cycles just around the transition point.

We have cooled through the transition temperature T_c on many occasions. We find that the weak link is characterized by two distinct $I(\phi)$ functions. Figure 1 shows the temperature dependence of these two functions. Both functions are sinelike near T_c but with maximum currents at a given temperature differing by more than a factor of 2. We refer to the higher critical current state as the **H** state and that with lower critical current as the **L** state. We find that the particular state, **H** or **L**, is set during the cooling passage through T_c. Once set, it is robust, and $I(\phi)$ is a reproducible function of temperature; no amount of acoustic excitation can change the state once set. However, rapid cooling through T_c and high levels of acoustic noise in the cell at the time of the superfluid transition favor the **L** state over **H**.

On four occasions we have performed detailed measurements of the complete family of $I(\phi)$ curves over the temperature interval $0.43 < T/T_c < 1$. Twice the curves followed the **H** set and twice they followed the **L** set. To confirm that there are only two states we performed a series of additional thermal cycles through T_c, and in each case measured the current-phase relation at a few reference temperatures. At each selected temperature, the $I(\phi)$ curves were always one of the two shown in Fig. 1. A convenient way to compare the families of $I(\phi)$ curves is to plot the temperature dependence of some characteris-

tic parameters of these curves. For example, the resonant frequency around a particular local stability point $\phi_{\min}$ is given by

$$f^2_{\phi_{\min}} = B\left(\frac{\partial I}{\partial \phi}\right)_{\phi_{\min}}, \qquad (4)$$

where B is a constant. Therefore the frequencies, f_o and f_π, although not revealing the complete $I(\phi)$ curve, give the slope of the $I(\phi)$ function at the stability points: $\phi_{\min} = 0$ and $\phi_{\min} = \pi$. (For both **H** and **L** states, oscillations around π stabilize only at temperatures substantially below T_c.)

Figure 2 shows these measured frequencies from ten different thermal cool-down cycles. The measured points for each oscillation mode cluster on two distinct temperature-dependent curves. At any given temperature, the value of f_0 for the **H** state lies above that for the **L** state, because the slope $(dI/d\phi)_0$ is greater for **H** than for **L**, as seen in Fig. 1. In contrast, at a given temperature the frequency of the π oscillation, f_π, is lower for the **H** state, because $(dI/d\phi)_\pi$ is smaller for **H** than for **L**. The fact that all ten thermal cycles result in all the data falling on two families of curves supports the observation that there are only two distinct states of the system.

Prior to our observations, no theory predicted both the bistability of $I(\phi)$ and the existence of the metastable π-state [13–18]. However it is well known that, due to the tensor nature of the order parameter, the state of a sample of ^{3}He-B can be characterized by a spatially varying vector field, referred to as an **n** texture, which should be oriented normal to a solid boundary [19]. It
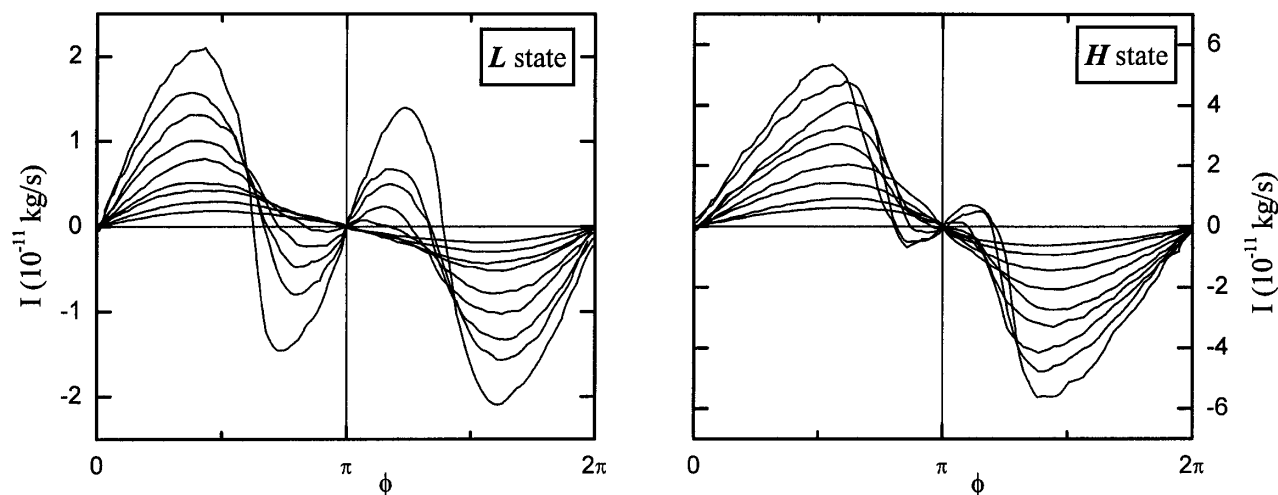

