High Rayleigh Number Turbulent Convection in a Gas near the Gas-Liquid Critical Point

Shay Ashkenazi and Victor Steinberg
Department of Physics of Complex Systems, The Weizmann Institute of Science, 76100 Rehovot, Israel

(Received 16 March 1999)

SF_6 in the vicinity of its critical point was used to study turbulent convection up to exceptionally high Rayleigh numbers, Ra (up to 5×10^{14}) and to verify for the first time the generalized scaling laws for the heat transport and the large scale circulation velocity as a function of Ra and the Prandtl number, Pr, in a very wide range of these parameters. Both scaling laws obtained are consistent with theoretical predictions by Shraiman and Siggia [Phys. Rev. A **42**, 3650 (1990)].

PACS numbers: 47.27.Te, 44.25.+f, 64.70.Fx

Turbulent convection has recently attracted a lot of attention due to the possibility to tune and control the relevant parameters with unbeatable precision. An important issue in our understanding of the convective turbulence is an experimental test of theoretical predictions on the Prandtl number, Pr, dependence of global characteristics, such as nondimensional heat transport, Nu(Ra, Pr), and Reynolds number of the large scale circulation flow, Re(Ra, Pr). The difficulty to measure the Pr dependence in turbulent convection arises from the fact that in conventional fluids it cannot be varied substantially but only by changing the fluid which is not always a simple task. The verification of these scaling relations will be a crucial test of the theory.

In this Letter we present results on the Pr dependence of global properties of the flow, namely, global heat transport, characterized by Nu, and large scale circulation velocity, characterized by Re.

There are two possibilities to reach the high Ra number convection regime: either to increase temperature difference across the cell or to vary physical parameters, which appear in the expression for $\mathrm{Ra} = \frac{g\alpha\Delta T L^3}{\kappa\nu}$, where ΔT is the temperature difference across the cell, g is the gravitational constant, α is the thermal expansion, and κ and ν are the thermal diffusivity and kinematic viscosity, respectively. The latter possibility was used successfully in compressed gases (He and SF_6) at an almost constant value of Pr at about 1 [1–4]. Another system in which this method can be used is a gas near its gas-liquid critical point (CP). It was realized a long time ago that heat transport is enhanced dramatically near T_c [5]. But it was only recently shown experimentally that the critical temperature difference for the convection onset decreases drastically by approaching T_c due to strong variations of thermodynamic and kinetic properties of a gas in the vicinity of CP [6]. Singular behavior of the thermodynamic and kinetic properties of the fluid near T_c provides the opportunity both to reach extremal values of the control parameter Ra and to scan Pr over an extremely wide range. All these features make the system unique in this respect [7,8].

However, the most exciting aspect of the system is the possibility to perform laser Doppler velocimetry (LDV) measurements that we recently demonstrated [7,8]. Small temperature differences used to reach high Ra lead to rather small fluctuations of the refraction index in the flow. It gives us the possibility to use a standard LDV technique. We discovered that the existence of the critical density fluctuations provided us the possibility to perform LDV measurements of the velocity field in a rather wide range of closeness to the CP between 3×10^{-4} and 10^{-2} in reduced temperature $\tau = (\overline{T} - T_c)/T_c$, where $\overline{T}$ is the mean cell temperature [7,8].

However, there are limitations and new features which should be taken into account and studied. A strong dependence of gas properties on the closeness to T_c manifests itself in nonuniform distribution of density in a gravitational field and variation of coefficients in the Navier-Stokes equation with temperature and density. The former, the well-known gravity effect [9], can be significant even at $\tau \approx 10^{-3}$ and 10 cm height cell: the density difference across the cell reaches 1% that leads to rather significant variations in the fluid properties. Fortunately, a temperature gradient compensates the gravity when heating from below [5], and strong convection reduces the characteristic size of nonuniformity to one of about the boundary layer height. This leads to a reduction of the density nonuniformity to a tolerant level much below 0.1%.

In the Boussinesq approximation fluid properties are assumed to be constant despite the temperature gradient across the cell, except for the buoyancy term. According to [10] the degree of deviation from the Boussinesq approximation is adequately described by the ratio of the temperature drop across the top boundary layer to the temperature drop across the bottom boundary layer. According to our estimates measurements at Ra up to 10^{15} can be done while non-Boussinesq effects are still relatively small. In the data presented the temperature drop ratio was above 0.7. So small deviations from the scaling behavior were observed at highest Pr and Ra.

