Modulation Effect with Realistic Velocity Dispersion of Supersymmetric Dark Matter

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The direct detection of supersymmetric dark matter is central to particle physics and cosmology. Since the expected event rates are very small, one may exploit the dependence of the event rate on the Earth's motion (modulation effect). We study this effect, on both nondirectional and directional experiments, with realistic (asymmetric) velocity distributions considering all components of the Earth's velocity. These defects combined lead to fivefold enhancement, i.e., a modulation amplitude as large as 46%.

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It is known that dark matter is needed to close the Universe [1,2]. Two kinds of such matter have been considered. One is composed of particles which were relativistic at the time of structure formation, which constitute the hot dark matter (HDM) component. The other is made up of particles which were nonrelativistic at the time of freeze-out, which constitute the cold dark matter (CDM) component. The COBE data [3] suggest that CDM is at least 60% [4]. On the other hand, recent data from the Supernova Cosmology Project suggest [5,6] that there is no need for HDM, and the situation can be adequately described by $\Omega < 1$, e.g., $\Omega_{\rm CDM} = 0.3$ and $\Omega_{\Lambda} = 0.6$. In any case the presence of CDM seems unavoidable.

Since the nonexotic component cannot exceed 40% of the CDM [1,7], there is room for exotic weakly interacting massive particles (WIMPs). The direct detection of such particles is thus of profound importance. Recently the claimed observation of one signal in the DAMA experiment [8] has been interpreted as a modulation signal [9].

In the currently favored supersymmetric extensions of the standard model the most natural WIMP candidate is the lightest supersymmetric particle (LSP). Since this particle, χ, is expected to be very massive, $m_{\chi} \geq 30$ GeV, and extremely nonrelativistic with average kinetic energy $T \leq 100$ keV; its most likely direct detection involves the observation of the recoiling nucleus (A, Z) following elastic scattering. In order to compute the event rate one proceeds with the following steps: (1) Write down the effective Lagrangian at the elementary particle (quark) level obtained in the framework of supersymmetry as described by Jungman *et al.* [1], Bottino *et al.* [10], and in Refs. [11,12]. (2) Go from the quark to the nucleon level using an appropriate quark model for the nucleon [13]. (3) Compute the relevant nuclear matrix elements [14], using as reliable as possible many body nuclear wave functions. (4) Calculate the modulation of the cross section due to the Earth's revolution around the Sun assuming a reasonable LSP velocity distribution.

In the present paper we will focus on items (3) and (4) of the above list. We will compute both the directional and nondirectional event rates, total as well as differential (with respect to energy transfer), employing a realistic velocity distribution. We will present our results in a way which can be easily understood by the experimentalists, focusing on those aspects, which do not depend on the details of supersymmetry (SUSY) models. We will specialize them to the case of the nucleus ^{127}I, which is one of the most popular targets [8,15].

For the evaluation of the differential rate it is convenient to use [16] the variables υ, the velocity of LSP in the laboratory frame, and a dimensionless quantity u, related to the experimentally measured energy transfer Q, and write

$$d\sigma(u,\upsilon) = F^2(u)\,\frac{du}{2(\mu_r b\upsilon)^2}\,\bar{\Sigma}\,, \tag{1}$$

where b is the size of the nucleus, $F(u)$ the nuclear form factor, and

$$\bar{\Sigma} = \sigma_0\left(\frac{\mu_r}{m_N}\right)^2 A^2\left[\left(f_S^0 - f_S^1\,\frac{A-2Z}{A}\right)^2\right], \tag{2}$$

$$\sigma_0 = \frac{1}{2\pi}\,(G_F m_N)^2 \simeq 0.77\times 10^{-38}\ \mathrm{cm}^2\,, \tag{3}$$

where μ_r is the reduced mass and f_S^0 (f_S^1) is the scalar isoscalar (isovector) elementary coupling [13]. In the above expression we have neglected the spin contribution, which is expected to be less important for this intermediate nucleus. The quantity u is defined by

$$Q = Q_0 u\,, \qquad Q_0 = \frac{1}{A m_N b^2}\,,$$
$$Q_0 = 60\ \mathrm{keV} \quad \text{for } ^{127}I\,. \tag{4}$$

The differential detection rate for a particle with velocity $\mathbf{v}$ with a target with mass m detecting in the direction $\mathbf{e}$ is given by

$$dR = \frac{\rho(0)}{m_{\chi}}\,\frac{m}{A m_N}\,\mathbf{v}\cdot\mathbf{e}\,d\sigma(u,\upsilon)\,, \tag{5}$$

where $\rho(0) = 0.3\ \mathrm{GeV/cm}^3$ is the LSP density in our vicinity, and $d\sigma(u,\upsilon)$ is given by Eq. (1).

