Spin Bottlenecks in the Quantum Hall Regime

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I report on a theory of time-dependent tunneling between a metal and a partially spin-polarized two-dimensional electron system (2DES). I find that the tunneling current which flows to screen an electric field between a metal and a 2DES is the sum of two exponential contributions whose relative weights depend on spin-dependent tunneling conductances, on quantum corrections to the electrostatic capacitance of the tunnel junction, and on the rate at which the 2DES spin polarization approaches equilibrium. For high mobility homogeneous 2DES's at filling factor $\nu = 1$, I predict a ratio of fast and slow tunneling rates equal to $(2K + 1)^2$ where K is the number of reversed spins in the Skyrmion excitations.

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There has been renewed [1] interest recently in nonequilibrium spin accumulation [2] due to electronic transport in spin-polarized electron systems, in part because these accumulations are important in giant magnetoresistance [3]. In this paper, I address spin accumulation in the linear-response tunneling current between a metal and a two-dimensional electron system (2DES). This work is motivated in part by recent experiments by Chan *et al.* [4] which have discovered unexplained two-rate tunneling currents in the quantum Hall regime, and also in part by the long [5] spin-relaxation times of 2DES's, especially long in the quantum Hall regime [6–8]. I find that spin accumulation depends subtly on the interplay of spin-dependent tunneling conductances, thermodynamic densities of states, and spin relaxation rates. According to this theory, the double-rate current found in experiment signals sizable quantum corrections to the effective capacitance of the junction *and* spin-relaxation processes which depart from those in conventional metals, both expected in the quantum Hall regime. Gapped quantum Hall states lead to rapid variation of the chemical potential [9] with density, and the presence of Skyrmion elementary charged excitations [10] requires an unusual spin-relaxation process. I predict that, for homogeneous 2DES's and thin tunneling barriers, the ratio of fast and slow relaxation rates at $\nu = 1$ will equal $(2K + 1)^2$, where K is the number of reversed spins in the lowest energy Skyrmion quasiparticles.

I start from the following phenomenological linear response equations for tunneling between a metal and a 2DES:

$$\begin{aligned} \dot{Q}_\uparrow &= -\mu_\uparrow G_\uparrow + (\mu_\downarrow - \mu_\uparrow) G_s \,, \\ \dot{Q}_\downarrow &= -\mu_\downarrow G_\downarrow + (\mu_\uparrow - \mu_\downarrow) G_s \,. \end{aligned} \tag{1}$$

Here Q_σ and μ_σ are the spin-σ particle-number and chemical potential, G_s characterizes linear-response spin relaxation in the 2DES, and G_σ is the spin-σ tunneling conductance. These equations express spin partitioning of the tunneling current; the assumption of separate chemical potentials for the 2DES spin subsystems is valid when the spin-relaxation time is much longer than other characteristic scattering times in the 2DES. In these equations, I have placed the zero of energy at the chemical potential of the metal. The two terms on the right-hand side of Eqs. (1) account, respectively, for tunneling across the junction into the 2DES and relaxation of the 2DES spin subsystems toward mutual equilibrium. A closed description of electron transport in the system requires, in addition to the above *conductance* equations, which relate currents to chemical potential differences, a set of *capacitance* equations which relate these chemical potentials to accumulated charges:

$$\mu_\uparrow = -V_0 + (C^{-1})_{\uparrow\uparrow} Q_\uparrow + (C^{-1})_{\uparrow\downarrow} Q_\downarrow \,, \tag{2}$$

$$\mu_\downarrow = -V_0 + (C^{-1})_{\downarrow\uparrow} Q_\uparrow + (C^{-1})_{\downarrow\downarrow} Q_\downarrow \,. \tag{3}$$