There are other aspects related to the proximity to T_c such as compressibility (besides adiabatic gradient) and breaking down of the hydrodynamic description due to the macroscopical size of thermodynamical fluctuations. Simple estimates show that both these factors do not play

0031-9007/99/83(18)/3641(4)$15.00

any significant role in the range of parameters under study [7].

Contrary to the compressed gas convection [2–4], here one cannot cover the wide range of Ra for one value of Pr. On the other hand, the data presented cover the range of Ra from 10^{10} at low values of Pr far from T_c and $\Delta T = 12$ mK up to the largest attainable in a laboratory $\mathrm{Ra} = 5 \times 10^{14}$ at high values of Pr close to T_c. The whole range of the reduced mean temperature was $1.6 \times 10^{-2} > \tau > 2 \times 10^{-4}$. And finally, as a result of large compressibility and relatively large cell height the adiabatic temperature gradient was observed [11]. This effect will be discussed below.

The experiment we present here was done with a high purity gas SF_6 (99.998%) in the vicinity of T_c and at the critical density ($\rho_c = 730$ kg/m^3). This fluid was chosen due to the relatively low critical temperature ($T_c = 318.73$ K) and pressure ($P_c = 37.7$ bars) and well-known thermodynamic and kinetic properties far away and in the vicinity of CP. This gas was widely used to study the equilibrium critical phenomena. The cell is a box of a cross section 76×76 mm^2 formed by 4 mm Plexiglass walls, which are sandwiched between a Ni-plated mirror-polished copper bottom plate and a 19 mm thick sapphire top plate of $L = 105$ mm apart.

The cell is placed inside the pressure vessel with two side thick plastic windows to withstand the pressure difference up to 100 bars. So the cell had optical accesses from above through the sapphire window and from the sides. The pressure vessel was placed inside a water bath which was stabilized with rms of temperature fluctuations at the level of 0.4 mK. The gas pressure was continuously measured with 1 mbar resolution by the absolute pressure gauge. Together with a calibrated 100 Ω platinum resistor thermometer they provide us the thermodynamic scale to define the critical parameters of the fluid and then to use a parametric equation of state developed recently for SF_6 [12].

We should point out here that it is not obvious at all that the critical parameters, defined at equilibrium conditions, can be used for a strongly nonequilibrium state to define closeness to CP. On the contrary, as suggested by theory [13] and experiment [14], turbulent flow in a binary mixture near its consolute CP can suppress the critical concentration fluctuations. At Re comparable with that achieved in our experiment, Pine *et al.* [14] observed the critical temperature depression up to 50 mK. We used the enhancement of the heat transport near CP as an indicator of the closeness to CP. The critical pressure and temperature obtained by this procedure agree within ±10 mK with those defined independently by light scattering at equilibrium conditions [7].

The heat transport measurements were conducted at the fixed heat flux and temperature of the top sapphire plate, while the bottom temperature was measured by a calibrated thermistor epoxied in it. Local temperature measurements in a gas were made by three 125 μm thermistors suspended on glass fibers in the interior of the cell (one at the center, and two about half way from the wall). Local vertical component velocity measurements at about $L/4$ from the bottom plate were conducted by using LDV on the critical density fluctuations [7,8]. Shadowgraph visualization was used mostly to get qualitative information about structures and characteristic time and length scales in the flow mostly of the top and bottom boundary layers.

From the measurements of the heat transport and, particularly, of the velocity we realized that at a small but finite temperature difference, much larger than defined by the critical Ra for convection onset, there exists a mechanically stable state. This temperature difference ΔT_{ad} is defined by the adiabatic temperature gradient for the convection onset $\Delta T_{\mathrm{ad}}/L = gT\alpha C_p^{-1}$ [11], where C_p is the heat capacity at constant pressure. As follows from thermodynamics and critical divergences of thermodynamical parameters as T approaches T_c, the adiabatic temperature gradient saturates at a finite value $\Delta T_{\mathrm{ad}}/L = g\rho(\partial P/\partial T)_v^{-1}$, which for SF_6 and $L = 10.5$ cm gives $\Delta T_{\mathrm{ad}} = 9.5$ mK for $\tau < 10^{-2}$. So with available temperature stability and resolution it can be measured. Then, as shown in [5], Ra should be modified as $\mathrm{Ra} = \frac{gL^3\alpha(\Delta T - \Delta T_{\mathrm{ad}})}{\nu\kappa}$.