We will now examine the consequences of the Earth's revolution around the Sun, i.e., the most important modulation effect, by convoluting the rate with the LSP velocity distribution.

In the present paper we will expand previous work [1] and assume an enhanced velocity dispersion in the galactocentric direction, i.e., a distribution of the form suggested by Drukier [17].

$$f(\upsilon', \lambda) = N(y_{\rm esc}, \lambda)(\sqrt{\pi}\,\upsilon_0)^{-3} f_1(\upsilon', \lambda). \quad (6)$$

$N(y_{\rm esc}, \lambda)$ is a normalization constant and

$$f_1(\upsilon', \lambda) = \exp\left(-\frac{(\upsilon_x')^2 + (1 + \lambda)[(\upsilon_y')^2 + (\upsilon_z')^2]}{\upsilon_0^2}\right). \quad (7)$$

The asymmetry parameter λ takes values between 0 (no asymmetry) and 1 (maximum asymmetry), and $\upsilon_0 = \sqrt{(2/3)\langle\upsilon^2\rangle} = 220$ km/s, i.e., it coincides with the velocity of the Sun around the center of the Galaxy. $\upsilon_{\rm esc}$ is the escape velocity in the gravitational field of the Galaxy, $\upsilon_{\rm esc} = 625$ km/s [17], which yields $y_{\rm esc} = \upsilon_{\rm esc}/\upsilon_0 = 2.84$.

In Eqs. (6) and (7) the z axis is chosen in the direction of the motion of the Sun, the y axis is perpendicular to the plane of the Galaxy, and the x axis is in the radial direction. Thus, the axis of the ecliptic [14] lies very close to the yz plane and the velocity of the Earth around the Sun is

$$\mathbf{v}_E = \mathbf{v}_0 + \mathbf{v}_1 = \mathbf{v}_0 + \mathbf{v}_1(\sin\alpha\,\hat{\mathbf{x}} - \cos\alpha\cos\gamma\,\hat{\mathbf{y}} + \cos\alpha\sin\gamma\,\hat{\mathbf{z}}), \quad (8)$$

where α is the phase of the Earth's orbital motion ($\alpha = 0$ around the 2nd of June).

We are now in a position to express the above distribution in the laboratory frame by setting $\mathbf{v}' = \mathbf{v} + \mathbf{v}_E$. The convoluted directional differential event rate is given by

$$\left\langle \frac{dR}{du} \right\rangle = \frac{\rho(0)}{m_\chi}\frac{m}{Am_N}\int f(\mathbf{v}, \mathbf{v}_E)\mathbf{v}\cdot\mathbf{e}\,\frac{d\sigma(u,\upsilon)}{du}\,d^3\mathbf{v}. \quad (9)$$

In evaluating the angular integrals we explored the fact that the velocity of the Earth around the Sun is small, $\delta = (2\upsilon_1/\upsilon_0) = 0.27$ so that we may keep terms up to linear in δ. Thus the differential event rate takes the form

$$\left\langle \frac{dR}{du} \right\rangle_{\rm dir} = \frac{\bar{R}}{2} R^0 t^0 \{[1 + \cos\alpha H_1(u)]\mathbf{e}_z\cdot\mathbf{e} - \cos\alpha H_2(u)\mathbf{e}_y\cdot\mathbf{e} + \sin\alpha H_3(u)\mathbf{e}_x\cdot\mathbf{e}\} \quad (10)$$

with

$$\bar{R} = \frac{\rho(0)}{m_\chi}\frac{m}{Am_N}\sqrt{\langle\upsilon^2\rangle}\,\bar{\Sigma}. \quad (11)$$