Here V_0 represents the electrostatic contribution from charges external to the 2DES. Elements of the inverse capacitance (C^{-1}) matrix have an electrostatic contribution proportional to the width of the tunnel barrier and a quantum "chemical potential" contribution due to the Fermi statistics and correlations of electrons in the 2DES;

$$(C^{-1})_{\sigma\sigma'} = \frac{1}{C_g} + \frac{1}{A} \frac{d\mu_\sigma}{dn_{\sigma'}} \equiv \frac{1}{C_g} + F_{\sigma,\sigma'} \,, \tag{4}$$

where $C_g = A\epsilon/(4\pi e^2 d)$ is the electrostatic capacitance of the junction, A is the cross-sectional area of the two-dimensional electron system, ϵ is the dielectric constant of the host semiconductor, d is the distance between the metallic electrode and the 2DES, and μ_σ is the spin-σ chemical potential of the 2DES relative to its electric subband energy. The notation above is motivated by the analogy between $F_{\sigma,\sigma'}$ and Fermi liquid theory interaction parameters. In the commonly employed Hartree mean-field approximation,

$$F_{\sigma,\sigma'} = \frac{\delta_{\sigma,\sigma'}}{AD^0_\sigma} \,, \tag{5}$$

where D^0_σ is the noninteracting 2DES density of states per area per spin; the chemical potential increases with density only because of the Pauli exclusion principle which operates separately on spin-up and spin-down quasiparticle

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states. More generally, however, the energy change on adding an electron depends on the density of the existing electrons of either spin with which the added electron must correlate. $d\mu_\sigma/d\eta_{\sigma'}$ is then nonzero for $\sigma \neq \sigma'$. The resulting correlation induced mixed-spin capacitance can be important in producing spin bottlenecks if it is large, as it is in the quantum Hall regime discussed below.

These equations can be used to describe various time-dependent or ac linear transport experiments involving tunneling from a metal to a 2DES. The phenomenology is readily generalized to the other cases, for example, to potentially interesting 2D to 2D tunneling experiments. I apply it here to the situation studied recently [4] by Chan *et al.* in which a chemical potential difference across the junction is created by external charges and the tunneling current which reestablishes equilibrium is measured as a function of time. In an obvious matrix notation the conductance and capacitance equations take the forms $\dot{\mathbf{Q}} = \mathbf{G}\boldsymbol{\mu}$ and $\boldsymbol{\mu} = -\mathbf{V}_0 + \mathbf{C}^{-1}\mathbf{Q}$. Eliminating the chemical potentials using the capacitor equations yields a set of two coupled first-order inhomogeneous linear differential equations for the time-dependent spin-up and spin-down charges in the 2DES. Solving these with the boundary condition $Q_\uparrow(t=0) = Q_\downarrow(t=0) = 0$ yields for the spin-dependent currents into the 2DES

$$\dot{Q}_\sigma(t) = I_{\sigma,+}\exp(-t/\tau_+) + I_{\sigma,-}\exp(-t/\tau_-), \quad (6)$$

where τ_+^{-1} and τ_-^{-1}, generalized RC relaxation rates, are the eigenvalues of $\mathbf{A} = \mathbf{G}\mathbf{C}^{-1}$. I have obtained the following explicit expressions for $I_{\uparrow,\pm}$:

$$\frac{I_{\uparrow,+}}{V_0} = \frac{G_\uparrow}{2}\left(1 + \frac{A_{\uparrow\uparrow} - A_{\downarrow\downarrow}}{\tau_+^{-1} - \tau_-^{-1}}\right) + \frac{G_\downarrow A_{\uparrow\downarrow}}{\tau_+^{-1} - \tau_-^{-1}}, \quad (7)$$

$$\frac{I_{\uparrow,-}}{V_0} = \frac{G_\uparrow}{2}\left(1 - \frac{A_{\uparrow\uparrow} - A_{\downarrow\downarrow}}{\tau_+^{-1} - \tau_-^{-1}}\right) - \frac{G_\downarrow A_{\uparrow\downarrow}}{\tau_+^{-1} - \tau_-^{-1}}. \quad (8)$$

The corresponding expressions for $I_{\downarrow,\pm}$ are obtained by interchanging spin labels. Figure 1 shows results for the dependence of tunneling current on 2DES chemical potential near Landau level filling factor $\nu = 1$ obtained from Eq. (8) using a noninteracting Skyrmion model described below. These replicate major features found in experiment [4]. The peak in both fast and slow relaxation rates is due to the sharp decrease in capacitance as the $\nu = 1$ incompressible quantum Hall state is approached. The tunneling current is dominated by the slow process, except in a narrow range very close to $\nu = 1$ where the fast process takes over. The origin of this crossover is explained below.

