Large Mass Hierarchy from a Small Extra Dimension

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We propose a new higher-dimensional mechanism for solving the hierarchy problem. The weak scale is generated from the Planck scale through an exponential hierarchy. However, this exponential arises not from gauge interactions but from the background metric (which is a slice of AdS_5 spacetime). We demonstrate a simple explicit example of this mechanism with two 3-branes, one of which contains the standard model fields. The phenomenology of these models is new and dramatic. None of the current constraints on theories with very large extra dimensions apply.

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If spacetime is fundamentally higher dimensional with $4 + n$ spacetime dimensions, then the effective four-dimensional (reduced) Planck scale, $M_{\rm Pl} = 2 \times 10^{18}$ GeV, is determined by the fundamental $(4 + n)$-dimensional Planck scale, M, and the geometry of the extra dimensions. In the simplest cases, where the higher-dimensional spacetime is approximately a product of a four-dimensional spacetime with a n-dimensional compact space,

$$M_{\rm Pl}^2 = M^{n+2} V_n \,, \tag{1}$$

where V_n is the volume of the compact space. Recently, it has been proposed that the large hierarchy between the weak scale and the fundamental scale of gravity can be eliminated by taking the compact space to be very large [1]. The fact that we do not see experimental signs of the extra dimensions despite the fact that the compactification scale, $\mu_c \sim 1/V_n^{1/n}$, would have to be much smaller than the weak scale, implies that the SM particles and forces with the exception of gravity are confined to a four-dimensional subspace within the $(4 + n)$-dimensional spacetime, referred to as a "3-brane." While this scenario does eliminate the hierarchy between the weak scale v and the Planck scale $M_{\rm Pl}$, it introduces a new hierarchy, namely, that between μ_c and v. In light of this, it is worthwhile to explore alternatives.

Here, we will present a distinct higher-dimensional scenario which provides an alternative approach to generating the hierarchy. We propose that the metric is not factorizable, but rather the four-dimensional metric is multiplied by a "warp" factor which is a rapidly changing function of an additional dimension. The dramatic consequences for the hierarchy problem that we identify in this Letter follow from the particular nonfactorizable metric,

$$ds^2 = e^{-2kr_c\phi} \eta_{\mu\nu} dx^\mu dx^\nu + r_c^2 d\phi^2, \tag{2}$$

where k is a scale of the order of the Planck scale, x^μ are coordinates for the familiar four dimensions, while $0 \leq \phi \leq \pi$ is the coordinate for an extra dimension, which is a finite interval whose size is set by r_c. We will show that this metric is a solution to Einstein's equations in a simple setup with two 3-branes and appropriate cosmological terms. In this space, four-dimensional mass scales are related to five-dimensional input mass parameters and the warp factor, $e^{-2kr_c\phi}$. To generate a large hierarchy does not require extremely large r_c. This is because the source of the hierarchy is an *exponential* function of the compactification radius. The small exponential factor above is the source of the large hierarchy between the observed Planck and weak scales.

Although designed to address the hierarchy problem by exploiting an additional dimension, this solution is quite distinct from that studied in Refs. [1–3]: (i) The hierarchy between the fundamental five-dimensional Planck scale and the compactification scale $\mu_c \equiv 1/r_c$ is only of order 10, as opposed to $(M_{\rm Pl}/{\rm TeV})^{2/n}$. (ii) There is one additional dimension, as opposed to $n \geq 2$. The experimentally distinctive consequences are as follows: (i) There are no light Kaluza-Klein (KK) modes. The excitation scale is of the order of a TeV. However, as with the scenario of Ref. [1], string/M-theoretic excitations are also expected to appear at the TeV scale. (ii) The coupling of an individual KK excitation to matter or to other gravitational modes is set by the weak, not the Planck scale. The KK modes are *not* invisible; they should be observable at high energy colliders as spin-2 resonances that can be reconstructed from their decay products.

The setup.—We work on the space $S^1/\mathbf{Z}_2$; that is, we take periodicity in ϕ, the angular coordinate for the fifth dimension, and identify (x, ϕ) with $(x, -\phi)$, though we use the range of ϕ $-\pi$ to π. The 3-branes, extending in the x^μ directions, are located at the orbifold fixed points $\phi = 0, \pi$. The 3-branes can support $(3 + 1)$-dimensional field theories. Both couple to the purely four-dimensional components of the bulk metric:

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$$g_{\mu\nu}^{\text{vis}}(x^\mu) \equiv G_{\mu\nu}(x^\mu, \phi = \pi),$$
$$g_{\mu\nu}^{\text{hid}}(x^\mu) \equiv G_{\mu\nu}(x^\mu, \phi = 0), \tag{3}$$

where $G_{MN}, M, N = \mu, \phi$, is the five-dimensional metric.

