

Watching a Superfluid Untwist Itself: Recurrence of Rabi Oscillations in a Bose-Einstein Condensate

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The order parameter of a condensate with two internal states can continuously distort in such a way as to remove twists that have been imposed along its length. We observe this effect experimentally in the collapse and recurrence of Rabi oscillations in a magnetically trapped, two-component Bose-Einstein condensate of ^{87}Rb .

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Quantization and persistence of current in superconductors and superfluids can be understood in terms of the topology of the order parameter. Current arises from a gradient in the phase of the order parameter. Quantization of flow around a closed path is a consequence of the requirement that the order parameter be single valued; metastability, or “persistence,” arises from the fact that the number of phase windings (the multiple of 2π by which the phase changes) around the path can be changed only by forcing the amplitude of the order parameter to zero at some point. If the energy this requires exceeds that available from thermal excitations, the current will then be immune to viscous damping. This familiar argument relies, however, on the order parameter’s belonging to a very simple rotation group. The order parameter in superfluid ^4He , for example, is a single complex number. Its phase can be thought of as a point lying somewhere on a circle, and is subject to the topological constraints mentioned above. The order parameter of a more complicated superfluid, on the other hand, will in general be capable of ridding itself of unwanted kinetic energy by moving continuously through a higher-dimensional order-parameter space in such a way as to reduce, even to zero, its winding number. Presumably this ability will reduce a superfluid’s critical velocity; in the limit that the order-parameter space is fully symmetric, the critical velocity may even vanish [1].

In this paper, we discuss experiments on a gas-phase Bose-Einstein condensate with two internal levels [2]. This is equivalent to a spin-1/2 fluid: the order parameter has SU(2) rotation properties. A differential torque across the sample is applied to the order parameter so that with time it becomes increasingly twisted. Eventually the sample distorts through SU(2) space so that the steadily applied torque now has the effect of untwisting the order parameter, which returns nearly to its unperturbed condition. The pattern of twisting and then untwisting is manifested experimentally as a washing out followed by a recurrence of an extended series of oscillations in the population between the spin states. Related behavior has been previously observed in A phase ^3He [3]; a major difference in

this paper is that we can directly observe components of the order parameter with temporal and spatial resolution.

Magnetically confined ^{87}Rb can exist in a superposition of two internal states, known as $|1\rangle$ and $|2\rangle$ [4]. The two internal states are separated by the relatively large ^{87}Rb hyperfine energy, but in the presence of a near-resonant coupling field the states appear, in the rotating frame, to be nearly degenerate. The condensate can then dynamically convert between internal states. The order parameter for the condensate is the pair of complex field amplitudes Φ_1 and Φ_2 of states $|1\rangle$ and $|2\rangle$. Evolution of these fields is governed by a pair of coupled Gross-Pitaevskii equations which model the coupling drive, the external confining potential, kinetic energy effects, and mean-field interactions [5–7]. The SU(2) nature of the order parameter (Φ_1, Φ_2) is more evident if we write

$$\Phi_1 = \cos(\theta/2)e^{-i\phi/2}n_t^{1/2}e^{i\alpha} \quad (1a)$$

and

$$\Phi_2 = \sin(\theta/2)e^{i\phi/2}n_t^{1/2}e^{i\alpha}, \quad (1b)$$

where θ , ϕ , n_t , and α are purely real functions of space and time. θ and ϕ give the relative amplitude and phase of the two internal components, and may be thought of, respectively, as the polar and azimuthal angles of a vector whose tip lies on a sphere in SU(2) space. The total density and mean phase, n_t and α , respectively, remain relatively constant [8] during the condensate evolution described in this paper.

The apparatus has been previously described [5,6,9]. The starting point for the measurements is a magnetically confined cloud of $\sim 8 \times 10^5$ evaporatively cooled, Bose-Einstein-condensed ^{87}Rb atoms near zero temperature. The combined gravitational and magnetic potentials [10] yield an axially symmetric, harmonic confining potential V_1 (V_2) for particles in the $|1\rangle$ ($|2\rangle$) state, in which the aspect ratio of the axial oscillation frequency in the trap to the radial frequency ω_z/ω_r can be varied from 2.8 to 0.95 [11]. V_1 and V_2 are nearly identical but can optionally be spatially offset a distance z_0 in the axial direction [12].