Similar results can be obtained for the instantaneous chemical potentials of the spin-up and spin-down subsystems:

$$\mu_\sigma(t) = -V_0 + \sum_{s=\pm}\mu_{\sigma,s}[1 - \exp(-t/\tau_s)], \quad (9)$$

where $\mu_{\sigma,\pm} = \sum_{\sigma'} C^{-1}_{\sigma,\sigma'} I_{\sigma',\pm}\tau_\pm$. I note that $\mu_{\sigma,+} + \mu_{\sigma,-} = V_0$; current flows until the electrochemical potential change for *each* spin cancels the electric potential from external charges. The two spin subsystems are in equilibrium at both the beginning and the end of the relaxation process, but are, in general, out of equilibrium at intermediate times. The nonequilibrium spin accumulation $\mu_\uparrow(t) - \mu_\downarrow(t) = (\mu_{\downarrow,-} - \mu_{\uparrow,-})[\exp(-t/\tau_-) - \exp(-t/\tau_+)]$. The fast and slow contributions to the current are readily separated experimentally, but their relationship to spin subsystem contributions is not always obvious. Nevertheless, we see that nonequilibrium spin accumulations occur in the system between times τ_- and τ_+ whenever both contributions are present. In Fig. 2 we plot time-dependent chemical potentials and accumulated charges for the noninteracting Skyrmion model at $\mu = 0.05(e^2/\ell)$. The majority spin chemical potential initially overshoots so that the two chemical potentials approach 0 from opposite sides at long times. I note from Fig. 1 that the slow tunneling current process actually dominates the capacitance at this value of μ; the naive view that the fast process is purely current flow to the 2DES while the slow process is purely spin equilibration is incorrect.

Before turning to the quantum Hall regime, where nonequilibrium spin accumulations are large, it is instructive to examine several limits for which spin accumulation does not occur. For $F_{\sigma,\sigma'}C_g \ll 1$ we find that $\tau_+^{-1} = (G_\uparrow + G_\downarrow)/C_g$, $\tau_-^{-1} \to 0$, $I_{\sigma,+} = G_\sigma V_0$, and $I_{\sigma,-} \to 0$. In

