Oscillons and Propagating Solitary Waves in a Vertically Vibrated Colloidal Suspension

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The observation of localized stationary structures, coined oscillons, in granular media has evoked much interest. By parametric excitation of clay suspensions, we demonstrate a hysteretic transition to oscillon-type states in a nongranular medium. When the symmetry of up-down reflection + time translation is lost, these states undergo a transition to propagating localized states previously seen in Newtonian fluids. These observations are in accord with recent theoretical predictions of sufficient conditions for oscillon formation. In addition, a novel measurement technique for the effective suspension viscosity demonstrates their shear-thinning properties.

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Recent experiments in dissipative 2D nonlinear pattern-forming systems [1,2] have observed stable highly localized states that coexist with a featureless, pattern-free state. A similar type of excitation generates both oscillating localized structures in granular media, coined oscillons [1], and propagating dissipative solitary states in Newtonian fluids [2]. Oscillons are localized, nearly stationary circular regions that oscillate between conical peaks and craters with a period of half of the external driving frequency. They appear with finite initial amplitude in a hysteretic region between patterns and featureless states. Propagating dissipative solitary states (DSS) are large amplitude, highly localized propagating states that appear in highly dissipative fluids and have the temporal periodicity of the driving.

How general are these states and what causes their localization? As their period has no amplitude dependence, both oscillons and DSS differ from localized states, such as in convecting binary mixtures, that are stabilized via the interplay between their amplitude and internal phase [3]. Stable oscillons were first observed theoretically in an extension of a Swift-Hohenberg (SH) model, with real coefficients, that exhibits a subcritical transition from patterns to stable oscillon structures [4]. Further work on real SH models suggested that large hysteresis together with reflection + discrete time translational symmetry are [5] sufficient conditions for oscillons to exist. These models suggest [4,5] a general (nonadiabatic) localization mechanism caused by the pinning of the large-scale envelope by the small-scale underlying periodic pattern [6,7].

The subsequent observation of oscillons in a variety of continuum and discrete models [8] suggests their generality. As described below, our experiments in vertically vibrated clay suspensions confirm this. Suspensions provide a bridge between Newtonian fluids and dry granular media as interactions between minute suspended particles are mediated by the fluid between them. They are non-Newtonian and their shear-thinning character will be demonstrated. We will show that both oscillons and DSS are observed in these fluids, and support for the above localization mechanism is indicated. We find the two states to be related; oscillons undergo a transition to DSS once the system's vertical reflection symmetry is broken.

Our experimental system [2] is characterized by the quantities f, h, ρ, and ν defined, respectively, as the externally imposed driving frequency, suspension depth, density, and kinematic viscosity. A layer of suspension is vertically (parallel to gravity $\mathbf{g}$) driven with a displacement $z(t) = a/\omega^2 \sin(\omega t)$, where a is the externally applied acceleration and ω the angular excitation frequency. The dimensionless acceleration amplitude, $\Gamma = a/g$, can be viewed as the system's control parameter. At a critical value Γ_c the initial spatially uniform fluid state loses its stability. In our experiments we used both 20 cm square and 29 cm in diameter circular containers with $0.4 < h < 4.0$ cm. The fluid rests on an aluminum plate with Plexiglas lateral boundaries. The working cell was mounted on a mechanical shaker providing vertical acceleration from 0 to $30g$. The range of driving frequencies used (between 10–60 Hz) was limited from above by the maximal output force of our shaker (225 N) and from below by its maximum stroke (1.25 cm). The acceleration, regulated to within $0.01g$, was monitored continuously by a calibrated accelerometer. Visualization was performed by the scattering of diffuse stroboscopic backlight (1 μsec illumination time) by the fluid surface. A video camera (JAI MV-30) with frame rates between 60–360 Hz was used to photograph the resulting wave states. The working suspensions were a mixture of water with commercial clay powder supplied by Negev Ceramics; this clay is composed of kaolinite ($\sim$60%) and quartz (25%–30%) with a mean grain size of 4.5 μm. The volume fraction, ϕ, of the dry clay was varied from 0.155 to 0.185 corresponding to density variations of 1.26 to 1.33 g/cm^3. The suspension density, ρ, is fit by $\rho = 0.98 + 0.7\phi + 6.31\phi^2$.