The parameter t^0 includes the effects of the convolution on the nonmodulated rate in the presence of the nuclear form factors. The factor of 1/2 is due to the fact that we have chosen to normalize our results to the nondirectional case in which case both directions are counted. The quantity $\bar{R}$ contains all SUSY parameters other than the LSP mass (see, e.g., Refs. [11] and [14]) and

$$R^0 = \frac{1}{t^0}\frac{dr^{(0)}}{du}, \qquad \frac{dr^{(0)}}{du} = a^2F^2(u)\psi^{(0)}(a\sqrt{u}), \quad (12)$$

$$H_l(u) = 0.135\,\frac{\psi^{(l)}(a\sqrt{u})}{\psi^{(0)}(a\sqrt{u})}, \qquad l = 1,3,$$
$$H_2(u) = 0.117\,\frac{\psi^{(2)}(a\sqrt{u})}{\psi^{(0)}(a\sqrt{u})}. \quad (13)$$

R^0 is the relative differential rate and is normalized to unity when integrated from $y_{\rm min} = a\sqrt{u_{\rm min}}$ to $y_{\rm esc}$. In the above expressions $a = (\sqrt{2}\,\mu_r b\upsilon_0)^{-1}$ and

$$\psi^{(l)}(x) = N(y_{\rm esc}, \lambda)e^{-(\lambda+1)}\Phi^{(l)}(x), \quad (14)$$

$$\Phi^{(l)}(x) = \frac{2}{\sqrt{6\pi}}\int_x^{y_{\rm esc}} dy\,y^{-1}\exp[-(1 + \lambda)y^2] \times [F_l(2y) + G_l(\lambda, y)], \quad (15)$$

the functions $G_l(\lambda, y)$, which contain the deviation from symmetric distribution $[G_l(0, y) = 0]$, are rather complicated and they will be described elsewhere [18]. The functions $F_l(2y)$ are given by

$$F_i(\chi) = \chi\cosh\chi - \sinh\chi, \qquad i = 0, 2, 3, \quad (16)$$

$$F_1(\chi) = 2\left[\left(\frac{\chi^2}{4} + 1\right)\sinh\chi - \chi\cosh\chi\right]. \quad (17)$$

Thus the modulated amplitude is described in terms of the parameters, $H_l(u)$, $l = 1$, 2, and 3. In the special case $\lambda = 0$ we have $H_2 = 0.117$ and $H_3 = 0.135$.

From the above equations one obtains the (usual) nondirectional rate by averaging over all orientations. Since the functions H_2 and H_3 are positive and $|H_1| < 1$ we find

$$\left\langle \frac{dR}{du} \right\rangle_{\rm undir} = \bar{R}t^0R^0[1 + \cos\alpha H_1(u) + |\cos\alpha|H_2(u) + |\sin\alpha|H_3(u)], \quad (18)$$

i.e., not a simple sinusoidal function of α.

The total rate is obtained by integrating the corresponding differential rate. Thus we obtain

$$R_{\rm dir} = \frac{1}{2}\bar{R}t^0\{[1 + h_1(a, Q_{\rm min})\cos\alpha]\mathbf{e}_z\cdot\mathbf{e} - h_2(a, Q_{\rm min})\cos\alpha\,\mathbf{e}_y\cdot\mathbf{e} + h_3(a, Q_{\rm min})\sin\alpha\,\mathbf{e}_x\cdot\mathbf{e}\}, \quad (19)$$

$$R_{\rm undir} = \bar{R}t^0[1 + h_1(a, Q_{\rm min})\cos\alpha + h_2(a, Q_{\rm min})|\cos\alpha| + h_3(a, Q_{\rm min})|\sin\alpha|], \quad (20)$$