This setup is similar to the scenario of Ref. [1], but we take into account the effect of the branes on the bulk gravitational metric and find a new solution to the hierarchy problem. As we will show, this requires nothing beyond the existence of the 3-branes in five dimensions and their compatibility with four-dimensional Poincaré invariance.

The classical action is

$$S = S_{\text{gravity}} + S_{\text{vis}} + S_{\text{hid}}, \qquad S_{\text{gravity}} = \int d^4x \int_{-\pi}^{\pi} d\phi \sqrt{-G}\{-\Lambda + 2M^3 R\},$$
$$S_{\text{vis}} = \int d^4x \sqrt{-g_{\text{vis}}}\{\mathcal{L}_{\text{vis}} - V_{\text{vis}}\}, \qquad S_{\text{hid}} = \int d^4x \sqrt{-g_{\text{hid}}}\{\mathcal{L}_{\text{hid}} - V_{\text{hid}}\}. \tag{4}$$

Note that from each 3-brane Lagrangian we have separated out a constant "vacuum energy" which acts as a gravitational source even in the absence of particle excitations. Other details of the 3-brane Lagrangian are not important (see Ref. [6]).

Classical solution.—In this section we solve the five-dimensional Einstein's equations for the above action:

$$\sqrt{-G}\left(R_{MN} - \frac{1}{2} G_{MN} R\right) = -\frac{1}{4M^3}[\Lambda\sqrt{-G}\, G_{MN} + V_{\text{vis}}\sqrt{-g_{\text{vis}}}\, g_{\mu\nu}^{\text{vis}} \delta_M^\mu \delta_N^\nu \delta(\phi - \pi)$$
$$+ V_{\text{hid}}\sqrt{-g_{\text{hid}}}\, g_{\mu\nu}^{\text{hid}} \delta_M^\mu \delta_N^\nu \delta(\phi)]. \tag{5}$$

We assume there exists a solution that respects *four*-dimensional Poincaré invariance in the x^μ directions. A five-dimensional metric satisfying this ansatz takes the form

$$ds^2 = e^{-2\sigma(\phi)} \eta_{\mu\nu} dx^\mu dx^\nu + r_c^2 d\phi^2. \tag{6}$$

The coefficient, r_c, is independent of ϕ, r_c being the constant of proportionality so that r_c is the "compactification radius" of the extra dimensional circle prior to orbifolding.

With this ansatz, the Einstein's equations following from Eq. (5) reduce to

$$\frac{6\sigma'^2}{r_c^2} = \frac{-\Lambda}{4M^3}, \tag{7}$$

$$\frac{3\sigma''}{r_c^2} = \frac{V_{\text{hid}}}{4M^3 r_c}\delta(\phi) + \frac{V_{\text{vis}}}{4M^3 r_c}\delta(\phi - \pi). \tag{8}$$

The solution to Eq. (7), consistent with the orbifold symmetry, is

$$\sigma = r_c|\phi|\sqrt{\frac{-\Lambda}{24M^3}}, \tag{9}$$

so we find $\Lambda < 0$. Note that the spacetime in between the two 3-branes is simply a slice of an AdS_5 geometry. (Note, this makes our bulk gravitational dynamics compatible with a supersymmetric extension.)

Recall that in computing derivatives we are to consider the metric a periodic function in ϕ. Equation (9), valid for $-\pi \leq \phi \leq \pi$, then implies

$$\sigma'' = 2r_c\sqrt{\frac{-\Lambda}{24M^3}}[\delta(\phi) - \delta(\phi - \pi)]. \tag{10}$$

From this, we see that we obtain a solution to Eq. (8) only if $V_{\text{hid}}, V_{\text{vis}}, \Lambda$ are related in terms of a single scale k:

$$V_{\text{hid}} = -V_{\text{vis}} = 24M^3 k, \qquad \Lambda = -24M^3 k^2. \tag{11}$$

These relations are necessary for four-dimensional Poincaré invariance. Note that these relations arise in the five-dimensional effective theory of the Horava-Witten scenario [4] if one were to interpret the expectation values of the background three-form field (but with frozen Calabi-Yau moduli) as cosmological terms in the effective five-dimensional theory after Calabi-Yau compactification [5]. We will assume that $k < M$ so that we trust our solution.

Our solution for the bulk metric is then

$$ds^2 = e^{-2kr_c|\phi|}\eta_{\mu\nu} dx^\mu dx^\nu + r_c^2 d\phi^2. \tag{12}$$

The compactification radius r_c is effectively an arbitrary integration constant for this solution.

Physical implications.—We can extract the physical implications with a four-dimensional effective field theory description. In this section, we derive the parameters of this low-energy theory, in terms of the five-dimensional scales, M, k, and r_c.