In Newtonian fluids the initial instability is generally to standing surface waves. In the suspensions used, the initially flat layer, upon losing stability, erupts into highly hysteretic, extremely large amplitude fingerlike

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states. To facilitate quantitative investigation, we limited ourselves to the study of the system's dynamics solely within the subcritical region. This was accomplished by mechanically introducing an $O(h)$ protrusion on the fluid surface by either striking or blowing compressed air on the suspension surface while increasing the acceleration at a fixed frequency until a $O(10\%)$ hysteretic transition from the featureless state to oscillons is observed. Despite the crude nature of the perturbation, the transition to oscillons was reproducible to $\sim 1\%-2\%$. We define Γ_{tran} as the threshold to the flat state upon *decreasing* Γ. Near Γ_{tran} either a single oscillon or oscillon pair is excited (Figs. 1a and 1b).

As in [1] oscillons oscillate with frequency $f/2$ with states π out of phase able to coexist. For $\Gamma > \Gamma_{\mathrm{tran}}$ oscillons can interact to form localized patterns such as the triad structures of in-phase oscillons, shown in Fig. 1c. Depending on both the driving amplitude and initial conditions, both short oscillon chains, composed of either like- or unlike-phase oscillons, and localized patterns of oscillons, with an internal hexagonal symmetry, have been observed. One important difference between oscillons observed in suspensions relative to those seen in granular media is [9] the range of their interactions. In suspensions, oscillons attract each other when separated by distances greater than their radius. In granular media the interaction distance is much shorter, typically a small fraction of an oscillon radius. (This may be due to the different degrees of shear thinning in the two media.) As observed in [1,5] increase of Γ leads to the formation of oscillon chains that tend to elongate by adding additional "links" in one direction while widening in the transverse direction. For large enough Γ this process eventually fills the entire cell with stripes. Further increase of Γ leads to distortion of the stripes and, eventually, to fingerlike states and droplet ejection. As the phase diagram in Fig. 2 shows, in contract to granular media, both oscillons and striped patterns are observed throughout the entire explored range of frequencies. The lower threshold, Γ_{pat}, for the formation of patterns is typically reproducible to within 2% whereas the upper threshold for patterns is strongly hysteretic and dependent on plate leveling. When Γ is decreased from a patterned state to below Γ_{pat}, the patterns decay into long chains of oscillons that gradually contract in length. Bending and rotation accompany the chain's contraction. Finally, the oscillons lose stability to the featureless state at Γ_{tran}. The most robust states are bound pairs of opposite "polarity" oscillons.

Changes in suspension depth do not lead to any qualitative changes in the system's behavior until a critical frequency where the oscillon amplitude A (typically, A is of order a/ω^2, the excitation amplitude) approaches h. Below this frequency, a transition from oscillons to propagating states occurs that corresponds to the breaking of the $A \to -A$ symmetry for $t \to t + 2/f$. Photographs of a typical propagating solitary structure are displayed in Fig. 3a. Whereas the oscillons are subharmonic standing waves oscillating at frequency $f/2$, these propagating solitary states are similar in appearance to those seen in highly dissipative Newtonian fluids (Fig. 3b). They are harmonic with the driving with their structure repeating itself over a basic period of $1/f$ [2]. The 3D structure of the states shown in the figure is representative and is independent of the driving frequency and fluid parameters. As Fig. 2 shows, the phase diagrams for the different heights are nearly identical until the transition to propagating states (DSS are observed below $f \sim 20$ Hz in the figure). The appearance of propagating states is accompanied by a smooth increase in Γ_{tran}, relative to the value observed for the oscillon state. A time sequence of a typical state appearing at the transition to propagating states is presented in Fig. 3c. Although these transition states do not propagate, they are seen to sway in the lateral direction. Their form suggests a mechanism by which the *subharmonic* oscillon becomes a propagating state whose time dependence is *harmonic* with the driving frequency.