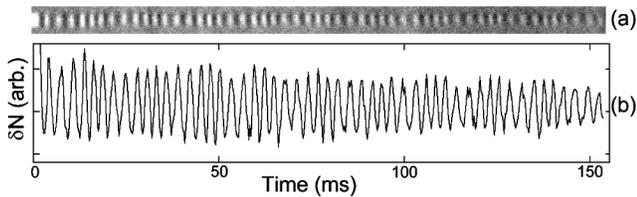


FIG. 1. (a) With the trap parameters adjusted for high spatial uniformity in Ω_{eff} , we drive the coupling transition and record a streak-camera image of 60 Rabi oscillations between the $|1\rangle$ (white) and $|2\rangle$ (black) states. The vertical dimension of the figure is $80 \mu\text{m}$. (b) The value of δN , the total number of atoms in $|2\rangle$ minus the total in $|1\rangle$, is extracted from the image in part (a). The contrast ratio remains near unity; the observed loss of signal is due to overall shrinkage of the condensate through collisional decay.

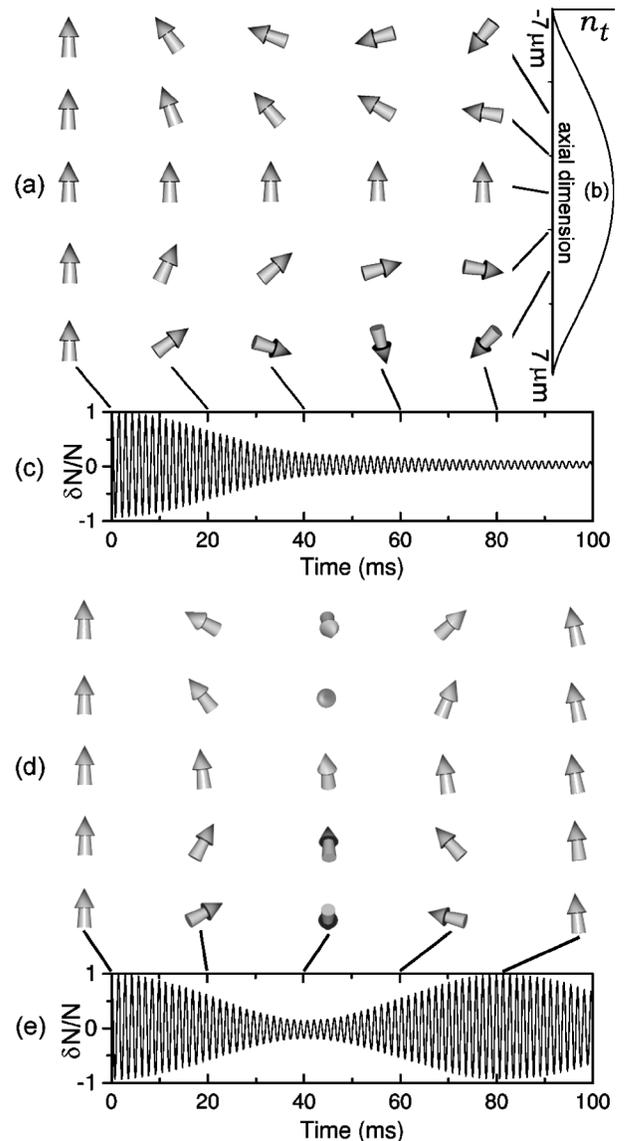
The coupling field has a detuning δ from the local $|1\rangle$ to $|2\rangle$ resonance. If z_0 is nonzero, δ depends on the axial position z , with $\delta(z) - \delta(z=0)$ linear in z and in z_0 . The strength, characterized by the Rabi frequency Ω , of the coupling field also varies with an axial gradient [13].