The first step is to identify the massless gravitational fluctuations about our classical solution [Eq. (12)]. They are the zero modes of our classical solution, and take the form

$$ds^2 = e^{-2kT(x)|\phi|}[\eta_{\mu\nu} + \overline{h}_{\mu\nu}(x)]dx^\mu dx^\nu + T^2(x) d\phi^2. \tag{13}$$

Here, $\overline{h}_{\mu\nu}$ represents tensor fluctuations about Minkowski space and is the physical graviton of the four-dimensional effective theory (and is the massless mode in the Kaluza-Klein decomposition of $G_{\mu\nu}$). Note that this metric is *locally* the same as our "vacuum" solution. The compactification radius, r_c, is the vacuum expectation value of the modulus field, $T(x)$. As with many higher dimensional theories, it will be critical that the T modulus is stabilized with a mass of at least 10^{-4} eV. This problem is not yet solved (see Refs. [7,8]); we assume we can replace T with r_c. In compactifying extra dimensions, one frequently encounters vector zero modes from $A_\mu dx^\mu d\phi$ fluctuations of the metric (that is the original Kaluza-Klein idea), corresponding to the continuous isometries of the higher dimensions, but in the present case there are no such isometries in the presence of the 3-branes. So all such off-diagonal fluctuations of the metric are massive and excluded from the low-energy effective theory.

The four-dimensional effective theory now follows by substituting Eq. (13) into the original action [Eq. (4)]. We focus on the curvature term from which we can derive the scale of gravitational interactions:

$$S_{\rm eff} \supset \int d^4x \int_{-\pi}^{\pi} d\phi\, 2M^3 r_c e^{-2kr_c|\phi|}\sqrt{-\overline{g}}\,\overline{R}\,, \tag{14}$$

where $\overline{R}$ denotes the four-dimensional Ricci scalar made out of $\overline{g}_{\mu\nu}(x)$, in contrast to the five-dimensional Ricci scalar, R, made out of $G_{MN}(x,\phi)$. We can explicitly perform the ϕ integral to obtain a four-dimensional action. From this, we derive

$$M_{\rm Pl}^2 = M^3 r_c \int_{-\pi}^{\pi} d\phi\, e^{-2kr_c|\phi|} = \frac{M^3}{k}[1 - e^{-2kr_c\pi}]. \tag{15}$$

This is an important result. It tells us that $M_{\rm Pl}$ depends only weakly on r_c in the large kr_c limit. Although the exponential has very little effect in determining the Planck scale, we will now see that it plays a crucial role in the determination of the visible sector masses.

From Eq. (3) we see that $g_{\rm hid} = \overline{g}_{\mu\nu}$. This is not the case for the visible sector fields; by Eq. (3), we have $g_{\mu\nu}^{\rm vis} = e^{-2kr_c\pi}\overline{g}_{\mu\nu}$. By properly normalizing the fields we can determine the physical masses. Consider, for example, a fundamental Higgs field,

$$S_{\rm vis} \supset \int d^4x \sqrt{-g_{\rm vis}} \times \{g_{\rm vis}^{\mu\nu} D_\mu H^\dagger D_\nu H - \lambda(|H|^2 - v_0^2)^2\}, \tag{16}$$

which contains one mass parameter v_0. Substituting Eq. (3) into this action yields

$$S_{\rm vis} \supset \int d^4x \sqrt{-\overline{g}}\, e^{-4kr_c\pi} \times \{\overline{g}^{\mu\nu} e^{2kr_c\pi} D_\mu H^\dagger D_\nu H - \lambda(|H|^2 - v_0^2)^2\}, \tag{17}$$

After wave-function renormalization, $H \to e^{kr_c\pi}H$, we obtain

$$S_{\rm eff} \supset \int d^4x \sqrt{-\overline{g}} \times \{\overline{g}^{\mu\nu} D_\mu H^\dagger D_\nu H - \lambda(|H|^2 - e^{-2kr_c\pi}v_0^2)^2\}. \tag{18}$$

A remarkable thing has happened. We see that the physical mass scales are set by a symmetry-breaking scale,

$$v \equiv e^{-kr_c\pi} v_0\,. \tag{19}$$

This result is completely general: any mass parameter m_0 on the visible 3-brane in the fundamental higher-dimensional theory will correspond to a physical mass,

$$m \equiv e^{-kr_c\pi} m_0\,, \tag{20}$$

when measured with the metric $\overline{g}_{\mu\nu}$, which is the metric that appears in the effective Einstein action, since all operators get rescaled according to their four-dimensional conformal weight. If $e^{kr_c\pi}$ is of order 10^{15}, this mechanism produces TeV physical mass scales from fundamental mass parameters not far from the Planck scale, 10^{19} GeV. Because this geometric factor is an exponential, we clearly do not require very large hierarchies among the fundamental parameters, v_0, k, M, and $\mu_c \equiv 1/r_c$; in fact, we require only $kr_c \approx 10$.