FIG. 1. The two measured current-phase functions of a superfluid ^{3}He-B array weak link. For both **L** and **H** states the current-phase relations are plotted in the temperature interval $0.450 < T/T_c < 0.850$ with the step of $(0.050 \pm 0.005)T/T_c$. The mass current through the weak link increases as the temperature is lowered. At temperatures close to T_c, the current-phase relations can be well fit with the sine function. At lower temperatures, the simplest three-parameter model involves a linear kinetic inductance in a series with a weak link having two terms: $I_1 \sin\phi$ and $I_2 \sin 2\phi$. Although this model can fit the data well, the fits are insensitive to the parameters chosen and, at the very lowest attainable temperatures, become inadequate. At a given temperature, the critical current [the maximum of the $I(\phi)$] is given by the expression $I_c^L = 6.8 \times 10^{-11}(1 - T/T_c)^2$ [kg/s] for the **L** state, and by $I_c^H = 1.3 \times 10^{-10}(1 - T/T_c)^{1.5}$ [kg/s] for the **H** state. The values for the critical current reported previously [1] were several times larger due to a calibration error in that earlier work.
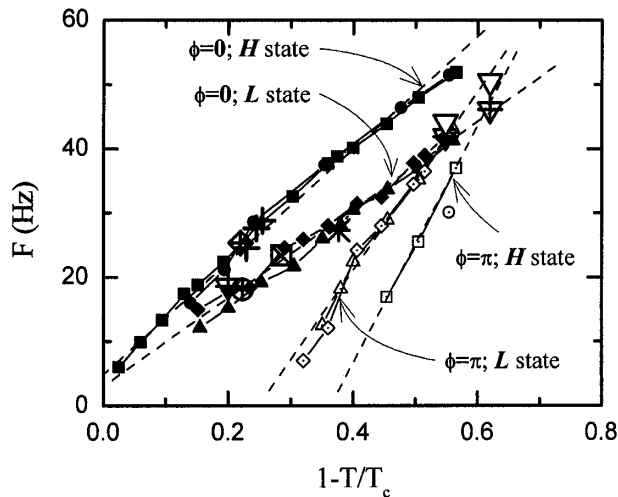
FIG. 2.   Two states of the ^{3}He weak link as seen through oscillation frequencies at $\phi = 0$ and $\phi = \pi$. We measured the complete temperature dependence of f_0 (solid symbols connected with solid line) and f_π (open symbols connected with solid line) for four passages through T_c. For six other passages we measured f_0 (large +-center symbols) and $f_\pi(\nabla)$ at several selected temperatures. Open symbols for the π-state oscillations correspond to the same solid or +-center symbols for the $\phi = 0$ oscillations. Dashed lines are just guides for the eye.

is tempting to suggest that the two states, **H** and **L**, are associated with parallel or antiparallel **n** textures on the opposite sides of the SiN membrane which contains the weak link. Indeed, previous calculations have shown that these two configurations could give rise to different $I(\phi)$ functions [15], although those calculations predict curves different from those reported here.

The existence of two stable (or metastable) states may be the strongest indication that it is the tensor nature of the order parameter in ^{3}He-B that is responsible for these new phenomena. In support of this conjecture, we find some lack of reproducibility in the observed $I(\phi)$ at temperatures close to T_c. It is known [20] that the **n** texture becomes "soft" for temperature above about $0.8T_c$, i.e., bending of the texture due to some local disturbances does not result in significant increase in the free energy of the superfluid. The observed irreproducibility of $I(\phi)$ at these elevated temperatures may be another indication that local **n**-textures play an important role in $I(\phi)$.

It can be seen in Fig. 1 that $I(\phi)$ in both **H** and **L** states becomes distorted from perfect sine functions and develop higher order harmonic terms (e.g., $\sin 2\phi$) when the temperature is lowered. The usual distortions from a $\sin\phi$ function for conventional superconducting weak links involve a steepening of the slope at π. That is simply the effect of series kinetic inductance, an inherent aspect of microbridge weak links [21]. In stark contrast, we observe that the distortion of the ^{3}He $I(\phi)$ is a *decrease* in steepness at π followed by a reverse of sign of the slope. In the standard analysis of conventional superconducting weak links and tunnel junctions no such feature appears [21].