We are able to measure the population of both spin states nondestructively using phase-contrast microscopy [14,15]. We tune the probe laser between the resonant optical frequencies for the $|1\rangle$ and $|2\rangle$ states. Since the probe detuning has an opposite sign for the two states, the resulting phase shift imposed on the probe light has an opposite sign, such that the $|1\rangle$ atoms appear white and the $|2\rangle$ atoms appear black against a gray background on the CCD array. We can acquire multiple, nondestructive images of the spatial distribution of the $|1\rangle$ and $|2\rangle$ atoms at various discrete moments in time, or we can acquire a quasicontinuous time record (streak image) of the difference of the populations in the $|1\rangle$ and $|2\rangle$ states, integrated across the spatial extent of the cloud.

FIG. 2. (a) We represent the polar-vector order parameter as an arrow in these simulations. The angle θ from the vertical axis determines the relative population, and the azimuthal angle ϕ is the relative phase of states $|1\rangle$ and $|2\rangle$ [see Eqs. (1)]. Each column in the arrow array is at fixed time, and each row at fixed axial location. $\hat{\Omega}$ is perpendicular to the plane of the page, so that a uniform, on-resonance Rabi oscillation would correspond to all the arrows rotating in unison, in the plane of the image. The tips of all the arrows are (on the relatively fast time scale of Ω_{eff}) tracing out circles nearly parallel to the plane of the page (in our rotating-frame representation, small excursions out of the page are a consequence of finite detuning). In the figures, we “strobe” the motion just as the central arrow approaches vertical, to emphasize the more slowly evolving “textural” behavior. (b) The total density of the condensate n_t maintains a Thomas-Fermi distribution (integrated through one dimension, as imaged), and changes only slightly during the evolution of the cloud. (c) In a simple model of individual, fixed atoms, a continuous inhomogeneity in Ω_{eff} will cause the Rabi oscillations in δN to wash out. (d) When a condensate is simulated [19], the kinetic energy causes the order parameter to precess through the full $\text{SU}(2)$ space, coming out of the page to cast off the winding and thus reduce its kinetic energy. (e) The corresponding plot of δN shows that, when the arrows are once more aligned, the Rabi oscillations recur.

The effect of the coupling drive is to induce a precession of the order parameter at the local effective Rabi frequency $\Omega_{\text{eff}}(z) \equiv [\Omega(z)^2 + \delta(z)^2]^{1/2}$. In a preliminary experiment, we chose parameters so as to make Ω_{eff} nearly uniform, with $\omega_z = 2\pi \times 63 \text{ Hz}$, $\omega_r = 2\pi \times 23 \text{ Hz}$, $\Omega \approx 2\pi \times 340 \text{ Hz}$, and $\delta(z) \approx 0$. A condensate at near-zero temperature was prepared in the pure $|1\rangle$ state. The coupling drive was then turned on suddenly, inducing an extended series of oscillations of the total population from the $|1\rangle$ to the $|2\rangle$ state (“Rabi oscillations”) [Fig. 1]. The robustness of the Rabi oscillations is proof that our imaging does not significantly perturb the quantum phase of the sample [16] (population transfer via Rabi oscillations is phase sensitive).

If there is an axial gradient to Ω_{eff} , then a relative torque is applied to the order parameter across the condensate, which can cause a twist to develop along the axial direction. If we naively model the sample as a collection of individual atoms, each held fixed at its respective location, then the order parameter at each point in space rotates