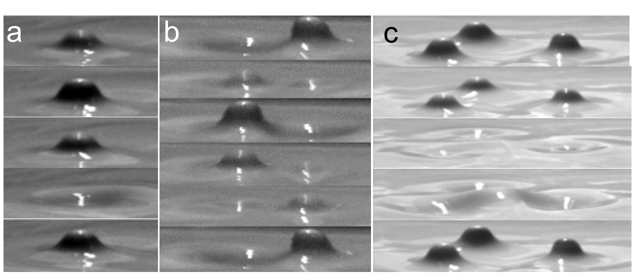

FIG. 1. Typical side view of localized states with $\rho = 1.28$ g/cm^3, $h = 4$ cm, frame widths = 4 cm. (a) Single oscillon, $f = 14$ Hz. (b) Oscillon pair, $f = 20$ Hz. (c) Oscillon triad $f = 25$ Hz. Two driving periods are shown for each sequence.

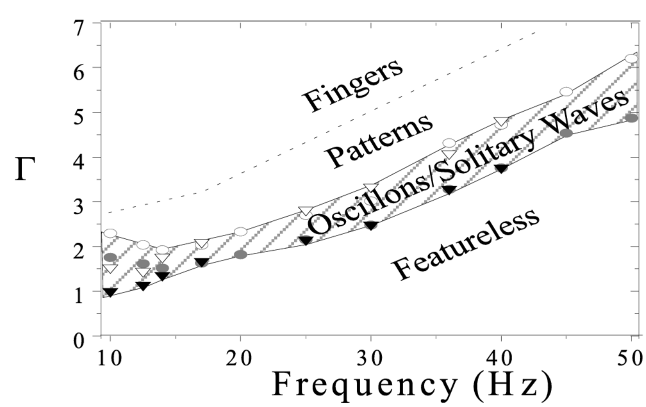

FIG. 2. Phase diagram of a clay suspension with $\rho = 1.28$ g/cm^3 for $h = 4.0$ cm (triangles) and $h = 0.8$ cm (circles). Shown are the lower stability boundaries, Γ_{pat} (open symbols), for patterns and Γ_{tran} (filled symbols), for oscillon or propagating solitary wave states. The dashed line schematically indicates the transition to highly hysteretic fingerlike states, the system's first instability. Both oscillons and patterns exist in the subcritical region of this state.

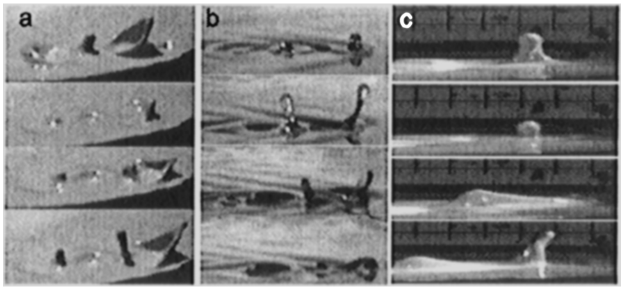

FIG. 3. Typical pictures of propagating solitary states in (a) a polydisperse colloidal suspension with $\rho = 1.2\ \mathrm{g/cm^3}$, $h = 0.7$ cm, $f = 15$ Hz; (b) in a Newtonian fluid (TKO-FF pump oil) $h = 0.15$ cm, $f = 30$ Hz, $\nu = 0.81$ St (1 St = 1 stoke = 1 $\mathrm{cm^2/s}$); (c) a transition state between oscillons and solitons $h = 0.8$ cm, $\rho = 1.28\ \mathrm{g/cm^3}$, $f = 14$ Hz. The sequences in (a) and (b) were taken over a single period of the driving frequency. The sequence in (c) was taken over 80 msec intervals. Frame widths are (a) 15.5, (b) 2.6, and (c) 6.3 cm. Time increases from top to bottom.

Let us first consider the motion of an oscillon whose amplitude is much smaller than the height of the system. Its peak occurs when the plate is at its minimum. In the rest frame of the upwardly accelerating plate, the material within the initial peak accelerates downward toward the bottom of the plate. The momentum within this material is sufficient to drive it below the fluid surface, creating a crater at the next minimum plate height. To create the crater, the material below the original peak is displaced in the lateral direction. When the fluid depth approaches the oscillon amplitude, the crater is again formed at the next minimum of the plate height, but now the displaced fluid acquires both an *upward* as well as a lateral displacement. As a result, the lateral boundaries of the crater now form an upward protrusion (see Fig. 3c), which is not evident in the pure oscillon state (Fig. 1a). Thus, the vertical symmetry breaking incurred by the interaction with the lower boundary leads to a protrusion of the fluid surface that is displaced laterally, by at least a crater radius, every period of the driving. As upon external perturbation of the fluid, when the disturbance is large enough, the protrusion can, itself, create an additional crater and propagation, which breaks the cylindrical symmetry of the initial crater, ensues. In *granular* materials, however, oscillons with amplitudes over $4h$ have been observed [1] while DSS were not seen, although large-amplitude oscillons will tend to sway periodically in the lateral direction [9]. One reason for their lack of lateral movement may be that, in granular materials, dissipation *increases* with the medium's height. This would tend to suppress the above propagation mechanism.