The most sensitive probe to detect the convection onset in our case was the local velocity measurements. The results of the mean vertical velocity V_m and rms of vertical velocity fluctuations V_s measurements at $\tau = 8 \times 10^{-4}$ are presented in Fig. 1. Both signals show a clear transition at $\Delta T_{\mathrm{ad}} = 9.5 \pm 0.5$ mK, which agrees well with the theoretical value [7,15]. These measurements were done for several values of the reduced temperature in the range $8 \times 10^{-3} > \tau > 2 \times 10^{-4}$. It was found that $\Delta T_{\mathrm{ad}}(\tau)$ is constant and independent of τ in agreement with the theory [5]. This value of ΔT_{ad} was used further to correct the Ra values. As a result of this correction for each Pr both Nu vs Ra (Fig. 2) and V_s vs Ra (Fig. 3) dependences show scaling laws in a rather wide range of Ra [7].

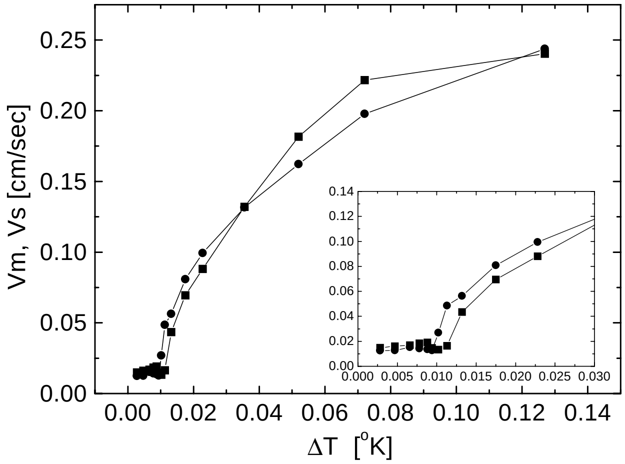

FIG. 1. LDV measurements of the mean V_m (squares) and rms V_s (circles) vertical velocity component. Inset: the same data with higher resolution.

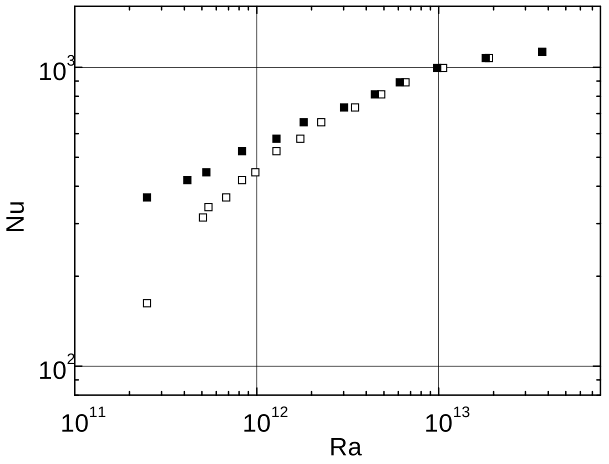

FIG. 2. Nu vs Ra: open squares (uncorrected) and full squares (corrected) data at Pr = 27.

The heat transport measurements were done in the range $1.6 \times 10^{-2} > \tau > 2 \times 10^{-4}$. In order to keep Pr constant, τ was kept constant, while ΔT changed during the measurements. The range of τ was limited from the higher temperature end by mechanical stability of constructing materials (mostly Plexiglass) and from the lower end by the temperature stabilization of the system. Heat transport measurements were also conducted far away from CP at $P = 20$ bars, $\overline{T} = 303$ K, and the density $\rho = 0.18$ g/cm^3, which corresponds to Pr = 0.9, and at $P = 50$ bars, $\overline{T} = 323$ K, and $\rho = 1.07$ g/cm^3, which corresponds to Pr = 1.5. The data far away from CP cover the range of Ra between 10^9 and 5×10^{12} with the temperature differences across the cell from about 0.1 till 10 K. By making all appropriate corrections for heat losses through the lateral walls, gas, and insulation outside the cell, and for a temperature drop across the top sapphire plate, we found that the data on the heat transport far away from CP are in good agreement with a 2/7 law for $10^9 \leq \text{Ra} \leq 5 \times 10^{12}$ [2]. These data combined with the heat transport measurements in the same cell in the vicinity of CP for higher values of Pr can be scaled and presented in a power law form: $\text{Nu} = 0.22\text{Ra}^{0.3\pm0.03}\text{Pr}^{-0.2\pm0.04}$ (Fig. 4). Just a few points at the highest Pr and Ra deviate from this scaling due to the non-Boussinesq effect. This scaling is consistent with the predictions of Ref. [16].