independently at the local effective Rabi frequency, Ω_{eff} . In Fig. 2(a) we see the implications of the “fixed-atom” model for $\Omega = 2\pi \times 700$ Hz and $\delta(z) = 2\pi \times (100 + 14z)$ Hz with z in microns: a twist develops in the order parameter which leads to a washing out of the Rabi oscillations [Fig. 2(c)]. In contrast, the kinetic energy provides stiffness for a true condensate. Simulation of the condensate [17] shows that, for early times, the order parameter begins twisting, as in the fixed-atom model, but the twisting process self-limits about 40 ms into the simulation. At this point, there is nearly a full winding across the condensate. Thereafter, although the two ends of the order parameter continue to twist with respect to one another, the order parameter has been sufficiently wrapped around the SU(2) sphere so that the effect of further torque is to return the condensate close to its unperturbed condition [Fig. 2(d)]. The Rabi oscillations exhibit a corresponding revival [Fig. 2(e)]. The factor driving the untwisting process is the increasing kinetic energy cost associated with an increasing twist in the order parameter. For a simple U(1) order parameter, continuously increasing the winding ultimately results in a “snap,” in which the order parameter is driven to zero and a discontinuous (and presumably dissipative [18]) process releases the excess windings. The revival in the present case is made possible by the larger rotation space available to a two-component cloud.

Under experimental conditions similar to those of the simulations in Fig. 2, we have observed as many as three complete cycles of Rabi-oscillation decay and recurrence. These data appear in Ref. [19]. In this paper we present data which correspond to the case of a more vigorous twisting. We increase the axial dimension of the condensate cloud by a factor of 4; the kinetic energy cost of twisting the condensate is correspondingly lower, so that at the point in time when the condensate is maximally distorted there are four windings across the cloud. The parameters of the experiment were as follows: $\omega_r = \omega_z = 2\pi \times 7.8$ Hz and mean $\Omega_{\text{eff}} = 2\pi \times 225$ Hz. There was a gradient in both δ and Ω across the $54 \mu\text{m}$ axial extent of the cloud, resulting in a $\sim 2\pi \times 60$ Hz difference in Ω_{eff} from the top to the bottom of the condensate. The result of the experiment is seen in Fig. 3. The observed recurrence of the Rabi oscillations at 180 ms [Fig. 3(a)], when corrected for overall decay of the cloud, corresponds to 60% contrast. We find it remarkable that the distorted order-parameter field seen in Fig. 3(b) at times 65 and 75 ms should find its own way back to a nearly uniform configuration.

The simulations qualitatively reproduce the integrated number and state-specific density distributions observed in the experiments. For large inhomogeneity in Ω_{eff} , however, the simulations predict the development of small-scale spatial structure not observed in the experiment.