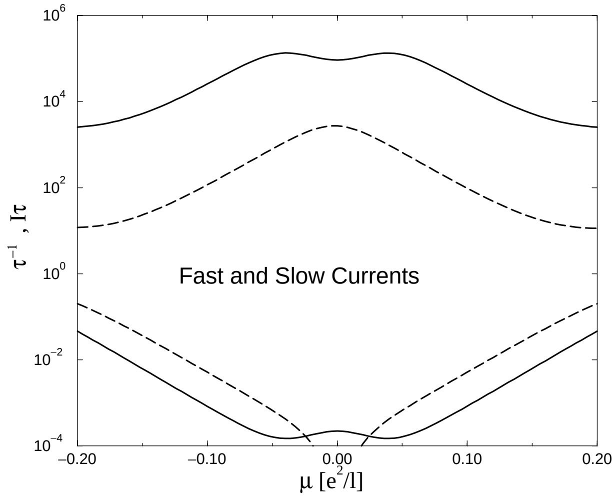

FIG. 1. Tunneling current between a metallic electrode and a 2DES as a function of chemical potential near $\nu = 1$. The chemical potential is in e^2/ℓ units, and the zero of energy is chosen so that $\mu = 0$ at $\nu = 1$. The solid and dotted lines are for fast and slow channels, respectively. The rate curves have maxima near $\nu = 1$ while the capacitance curves ($I_+\tau_+$ and $I_-\tau_-$) have minima near $\nu = 1$. These curves were calculated using the noninteracting Skyrmion model explained in the text to evaluate the quantum inverse capacitance contributions and a separation $d = 5\ell$ between the metal and the 2DES. G_σ and G_s were assumed to have the form $1000/F_{\sigma,\sigma} + 10$ and $1/(F_{\uparrow\uparrow}F_{\downarrow\downarrow}) + 0.01$ in arbitrary units; the two terms represent uniform system golden rule and inhomogeneity contributions, respectively. The tunneling rates are in units of C_g^{-1} times the arbitrary conductance unit, while the capacitances per unit area are in units of ℓ^{-1}. This figure is for $k_BT = 0.025e^2/\ell$.

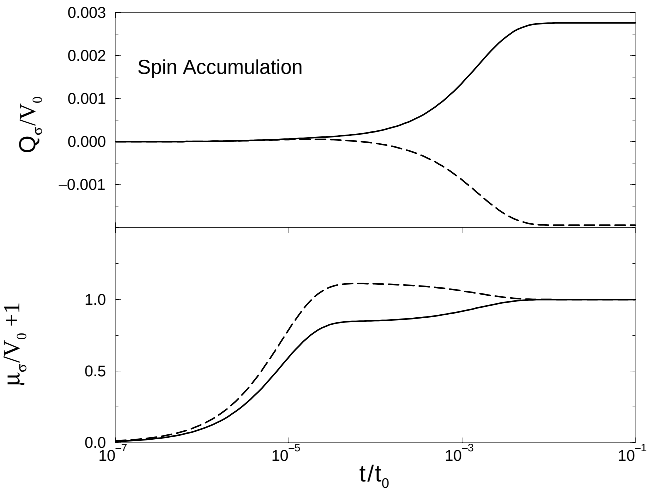

FIG. 2. $\mu_\sigma + V_0$ in units of V_0 and partial accumulated charge Q_σ/V_0 as a function of time for the model of Fig. 1 at $\mu = 0.05e^2/\ell$. The dashed lines show majority spin quantities, while the solid lines show minority spin quantities. t_0 is C_g divided by the arbitrary conductance unit. At short times both minority and minority spins tunnel into the 2DES. At long times equilibrium is reestablished with the added charge in the form of Skyrmions consisting of three majority spin holes and four minority spin electrons. At intermediate times the chemical potential of the majority spins exceeds that of minority spins, causing flow from majority spins to minority spins within the 2DES. Note that slow and fast processes include both net current flow to the 2DES and spin reversal in the 2DES, although the relative importances differ in the two cases.