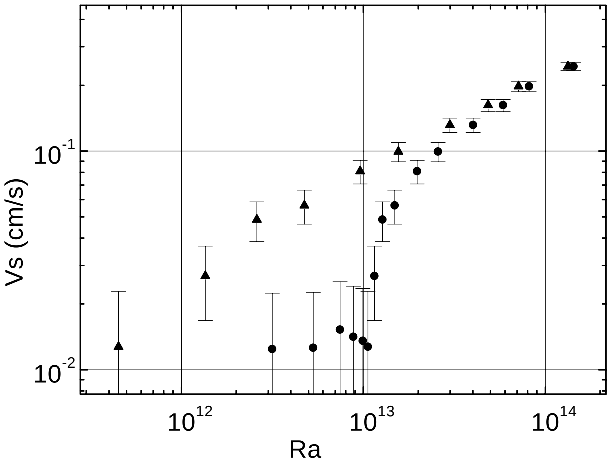

FIG. 3. V_s vs Ra: circles (uncorrected) and triangles (corrected) data at Pr = 93.

Recently the Pr dependence of Nu was the subject of experimental studies [17,18]. In Ref. [17] by comparison of various experimental data on the heat transfer in water and mercury the authors came to the conclusion that Nu should increase with Pr in odds with the theories and our results presented. These data were taken on different convection cells of different aspect ratios. On the other hand, rather accurate measurements of Nu vs Ra in water with Pr between about 4 and 7 show a slight decrease of Nu but with a much smaller exponent than predicted [18]. So the problem is still controversial, particularly in light of the recent theoretical reconsiderations of the scaling laws [19].

We visualized a structure of the large scale flow by using the shadowgraph technique through the top and side windows. The large diameter with narrow focal width imaging optics enabled us to visualize narrow slices across the cell and scan it in both vertical and horizontal directions. The flow was recorded at various Ra and Pr. This visualization confirmed the picture of up- and down-going circulation jet flow along the cell diagonal, which forms the top and bottom turbulent boundary layers. The bulk of the cell appears homogeneous. The turbulent character of the flow at the boundary layers is seen as a rapid and "violent" horizontal motions [7,8].

The circulation frequency, which corresponds to the travel time of a fluid element passing through one cycle of the large eddy, is observed as a peak in the power spectra for both the velocity and temperature fluctuations (see Fig. 5 inset). We also measured directly by LDV the large scale circulation velocity. However, the best results were obtained by extracting the peak frequency from the velocity power spectra at various Ra and Pr which can be presented in a power law form: $\text{Re} = 2.6\text{Ra}^{0.43\pm0.02}\text{Pr}^{-0.75\pm0.02}$ (Fig. 5). Here $\text{Re} = 4f_pL^2/\nu$,

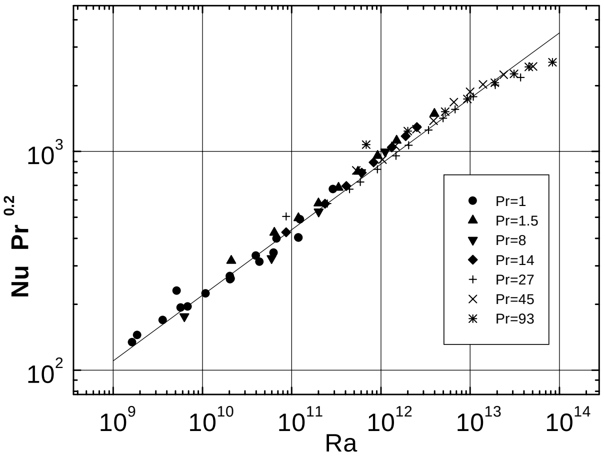

FIG. 4. Nu vs Ra for different Pr.

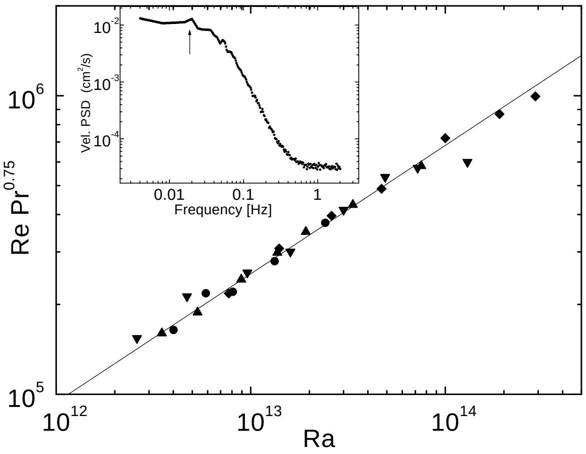

FIG. 5. Re of the large scale circulation vs Ra for different Pr: circles–27; up-triangles–45; down-triangles–93, diamonds–190. Inset: power spectra density of vertical velocity fluctuations at Ra $= 3 \times 10^{12}$ at Pr $= 93$.

and f_p is the peak frequency. The data from the low frequency peak in the temperature spectra far away and close to CP and from LDV measurements are consistent with this scaling law. We point out that Re reaches values up to 10^5 at the highest values of Ra. The dependence of Re on Pr has almost the same exponent (within the measurement uncertainty) as one found recently in convection in helium for the much more narrow range of Pr [20] and agrees rather well with the theoretical prediction $5/7$ in Ref. [16]. However, the scaling of Re with Ra differs significantly and is close to $3/7$ rather than to 0.5 [20].