The observed distortion leads to a metastable state around $\phi = \pi$. The energy, $E(\phi)$, associated with the flow of mass current through a weak link is given by

$$E(\phi) = \frac{\hbar}{2m_3} \int_0^\phi I(\phi') \, d\phi'. \qquad (5)$$

Figure 3 shows this energy computed by integrating the $I(\phi)$ curves shown in Fig. 1. There is a clear local minimum in the energy at π which continuously develops at lower temperatures for both the **L** and **H** states. The figure shows that the **L** state develops a minimum below about $0.65T_c$, and the **H** state exhibits a similar feature below about $0.55T_c$. These are the temperatures where we experimentally observe the onset of stability of mass current oscillations with the phase difference of π; these oscillations are the direct manifestation of a metastable energy state.

In the paper first reporting the existence of the π state [7], the data focused on the lowest achievable temperatures, near $0.3T_c$, the regime where the state was most stable. There the $I(\phi)$ function appeared to be discontinuous. An important conclusion of the present acoustically shielded experiment is that even at temperatures as high as $0.65T_c$, a minimum in the energy occurs at $\phi = \pi$ without the appearance of a hysteretic $I(\phi)$ relation. Therefore, theories of the π state that require hysteretic phase slippage [10] are probably not an appropriate description of this system.

As with the existence of two states, the origin of the π state may lie in the tensor nature of the ^{3}He order parameter. Current-phase relations of the shape similar to those shown in Fig. 1 were recently predicted for Josephson junctions in high-T_c superconductors [22,23], assuming that the tensor order parameter is purely $d_{x^2-y^2}$. Alternatively, recent theoretical studies of weakly coupled Bose-Einstein condensate (BEC) samples [24], extended [8,9] to superfluid ^{3}He, have shown a continuously developing minimum in energy of the condensate at $\phi = \pi$ as the temperature decreases. This feature is a consequence of a fixed number of particles in the system. Our experiment contains 10^{20} particles which is about 10^{14} times greater than a typical BEC. Therefore the applicability of these calculations to our system has to be verified. Although a BEC system with fixed particle number may not be directly analogous to ^{3}He, we note that the diagrams shown in Fig. 3 look similar to those in Ref. [8].

To summarize, we have discovered that a superfluid ^{3}He-B aperture-array weak link can exist in two distinct states. We emphasize that there are apparently two possible order parameter configurations that can give rise to distinct $I(\phi)$ functions. For temperatures above $0.45T_c$, both of the $I(\phi)$ functions are single valued and continuous. Both give rise to an evolving energy minimum at the π phase difference as the temperature decreases. These observations shed new light on the properties of weak links in complex order parameter
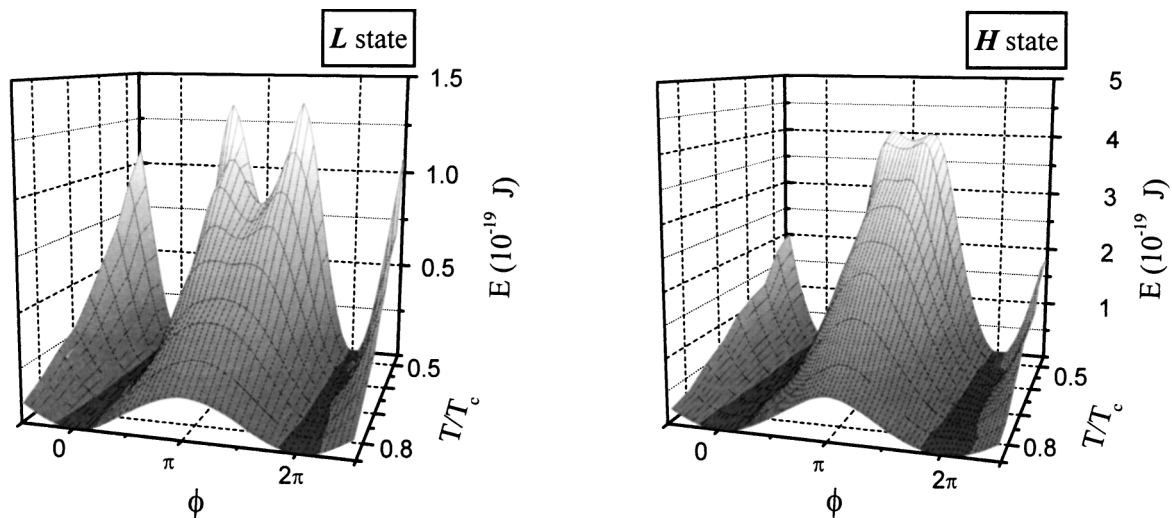
FIG. 3.   Energy of the superfluid ^{3}He-B weak link calculated from the current-phase relations shown in Fig. 1. Since the **L** state develops a relatively deep minimum at $\phi = \pi$, it is not surprising that we find that in the **L** configuration at low temperatures almost any applied impulsive disturbance can result in a transient that shows π-state oscillation. By contrast, in the **H** configuration the π state usually can be reached only by careful resonant excitation about $\phi = 0$ and this is the state whose properties were reported in our earlier work [7]. It is interesting to point out that the depths of the π minima are large compared to thermal energy, $k_b T$, but small compared to macroscopic energy. It therefore seems plausible that before our acoustic shield was installed, environmental vibration would be sufficient to excite the system out of the π state, thus explaining why this feature was originally rather elusive.