this limit, which usually holds for metallic electrodes, spin-independent electrostatic contributions dominate electrochemical potential changes; no nonequilibrium spin accumulation occurs because the spin subsystems are never driven from equilibrium. For strong tunnel barriers ($G_\sigma \ll G_s$), on the other hand, $\tau_+^{-1} = G_s(F_{\uparrow,\uparrow} + F_{\downarrow,\downarrow} - 2F_{\uparrow,\downarrow})$, $\tau_-^{-1} = (G_\uparrow + G_\downarrow)(F_{\uparrow,\uparrow}F_{\downarrow,\downarrow} - F_{\uparrow,\downarrow}^2)/(F_{\uparrow,\uparrow} + F_{\downarrow,\downarrow} - 2F_{\uparrow,\downarrow})$, and the fast relaxation current $I_+ \equiv I_{\uparrow,+} + I_{\downarrow,+} = 0$. For this limit spin accumulation does not occur because the relaxation processes are fast enough to maintain instantaneous equilibrium. Unlike the electrostatic-dominance case discussed first, the tunneling current flows at the slow rate τ_-^{-1}. A third more subtle limit in which spin accumulation does not occur applies to Fermi gas 2DES's in which we may ignore correlation contributions to the chemical potential and the commonly adopted forms $G_\sigma = cAD_\sigma$, $G_s = c_sAD_\uparrow D_\downarrow$ hold. These expressions result from golden rule estimates of quasiparticle tunneling and spin-flip transition rates, respectively, c is a constant which declines exponentially with the thickness of the tunneling barrier, and c_s is a constant dependent on spin-orbit scattering strength in the 2DES. For this model we find that $\tau_+^{-1} = c[1 + (D_\uparrow + D_\downarrow)/C_g]$, $\tau_-^{-1} = c + c_s(D_\uparrow + D_\downarrow)$, and all the weight is in the fast tunneling current. No spin accumulation occurs because the spin subsystems are not coupled by interactions and the ratio of tunneling conductances equals the ratio of the rates at which the chemical potentials increase with density. Finally, we mention the case in which the 2DES is paramagnetic to which we return below. For $G_\uparrow = G_\downarrow$ and $F_{\uparrow,\uparrow} = F_{\downarrow,\downarrow}$, symmetry forbids spin accumulations. An explicit calculation finds no weight for the slow tunneling current and the rate ratio

$$\frac{\tau_-}{\tau_+} = \frac{G_\uparrow(2/C_g + F_{\uparrow,\uparrow} + F_{\uparrow,\downarrow})}{(G_\uparrow + 2G_s)(F_{\uparrow,\uparrow} - F_{\uparrow,\downarrow})}. \tag{10}$$

None of these limits apply throughout the quantum Hall regime. Near integer Landau level filling factors, Fermi statistics and correlations in the 2DES, not electrostatics, dominate the electrochemical potential changes with density [9]. Equilibrium electronic states contain [10] complex Skyrmion quasiparticles whose formation from the fully spin-polarized $\nu = 1$ ground state cannot be achieved by a single-particle process. Spin equilibration will therefore be slow [10]. The two spin systems are intricately coupled so that the Fermi gas limit does not apply. Furthermore, the 2DES will generally be strongly spin polarized.

A simple model of the 2DES valid at low temperature for ν near one is obtained by ignoring interactions between Skyrmions. I obtain the following grand-canonical ensemble expressions for the occupation probabilities of the $N_\phi = A/(2\pi\ell^2)$ Skyrmion quasielectron and quasihole states with K excess reversed spins:

$$n_{Ke} = f[\epsilon_K + K\mu_\uparrow - (K + 1)\mu_\downarrow], \tag{11}$$

$$n_{Kh} = f[\epsilon_K + (K + 1)\mu_\uparrow - K\mu_\downarrow]. \tag{12}$$

Here $f(\epsilon) = [\exp(\epsilon/k_BT) + 1]^{-1}$ is a Fermi factor [11], ϵ_K is the energy of a Skyrmion quasiparticle, $(2\pi\ell^2)^{-1}$ is the density of a full Landau level, and we have chosen the zero of energy so that quasielectron and quasihole Skyrmion states have the same energy [12]. When the spin subsystems are in equilibrium ($\mu_\uparrow = \mu_\downarrow = \mu$) we can use Eqs. (12) to calculate the chemical potential, given the Landau level filling factor. Equations (12) express the property that the Kth quasielectron Skyrmion is formed by adding $K + 1$ spin-down electrons to and removing K spin-up electrons from the $\nu = 1$ ground state. For noninteracting electrons only $K = 0$ quasiparticles occur; for typical 2DES's, on the other hand, the lowest energy quasiparticles have $K = 3$ [13]. From Eqs. (12) we calculate the following thermodynamic densities of states, $D_{\sigma,\sigma'} \equiv dn_\sigma/d\mu_{\sigma'}$ for the coupled spin systems:

$$2\pi\ell^2 D_{\uparrow,\uparrow} = \sum_K [(K + 1)^2\Delta(\epsilon_K + \mu) + K^2\Delta(\epsilon_K - \mu)],$$

$$2\pi\ell^2 D_{\uparrow,\downarrow} = -\sum_K K(K + 1) \times [\Delta(\epsilon_K + \mu) + \Delta(\epsilon_K - \mu)],$$

where $\Delta(x) = \mathrm{sech}^2(x/2k_BT)/4k_BT$. In Fig. 3 we plot quantum contributions to the model's inverse capacitance, obtained by inverting this density-of-states matrix. The