where $Q_{\rm min}$ is the energy transfer cutoff imposed by the

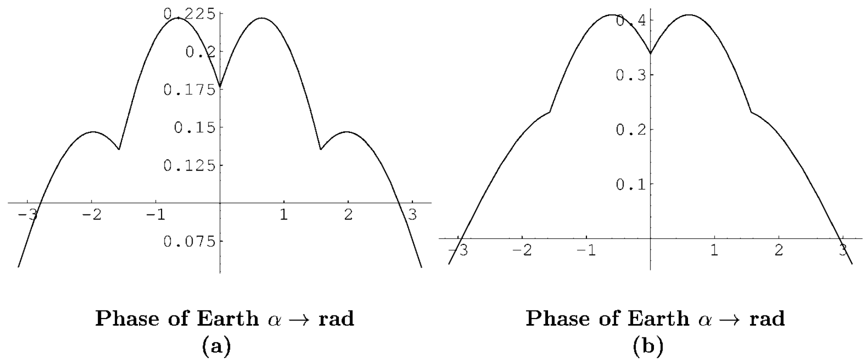

FIG. 1. The modulation amplitude as a function of the phase of the Earth α in two cases: for (a) $h_1 = 0.059$, $h_2 = 0.117$, and $h_3 = 0.135$; (b) $h_1 = 0.192$, $h_2 = 0.146$, and $h_3 = 0.231$. Note that in case (b) the minimum is negative. The results shown are for the target $_{53}I^{127}$ (for definitions, see text).

detector. In this Letter we will consider only partial results for $Q_{\min} = 0, 10$ keV. A more complete presentation will be given elsewhere [18].

Given the functions $h_l(a, Q_{\min})$, one can plot the modulation in Eq. (20) as a function of the phase of the Earth α; see Fig. 1. For a gross description we notice that the modulation has a maximum at $\pm\alpha_h$ with

$$\alpha_h = \tan^{-1}\left[\frac{h_3(a, Q_{\min})}{h_1(a, Q_{\min}) + h_2(a, Q_{\min})}\right], \qquad (21)$$

which corresponds to a shift of about 35 days from the naively expected maximum on June 2nd. The difference between the maximum and the minimum is now given by

$$h_m = [(h_1 + h_2)^2 + h_3^2]^{1/2} - \min(h_1 - h_2, h_3). \quad (22)$$

The values of t_0 and h_1, h_2, h_3, and h_m are given in Table I. We see that in the presence of asymmetry the modulation can be quite sizable exceeding 45%. It also increases as a function of $Q_{\min}$ at the expense of the total rate.

Finally we present the modulation amplitudes entering the differential rate in Fig. 2. We notice that the modulation is sizable even at low u. It tends to increase as a function of u, but, at the same time, the number of events decreases due to the nuclear form factor as manifested by t^0 (see Table I).

The curves shown in Fig. 2 correspond to LSP masses as follows: (i) solid line: $m_\chi = 30$ GeV. (ii) Dotted line: $m_\chi = 50$ GeV. (iii) Dashed line: $m_\chi = 80$ GeV. (iv) Intermediate dashed line: $m_\chi = 100$ GeV. (v) Fine solid line: $m_\chi = 125$ GeV. (vi) Long dashed line: $m_\chi = 250$ GeV. If some curves of the above list seem to have been omitted, it is understood that they fall on top of (vi).

The results shown were obtained numerically due to the complications introduced by the nuclear form factor and the fact that only velocities up to $v_{\rm esc}$ were considered. Ignoring the nuclear form factor, which involves no approximation for the differential rate and is a good approximation in the case of the total rate for small reduced mass, and setting $v_{\rm esc} = \infty$, we find from the form of Eq. (7) the following dependence on λ:

TABLE I. The quantities t^0, h_l, $l = 1, 2, 3$, and h_m for various values of the asymmetry parameter λ in the case of the target $_{53}I^{127}$ for various LSP masses and $Q_{\min} = 0, 10$ keV (for definitions, see text). Only the scalar contribution is considered. Note that in the case $\lambda = 0$, h_2 and h_3 are constants equal to 0.117 and 0.135, respectively. The results for $\lambda = 0.5$ are between the two presented sets.