Let us consider the localization mechanism of the oscillon state. In [4,5] localization is due to a mechanism, originally suggested by Pomeau [6], whereby a front separating bistable periodic and featureless states is pinned. The stability of such fronts was calculated in a subcritical SH model by Bensimon *et al.* [7]. They predicted that the front should be stabilized over a finite width, $\Delta\epsilon \sim \exp[-\alpha/\epsilon^{1/2}]$, of the control parameter ϵ. Estimating ϵ by $(\Gamma_{\mathrm{pat}} + \Gamma_{\mathrm{tran}})/2$ and $\Delta\epsilon$ by $(\Gamma_{\mathrm{pat}} - \Gamma_{\mathrm{tran}})$, Crawford and Riecke [5] observed this scaling in numerical studies of oscillon states. They suggested that the scaling, which was not seen in [1], is masked in granular materials. We have checked the predicted scaling in suspensions for a number of different values of ϕ and h (Fig. 4). Although our range in both $\Delta\epsilon$ and ϵ is limited, all of our measurements are compatible with this prediction, with α consistently within the range 1–3.

Let us now consider the characterization of the clay suspensions. As ν is a strongly dependent function of the shear rate, it is difficult to accurately characterize the effective viscosity of a suspension. Since commercial viscometers rely on the knowledge of a known flow state, the highly non-Newtonian character of suspensions precludes their use. Studies of parametrically driven surface waves in Newtonian fluids have established [10] the accuracy of the numerical code developed by Kumar and Tuckerman [11] for calculation of the critical acceleration, Γ_c, and wave numbers, k_c, at the instability threshold. For given values of h and ρ there is only a single combination of f and ν that yields a given k_c and Γ_c. We propose to use these calculations to determine ν for our suspensions. As input to the code we use Γ_{pat}, corresponding to the lowest amplitude steady-state wave observed, together with f, h, and ρ. We then solve for both ν and k_c. We use Γ_{pat} instead of the suspension's measured threshold value of Γ_c for the following reason. As we shall see, in the suspensions used ν is a strongly decreasing function of f. At Γ_c there is no relative motion between the fluid and the container and the effective viscosity of the suspension is at the $f = 0$ limit. Once motion is initiated, ν decreases significantly and this lower effective dissipation provides *positive* feedback to the motion. Thus, the suspensions

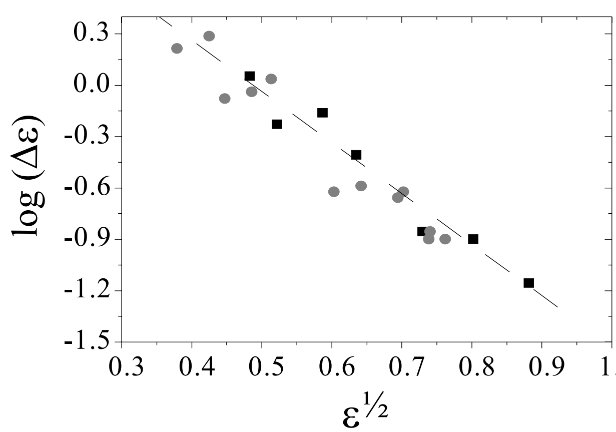

FIG. 4. Comparison of the region of existence of oscillons with the scaling prediction $\Delta\epsilon \sim \exp[-\alpha/\epsilon^{1/2}]$ of [5,7] (due to nonadiabatic pinning of the front separating patterns and flat states). Here $\epsilon = (\Gamma_{\mathrm{pat}} + \Gamma_{\mathrm{tran}})/2$ and $\Delta\epsilon = (\Gamma_{\mathrm{pat}} - \Gamma_{\mathrm{tran}})$. Data sets for $\rho = 1.28\ \mathrm{g/cm^3}$, $h = 0.8$ cm (circles) and $\rho = 1.28\ \mathrm{g/cm^3}$, $h = 4.0$ cm (squares) are shown.

