Chiral Fluid Dynamics and Collapse of Vacuum Bubbles

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We study the expansion dynamics of a quark-antiquark plasma droplet from an initial state with restored chiral symmetry. The calculations are made within the linear σ model scaled with an additional scalar field representing the gluon condensate. We solve numerically the classical equations of motion for the meson fields coupled to the fluid-dynamical equations for the plasma. Strong space-time oscillations of the meson fields are observed in the course of the chiral transition. A new phenomenon, the formation and collapse of vacuum bubbles, is predicted. The particle production due to the bremsstrahlung of the meson fields is estimated.

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It is commonly believed that the conditions for chiral symmetry restoration and color deconfinement can be reached in the course of an ultrarelativistic heavy-ion collision. The quark-gluon plasma (QGP) is expected to be formed at some intermediate stage of the reaction. Since strong collective expansion may develop already in the QGP, its subsequent transition to the hadronic phase should be treated dynamically [1]. At present this is possible only on the basis of effective models obeying the symmetry properties of QCD.

Our considerations below are based on the linear σ model which respects approximate chiral symmetry. In addition to the usual chiral fields, σ and $\boldsymbol{\pi} = \{\pi_1, \pi_2, \pi_3\}$, the model includes the dilaton or glueball field, χ, to simulate the trace anomaly of QCD [2]. The σ and χ represent the quark and gluon condensates as effective meson fields. Models of this kind were used earlier for nuclear matter (see, e.g., [3]).

The dynamical model.—The effective chiral Lagrangian for constituent quarks interacting with the background meson fields is written as

$$\begin{aligned} \mathcal{L} &= \bar{q}[i\gamma_\mu \partial^\mu - g(\sigma + i\gamma_5 \boldsymbol{\tau} \cdot \boldsymbol{\pi})]q \\ &\quad + \frac{1}{2}[\partial_\mu \sigma \partial^\mu \sigma + \partial_\mu \boldsymbol{\pi} \cdot \partial^\mu \boldsymbol{\pi}] - U(\sigma, \boldsymbol{\pi}, \chi) \\ &\quad + \frac{1}{2}\partial_\mu \chi \partial^\mu \chi - W(\chi) - J\chi^2. \end{aligned} \tag{1}$$

$$\begin{aligned} U(\sigma, \boldsymbol{\pi}, \chi) &= \frac{\lambda_1^2}{4}\left[\sigma^2 + \boldsymbol{\pi}^2 - \sigma_0^2\left(\frac{\chi}{\chi_0}\right)^2\right]^2 \\ &\quad - f_\pi m_\pi^2 \sigma \left(\frac{\chi}{\chi_0}\right)^n, \\ W(\chi) &= \frac{\lambda_2^2}{4}\chi^4 \ln\left(\frac{\chi^4}{\Lambda^4}\right). \end{aligned} \tag{2}$$

Here U is the usual Mexican Hat potential scaled by the glueball field χ (below we take $n = 3$), and W is the effective potential responsible for the scale invariance breaking. The trace of the energy-momentum tensor for the above Lagrangian, $T^\mu_\mu = -\lambda_2^2 \chi^4$, is assumed to be proportional to the gluon condensate, $\langle G^2_{\mu\nu} \rangle$. The scale parameter Λ is of the order of $\Lambda_{\rm QCD} \approx 200$ MeV.

This Lagrangian leads to the normal vacuum state, where chiral symmetry is spontaneously broken: $\sigma = f_\pi = 93$ MeV, $\boldsymbol{\pi} = 0$, and $\chi = \chi_0 = 136$ MeV. The parameters of the Lagrangian are chosen so that in the normal vacuum the constituent quark mass $m_q = g f_\pi = 313$ MeV, the σ-meson mass $m_\sigma^2 = 2\lambda_1^2 f_\pi^2 + m_\pi^2 \approx (0.6 \text{ GeV})^2$, and the glueball mass $m_G^2 = 4\lambda_2^2 \chi_0^2 + \mathcal{O}(m_\pi^2) \approx (1.7 \text{ GeV})^2$. The energy density associated with breaking the gluon condensate $B = \lambda_2^2 \chi_0^4/4$ is fixed to 0.5 GeV/fm^3 [4]. In the case of thermal equilibrium and a frozen χ field, $\chi = \chi_0$, the model leads to a chiral transition at temperatures around 130 MeV [5].