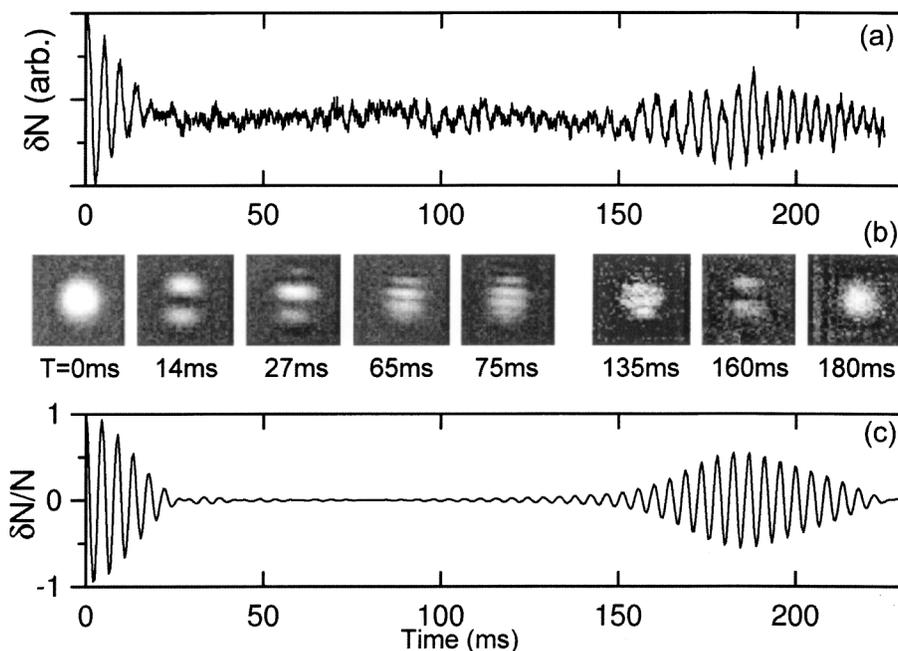


FIG. 3. A condensate with large axial extent undergoes twisting. (a) The streak-camera data show a rapid decay in the Rabi oscillation in the integrated population difference, from full contrast at $t = 0$ to near zero by $t = 20$ ms. The oscillations recur at 180 ms. (b) Individual phase-contrast images (at distinct moments in time) of the spatial distribution of $|1\rangle$ -state atoms show that the spatially inhomogeneous Rabi frequency is twisting the order parameter, cranking successively more windings into the condensate until, by ~ 75 ms, four distinct windings are visible. Further evolution results not in more but in fewer windings until, at time 180 ms, the order parameter is once more uniform across the cloud. Each image block is $100 \mu\text{m}$ on a side, and the probe laser is tuned much closer to the $|1\rangle$ state than to the $|2\rangle$ state. (c) The numerical simulation reproduces the qualitative features of the corresponding experimental plot (a). The simulation used $\delta(z) = 0$, $\Omega(z = 0) = 2\pi \times 225$ Hz and a $2\pi \times 60$ Hz spread in Ω across the extent of the condensate.

The simulations contain no dissipation, whereas finite-temperature damping may occur in the experiment [20].

Heuristically, what value do we expect for the recurrence time t_{recur} for the data in Fig. 3? The difference in Ω_{eff} from the top to the bottom of the condensate is about 60 Hz. From the data in Fig. 3(b) we see that the recurrence occurs only after four windings have one by one been twisted in and then twisted out of the condensate. A rough estimate then would be $t_{\text{recur}} = (4 + 4)/60 \text{ Hz} = 133 \text{ ms}$, shorter than the observed value of 180 ms, but reasonable given that edge effects have been neglected.

An interesting theoretical challenge would be to develop simple arguments that would allow an *a priori* prediction of the spacing of the windings at the instant of maximum twist. Also, for particularly strong torques, should not one expect suppression of the total density, rather than continuous evolution through SU(2) space, to be the preferred mode of relieving accumulated stress?

We have observed that the presence of the coupling drive need not result in population oscillations. For any given frequency and coupling strength, there are two steady-state solutions which are completely analogous to the dressed-state solutions of the single-atom problem [21]. We have been able experimentally to put the condensate in such states via an adiabatic process: the strength of the drive is increased gradually from zero, and the frequency (initially far detuned) is gradually ramped onto resonance. The resulting “dressed condensate” is extremely stable—after the ramp is complete, the cloud remains motionless in a near-equal superposition of the two bare states. The twisting experiment discussed in this paper can be thought of as the evolution of a highly nonlinear superposition of the two dressed states. It would be interesting to work in the opposite limit and explore the spectrum of *small*-amplitude excitations on a dressed state. Analogies to (i) the transverse zero sound in the He³ *B* phase [22] and (ii) spin waves in spin-polarized atomic hydrogen [23] should also be studied [24].

The ability to fundamentally alter the topological properties of a condensate has already proven useful. With the coupling drive off, the SU(2) properties vanish, and states $|1\rangle$ and $|2\rangle$ become distinct species, each forced to live in its separate U(1) space. With a variation [25] on this technique we have created a vortex-state condensate [26].

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- [1] P. Bhattacharyya, T. Ho, and N. Mermin, Phys. Rev. Lett. **39**, 1290 (1977); T.-L. Ho, Phys. Rev. Lett. **49**, 1837 (1982).
 [2] For other recent experimental work on multicomponent Bose-Einstein condensate, see J. Stenger *et al.*, Nature