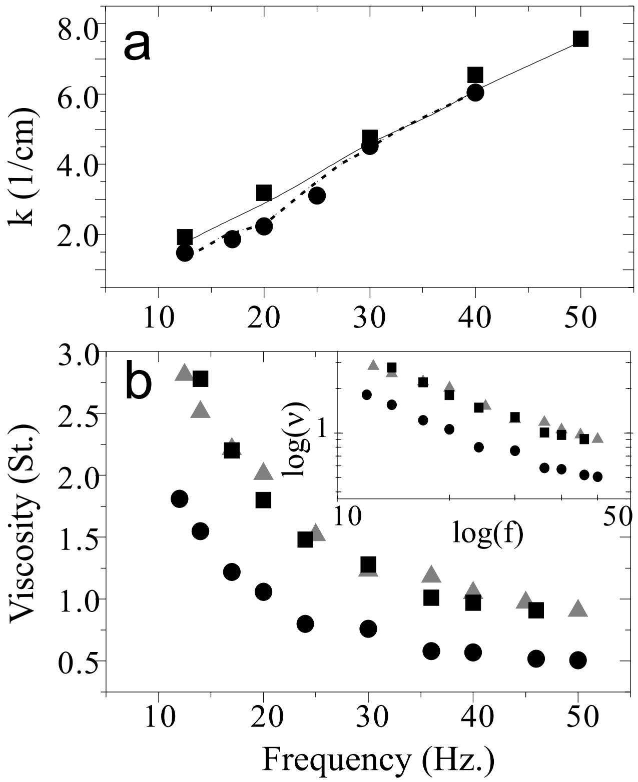

FIG. 5. (a) Measured wave numbers for $h = 0.8$ cm, $\rho = 1.28$ g/cm^3 (squares) and $h = 4.0$ cm, $\rho = 1.28$ g/cm^3 (circles). The solid and dashed lines are the respective wave numbers calculated for a Newtonian fluid [11]. (b) Calculated kinematic viscosity (with $\Gamma_{\rm pat}$ as input) as a function of f for $h = 3.0$ cm, $\rho = 1.33$ g/cm^3 (squares), $h = 4.0$ cm, $\rho = 1.26$ g/cm^3 (circles) and for $h = 0.8$ cm, $\rho = 1.28$ g/cm^3 (triangles). (Inset) Log-log plots of data in (b) demonstrating $\sim 1/f$ behavior.

shear-thinning character may lead to both the large hysteresis in Γ and the high amplitude fingerlike eruptions that occur immediately at the onset of motion.

We check the validity of this approach by using the value of ν obtained above to predict k_c. The calculated and measured values of k_c, as shown in Fig. 5a, agree to within 10%. This provides *a posteriori* justification for the use of the calculated viscosity as an accurate measure of ν. The frequency dependence of ν (Fig. 5b) indicates the shear-thinning behavior of these suspensions. Our experiments using $0.155 < \phi < 0.185$ indicate an approximate $1/f$ dependence, with some small systematic variations. These observations agree with the predictions for ν obtained in recent calculations [12] of the shear-rate dependence of a 30% suspension of spheres. Shear thinning with a $1/f$ dependence has also been observed in dynamic viscosity measurements of suspensions of hard spheres for $\phi > 0.30$, within an oscillating flow [13].

In conclusion, our experiments indicate that the existence of highly localized states such as oscillons and propagating solitary states may be quite general. Oscillons are observed for the first time in a medium that is significantly different from granular media. Our results are consistent with recent theoretical predictions of sufficient conditions for oscillons' appearance, i.e., hysteresis together with reflection + discrete time translational symmetry. In the suspensions studied, when this symmetry is broken, a transition occurs from oscillons to the highly localized propagating solitary waves, observed in highly dissipative Newtonian fluids. The region of existence of these localized states is also found to be consistent with the nonadiabatic pinning mechanism [6] that recent studies of model systems [4,5] have indicated to be the mechanism governing oscillon localization.

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