superfluids. We hope that the results reported here will stimulate theories to explain these intriguing questions.

Note added.—After submitting this paper, we learned about new theoretical investigations of superfluid ^{3}He weak links [25–27] which are based on the results of this experiment and those of Ref. [1]. These new theories provide mechanisms for π states and/or bistability. We thank the authors for sending us the preprints.

[1] S. Backhaus, S. Pereverzev, A. Loshak, J. C. Davis, and R. E. Packard, Science **278**, 1435 (1997).

[2] Single-aperture weak links have also been shown to be consistent with the dc-Josephson relation: O. Avenel and E. Varoquaux, Phys. Rev. Lett. **60**, 416 (1988).

[3] S. V. Pereverzev, A. Loshak, S. Backhaus, J. C. Davis, and R. E. Packard, Nature (London) **388**, 449 (1997).

[4] A. Marchenkov, R. W. Simmonds, A. Loshak, J. C. Davis, and R. E. Packard, e-print, xxx.lanl.gov/abs/cond-mat/9910120.

[5] R. W. Simmonds *et al.,* Phys. Rev. Lett. **81**, 1247 (1998).

[6] R. W. Simmonds, A. Marchenkov, J. C. Davis, and R. E. Packard (to be published).

[7] S. Backhaus *et al.,* Nature (London) **392**, 687 (1998).

[8] N. Hatakenaka, J. Phys. Soc. Jpn. **67**, 3672 (1998).

[9] N. Hatakenaka, Phys. Rev. Lett. **81**, 3753 (1998).

[10] O. Avenel, Y. Mukharsky, and E. Varoquaux, Nature (London) **397**, 484 (1999).

[11] Both the aperture's diameter and the weak link length are of the order of the temperature-dependent coherence length. At our standard condition (zero ambient pressure), the superfluid coherence length, ξ, is given by $\xi = 65/\sqrt{1 - T/T_c}$ [nm].

[12] H. J. Paik, J. Appl. Phys. **47**, 1168 (1976).

[13] H. Moninen and L. Tewordt, J. Low Temp. Phys. **62**, 277 (1986).

[14] J. R. Hook, Jpn. J. Appl. Phys. **26**, 159 (1987).

[15] E. V. Thuneberg, Europhys. Lett. **7**, 441 (1988).

[16] J. Kurkijärvi, Phys. Rev. B **38**, 11 184 (1988).

[17] S. Ullah and A. L. Fetter, Phys. Rev. B **39**, 4186 (1989).

[18] E. V. Thuneberg, J. Kurkijärvi, and J. A. Sauls, Physica (Amsterdam) **165B/166B**, 755 (1990).

[19] D. Vollhardt and P. Wölfle, *The Superfluid Phases of Helium 3* (Taylor & Francis, London, 1990), Chap. 7.

[20] A. I. Ahonen, M. Krusius, and M. A. Paalanen, J. Low Temp. Phys. **25**, 421 (1976).

[21] K. K. Likharev, Rev. Mod. Phys. **51**, 101 (1979).

[22] S.-K. Yip, J. Low Temp. Phys. **109**, 547 (1997).

[23] M. Fogelström, S. Yip, and J. Kurkijärvi, Physica (Amsterdam) **294C**, 289 (1998).

[24] A. Smerzi, S. Fantoni, S. Giovanazzi, and S. R. Shenoy, Phys. Rev. Lett. **79**, 4950 (1997).

[25] A. Smerzi, S. Raghavan, S. Fantoni, and S. R. Shenoy (to be published).

[26] S.-K. Yip, Phys. Rev. Lett. **83**, 3864 (1999).

[27] J. K. Viljas and E. V. Thuneberg, Phys. Rev. Lett. **83**, 3868 (1999).