- (London) **396**, 345 (1998); H.-J. Miesner *et al.*, Phys. Rev. Lett. **82**, 2228 (1999).
 [3] D. Paulson, M. Krusius, and J. Wheatley, Phys. Rev. Lett. **37**, 599 (1976).
 [4] The states $|1\rangle$ and $|2\rangle$ correspond to the two Zeeman sub-levels, $F = 1, m = -1$, and $F = 2, m = 1$, respectively. For further details on the system, see Ref. [9].
 [5] E. A. Cornell, D. S. Hall, M. R. Matthews, and C. E. Wieman, J. Low Temp. Phys. **113**, 151 (1998).
 [6] D. S. Hall, M. R. Matthews, C. E. Wieman, and E. A. Cornell, Phys. Rev. Lett. **81**, 1543 (1998).
 [7] R. Dum, J. I. Cirac, M. Lewenstein, and P. Zoller, Phys. Rev. Lett. **80**, 2972 (1998); R. J. Ballagh, K. Burnett, and T. F. Scott, Phys. Rev. Lett. **78**, 1607 (1997); K.-P. Marzlin, W. Zhang, and E. M. Wright, Phys. Rev. Lett. **79**, 4728 (1997); J. Williams *et al.*, Phys. Rev. A **57**, 2030 (1998).
 [8] D. S. Hall *et al.*, Phys. Rev. Lett. **81**, 1539 (1998).
 [9] M. R. Matthews *et al.*, Phys. Rev. Lett. **81**, 243 (1998).
 [10] W. Petrich, M. H. Anderson, J. R. Ensher, and E. A. Cornell, Phys. Rev. Lett. **74**, 3352 (1995).
 [11] J. R. Ensher, Ph.D. thesis, University of Colorado, 1998.
 [12] D. S. Hall *et al.*, cond-mat/9903459.
 [13] The microwave coupling drive is actually accomplished by a two-photon transition. The quoted values of Ω and δ refer to the combined effect of a two-frequency drive (see Ref. [9]). Since Ω depends on the intermediate-state detuning, it can in general have a spatial dependence due to Zeeman shifts. Ω also depends on the (poorly characterized) polarization of the coupling fields, whose projection onto the local quantization axis varies weakly across the cloud. In our comparisons with theory, the gradient in Ω is an adjustable parameter.
 [14] F. Zernike, in *Nobel Lectures, Physics* (Elsevier, Amsterdam, 1964), p. 239 (text of 1953 Nobel Prize lecture).
 [15] M. R. Andrews *et al.*, Science **273**, 84 (1996).
 [16] The probe beam does impose a small, reproducible shift on the energy difference between the $|1\rangle$ and $|2\rangle$ states, a measurable shift for which we correct.
 [17] The simulation is a numerical integration of the coupled Gross-Pitaevski equations in [6]. See [19] for more detail.
 [18] K. Schwab, N. Bruckner, and R. Packard, Nature (London) **386**, 585 (1997); O. Avenel, P. Hakonen, and E. Varoquaux, Phys. Rev. Lett. **78**, 3602 (1997).
 [19] J. Williams, R. Walser, J. Cooper, E. A. Cornell, and M. Holland, cond-mat/9904399.
 [20] P. O. Fedichev, G. V. Shlyapnikov, and J. T. M. Walraven, Phys. Rev. Lett. **80**, 2269 (1998); L. P. Pitaevskii and S. Stringari, Phys. Lett. A **235**, 398 (1997); W. V. Liu, Phys. Rev. Lett. **79**, 4056 (1997).
 [21] P. B. Blakie, R. J. Ballagh, and C. W. Gardiner, cond-mat/9902110.
 [22] Y. Lee, T. M. Haard, J. B. Kycia, J. A. Sauls, and W. P. Halperin, Nature (London) **400**, 431 (1999).
 [23] N. Bigelow, J. Freed, and D. Lee, Phys. Rev. Lett. **63**, 1609 (1989).
 [24] Obviously, oscillatory behavior in coupled spin systems is a recurring theme. At present it is unclear to us how deep the analogy runs.
 [25] J. E. Williams and M. J. Holland, cond-mat/9909163.
 [26] M. R. Matthews *et al.*, Phys. Rev. Lett. **83**, 2498 (1999).