We now study the gravitational modes. This gives rise to a rich and very distinctive phenomenology. To determine the parameters of the gravitational modes in detail requires an explicit Kaluza-Klein decomposition. We will do this in Ref. [9]. The result is that the masses *and* couplings of the Kaluza-Klein modes are determined by the TeV scale. This result can be readily understood.

Until this point, we have viewed $M \approx M_{\rm Pl}$ as the fundamental scale, and the TeV scale as a derived scale as a consequence of the exponential factor appearing in the metric. However, one could equally well have regarded the TeV scale as fundamental, and the Planck scale of 10^{19} GeV as the derived scale. That is, the ratio is the physical dimensionless quantity. From this viewpoint, which is the one naturally taken by a four-dimensional observer residing on the visible brane, the large Planck scale (the weakness of gravity) arises because of the small overlap of the graviton wave function in the fifth dimension (which is the warp factor) with our brane. In fact, this is the *only* small number produced. All other scales are set by the TeV scale.

Technically, this change in viewpoint is established by the change of coordinates, $x^\mu \to e^{kr_c\pi}x^\mu$. In this case, the warp factor at $\phi = \pi$ is unity, whereas that at $\phi = 0$ is $e^{2kr_c}\pi$. In this language, since there is no rescaling of the "v" parameter in the Higgs potential because the Higgs is already canonically normalized, the scale v should take its physical value. Because we are assuming all fundamental mass parameters are of the same order, all these parameters are also of order TeV. (Note that the relation between the mass parameters in the new coordinates and the old mass parameters is due to the spacetime coordinate rescaling.)

This result contrasts sharply with the scenario of large extra dimensions for solving the hierarchy problem with a product structure for the full spacetime, where the Kaluza-Klein splittings are much smaller than the weak scale, possibly smaller than an eV. The dangerous astrophysical and cosmological effects of very light Kaluza-Klein [3] states are absent in our model.

The phenomenological implications of this scenario for future collider searches are very distinctive. For a product spacetime, each excited state couples with gravitational strength, and the key to observing these states in accelerator experiments is the large multiplicity of states due to their fine splittings. In our model, with roughly weak scale splittings a relatively small number of excitations will be kinematically accessible at accelerators. However, their couplings to matter are set by the weak scale rather than the Planck scale. Instead of gravitational strength couplings $\sim$energy$/M_{\rm Pl}$, each excited state coupling is of the order of energy/TeV, and therefore each can be *individually* detected. These resonances can be detected via their decay products. This should allow detailed reconstruction, permitting mass and spin determination of these *gravitational* modes.

From the above discussion it should be clear that, at energies somewhat larger than the weak scale, the excited gravitons are strongly coupled. This regime should likely open up the production of string/M-theoretic excitations which lie outside the domain of even our starting five-dimensional field theory. This means that, although the fundamental scales of the higher dimensional theory are of order $M_{\rm Pl}$, the *apparent* scale where the theory becomes strongly coupled and the string/M excitations appear is of the order of the weak scale according to a four-dimensional observer. This is an important result for the consistency of our scenario beyond tree level. As with Ref. [1], the TeV-scale strings will cut off large renormalization of the weak scale.

Conclusions.—In the 3-brane scenario, where extra-dimensional translational symmetry is necessarily broken, nontrivial warp factors naturally arise upon solving Einstein's equations. The Kaluza-Klein reduction is considerably more subtle than in product spacetimes, as we will detail in a following paper [9]. This has important phenomenological and theoretical implications.

In this Letter, we focused on a potential phenomenological implication of this scenario, namely, an exponential generation of the hierarchy. Remarkably, the four-dimensional masses on the visible brane depend on the background metric in such a way that their physical values differ significantly from the input mass parameters, even without invoking a large compactification volume. This is a potential resolution to the hierarchy problem akin in spirit to the ideas of strongly coupled gauge theories which generate the low scale through an exponential times a fundamental high-energy scale. As an aside, we mention that the exponential we exploited could generate other scales, such as the low-energy supersymmetry breaking scale. However, it is important to the viability of our mechanism that it is possible to stabilize the compactification radius roughly 2 orders of magnitude larger than the fundamental five-dimensional Planck length. Issues such as flavor violation and proton decay, in the face of the low scale of new physics [10], also remain important challenges.

Fortunately, this solution to the hierarchy problem is subject to experimental verification. The phenomenology is quite distinct from the scenario of large radius compactification. The gravitational resonances are of the order of a TeV, and couple with TeV-suppressed, rather than Planck-suppressed, strength. Furthermore, there are no experimental bounds pushing this scale very high. Should this solution prove correct, there is a rich spectroscopy awaiting us at the LHC.

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