We can also verify consistency of the exponents of both scaling laws obtained by using one of them, e.g., for Re, and the exact relation for the dissipation in a bulk turbulent regime $\mathrm{PrRa}(\mathrm{Nu} - 1) \sim \mathrm{Re}^3\mathrm{Pr}^3$ [16]. The resulting scaling relation $\mathrm{Nu} \sim \mathrm{Ra}^{0.29}\mathrm{Pr}^{-0.25}$ is consistent with that found experimentally. Together with the unified scaling law in the whole range of Ra and Pr under studies for both transport mechanisms it suggests that a single mechanism is responsible for these scaling laws. Moreover, our data did not indicate any signature of the transition to the asymptotic regime in the heat transport, discovered recently at relatively low Pr and Ra $> 10^{11}$. There are two possible explanations to this fact: either at higher Pr the transition occurs at higher Ra or the scatter in the data for different Pr does not allow us to observe the transition.

This work was partially supported by the Minerva Foundation and the Minerva Center for Nonlinear Physics of Complex Systems. V. S. is grateful for support of the Alexander von Humboldt Foundation.

◆

[1] F. Heslot, B. Castaing, and A. Libchaber, Phys. Rev. A **36**, 5870 (1987).
[2] B. Castaing *et al.*, J. Fluid. Mech. **204**, 1 (1989).
[3] X. Wu, L. Kadanoff, A. Libchaber, and M. Sano, Phys. Rev. Lett. **64**, 2140 (1990).
[4] A. Tilgner, A. Belmonte, and A. Libchaber, Phys. Rev. E **47**, R2253 (1993).
[5] M. Gitterman and V. Steinberg, High Temp. (USSR) **8**, 754 (1970); J. Appl. Math. Mech. (USSR) **34**, 305 (1971).
[6] M. Assenheimer and V. Steinberg, Phys. Rev. Lett. **70**, 3888 (1993).
[7] Sh. Ashkenazi, Ph.D. thesis, Weizmann Institute of Science, Rehovot, Israel, 1997.
[8] Sh. Ashkenazi and V. Steinberg (to be published).
[9] M.R. Moldover, J.V. Sengers, R.W. Gammon, and R.J. Hocken, Rev. Mod. Phys. **51**, 79 (1979).
[10] X.Z. Wu and A. Libchaber, Phys. Rev. A **43**, 2833 (1991).
[11] L.D. Landau and E.M. Lifshitz, *Fluid Mechanics* (Pergamon Press, Oxford, 1984).
[12] A. Abbaci and J. Sengers, Institute for Physical Sciences and Technology, University of Maryland Technical Report No. BN1111, 1990.
[13] R. Ruiz and D.R. Nelson, Phys. Rev. A **23**, 3224 (1981); A. Onuki, Phys. Lett. **101A**, 286 (1984).
[14] D.J. Pine, N. Eswar, J.V. Maher, and W.I. Goldburg, Phys. Rev. A **29**, 308 (1984).
[15] It was the first direct laboratory measurements of the adiabatic temperature gradient (see Ref. [7]). Simultaneously it was also observed in low temperature convection in helium (X. Chavanne *et al.*, J. Low Temp. Phys. **104**, 109 (1996); B. Castaing (private communication). Recently more precise data on the convection onset in a compressible fluid—He^3 near CP—were presented in A. Kogan, D. Murphy, and H. Meyer, Phys. Rev. Lett. **82**, 4635 (1999).
[16] B.I. Shraiman and E.D. Siggia, Phys. Rev. A **42**, 3650 (1990); E.D. Siggia, Annu. Rev. Fluid Mech. **26**, 137 (1994).
[17] S. Cioni, S. Ciliberto, and J. Sommeria, J. Fluid Mech. **335**, 111 (1997).
[18] Y. Liu and R.E. Ecke, Phys. Rev. Lett. **79**, 2257 (1997).
[19] S. Grossmann and D. Lohse, University of Marburg report, 1999.
[20] X. Chavanne *et al.*, Phys. Rev. Lett. **79**, 3648 (1997).