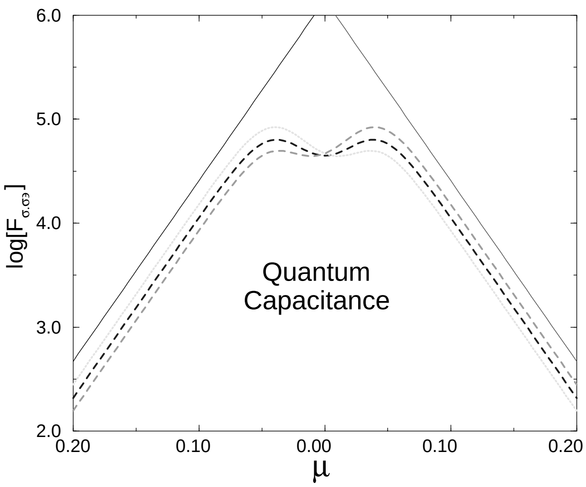

FIG. 3. Quantum contributions to the inverse capacitance for a 2DES near $\nu = 1$. The solid lines show the majority and minority spin results for $F_{\sigma,\sigma}$ for a noninteracting model which contains only $K = 0$ quasiparticles. For the Skyrmion model, $F_{\uparrow,\uparrow}$, $F_{\downarrow,\downarrow}$, and $F_{\uparrow,\downarrow}$ are shown by dotted, short-dashed, and long-dashed lines, respectively.

large peaks in inverse capacitance near $\nu = 1$ are responsible for the peaks in both tunneling current rates. In the low-temperature limit, only the lowest energy $K = 3$ Skyrmion will contribute so that for $\nu > 1$, $F_{\uparrow,\uparrow}$, $F_{\downarrow,\downarrow}$, and $F_{\uparrow,\downarrow}$ occur in the ratio $(K + 1)^2 \colon K^2 \colon K(K + 1) = 16 \colon 9 \colon 12$. This contrasts with the noninteracting electron case for which $F_{\uparrow,\uparrow}$ is much larger than $F_{\downarrow,\downarrow}$ and $F_{\uparrow,\downarrow}$ vanishes. (The same ratios apply for $\nu < 1$ with inverted spin indices.) For $\nu = 1$ the low-temperature equilibrium state charge is added to the $\nu = 1$ state in the form of $K = 3$ Skyrmions; i.e., for each four up spins added to the 2DES, three down spins are removed. It follows that the time-integrated spin-up and spin-down tunneling currents are approximately equal in magnitude and opposite in sign.

In contrast to the partitioning of total tunneling charge between spins, which is determined purely by thermodynamic considerations, the observable partitioning between fast and slow components is difficult to understand intuitively in the general case. Simplification occurs, however, when $\nu \equiv 1$. It follows from particle-hole symmetry [12] that at $\nu = 1$ the 2DES can be considered to be a paramagnetic system of spin-down electrons and spin-up holes, explaining the vanishing weight in the slow tunneling current channel seen in Fig. 1 as the filling factor approaches one. The ratio of fast to slow tunneling rates, Eq. (10) depends only on thermodynamic quantities provided that the tunneling barrier is thin enough that $G_\sigma \gg G_s$. Then, provided that the temperature is sufficiently low that quantum terms dominate the inverse capacitance we find that $\tau_-/\tau_+ = (2K + 1)^2 = 49$, in rough agreement with the findings of Chan *et al.* [4]. These considerations do not account for the inhomogeneity present at integer filling factors in all current samples.

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