			LSP mass in GeV						
Quantity	λ	$Q_{\min}$	10	30	50	80	100	125	250
t^0			1.960	1.355	0.886	0.552	0.442	0.360	0.212
h_1	0.0	0.0	0.059	0.048	0.037	0.029	0.027	0.025	0.023
h_m			0.164	0.144	0.124	0.111	0.107	0.104	0.100
t^0			2.429	1.825	1.290	0.837	0.678	0.554	0.330
h_1			0.192	0.182	0.170	0.159	0.156	0.154	0.150
h_2	1.0	0.0	0.146	0.144	0.141	0.139	0.139	0.138	0.138
h_3			0.232	0.222	0.211	0.204	0.202	0.200	0.198
h_m			0.456	0.432	0.404	0.382	0.375	0.379	0.361
t^0			0.000	0.354	0.502	0.410	0.349	0.295	0.184
h_1			0.000	0.241	0.197	0.174	0.167	0.162	0.154
h_2	1.0	10.0	0.000	0.157	0.146	0.142	0.140	0.139	0.138
h_3			0.000	0.273	0.231	0.213	0.208	0.205	0.200
h_m			0.000	0.565	0.464	0.413	0.398	0.387	0.370

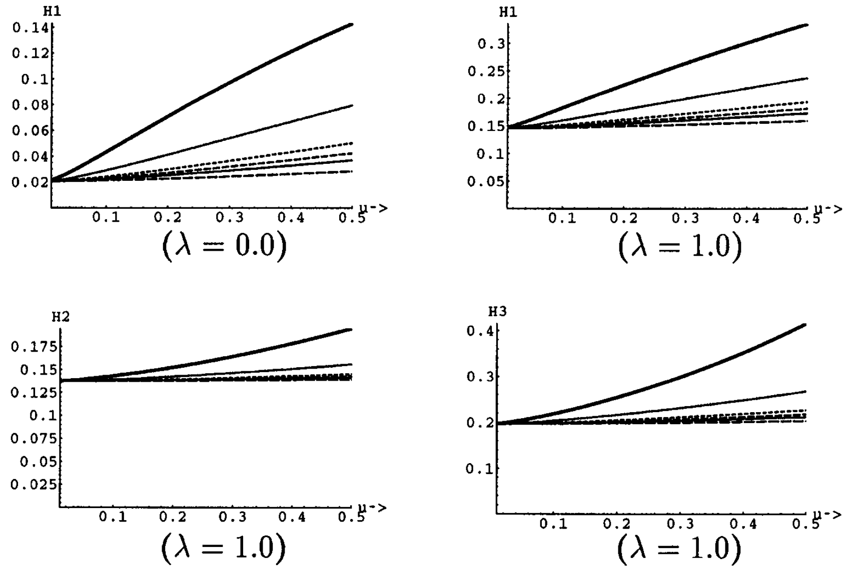

FIG. 2. The quantities H_1, H_2, and H_3 entering the differential rate. For definitions, see text. The energy transfer Q is given by $Q = uQ_0$, $Q_0 = 60$ keV.

$$\frac{h_1(\lambda)}{h_1(0)} = (\lambda + 1)\left[\frac{\langle v_z^2\rangle}{v_0\langle v_z\rangle} + 1\right] = (\lambda + 1) + \sqrt{\lambda + 1}\,, \tag{23}$$

$$\frac{h_2(\lambda)}{h_1(0)} = (\lambda + 1)\frac{\langle v_y^2\rangle}{v_0\langle v_z\rangle} = \sqrt{\lambda + 1}\,,$$

$$\frac{h_3(\lambda)}{h_1(0)} = \frac{\langle v_x^2\rangle}{v_0\langle v_z\rangle} = \sqrt{\lambda + 1}\,. \tag{24}$$

These predictions, when applicable, agree very well with the exact results.

To conclude, in the present paper we calculated all parameters which describe the annual modulation of the direct detection rate for supersymmetric dark matter. All components of the Earth's velocity were taken into account, not just its component along the Sun's direction of motion previously considered. These extra components have little effect on the total rate, but they are very important for the modulation. Thus we find that the modulation is no longer a simple sinusoidal function. Its maximum is shifted about ± 35 days from June 2nd. The realistic axially symmetric velocity dispersion does not significantly modify the total rates, but it increases the modulated amplitude from 16% to 46% for $Q_{\min} = 0$ or even more for $Q_{\min} \neq 0$. In the case of differential event rate, $Q > Q_{\min}$, the increase is from about 10% to about 35%.

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