The form of the effective glueball potential Eq. (1) is motivated also by the instanton liquid model (see the recent review [6]) if χ^4 is identified with the instanton density. As predicted by this model, the instanton density is significantly suppressed at high temperatures. In our calculations the coupling of the gluon condensate to the thermal bath is parametrized in a simple form $J(x)\chi^2(x)$ [the last term in Eq. (2)], where $J(x) = AT^2(x)$ and $T(x)$ is the local temperature. The coupling strength $A \approx 2.4$ is chosen so that the gluon condensate, χ^4, is reduced by about 60% at $T = 280$ MeV.

The model presented above is not fully consistent since it does not include explicitly the thermal gluon excitations. In doing so we were motivated by the recent analysis [7] of lattice data demonstrating that at temperatures of interest here the gluons have a rather large effective mass of about 0.6–0.8 GeV. Therefore, the contribution of thermal gluons to all thermodynamical quantities is significantly suppressed by the Boltzmann factor. On the other hand, their contribution is included implicitly in the coupling term $J\chi^2$ determining the degree of reduction of the gluon condensate at high temperatures.

Below we adopt the mean field approximation considering σ, $\boldsymbol{\pi}$, and χ as classical fields. The equations of

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motion for these fields are obtained by applying the variational principle to the above Lagrangian. The source terms in these equations are determined by the distribution of quarks and antiquarks, which in principle should be found by solving the Dirac equation. Because of the interaction with meson fields, quarks acquire an effective mass [8]

$$m_q(x) = g\sqrt{\sigma^2(x) + \boldsymbol{\pi}^2(x)}, \tag{3}$$

which, in general, is space and time dependent. This makes an exact solution of the Dirac equation very difficult. To avoid this problem one should make further approximations. A reasonable starting point is the Vlasov-type kinetic equation for the scalar part of the quark-antiquark Wigner function $f(x, p)$,

$$\left[p^\mu \frac{\partial}{\partial x^\mu} + \frac{1}{2}\frac{\partial m_q^2(x)}{\partial x^\mu}\frac{\partial}{\partial p^\mu}\right] f(x,p) = I_{\mathrm{coll}}[f], \tag{4}$$

where I_{coll} is the collision term. In Refs. [8–10] the collisionless ($I_{\mathrm{coll}} = 0$) approximation was used for the propagation of quarks in background meson fields. Obviously this approximation can be justified only for the late stages of the expansion.

Here we consider another approximation which is more appropriate to high temperatures. Namely, we assume that the partonic collisions are frequent enough to maintain local thermodynamical equilibrium. In this case $f(x, p)$ can be represented in terms of the equilibrium distribution functions characterized by local temperature $T(x)$ and chemical potential $\mu(x)$.

By multiplying both sides of Eq. (4) with p^μ, projecting on the mass shell, $p^\mu p_\mu = m^2(x)$, and integrating over 4-momenta one arrives at the equations of relativistic hydrodynamics [11] (see also [12])

$$\frac{\partial}{\partial x^\mu} T^{\mu\nu}(x) + \rho_s(x)\frac{\partial}{\partial x_\nu} m_q(x) = 0, \tag{5}$$

where $T^{\mu\nu}(x)$ is the energy-momentum tensor, $T^{\mu\nu} = (\mathcal{E} + P)u^\mu u^\nu - Pg^{\mu\nu}$, and $u^\nu(x)$ is the collective 4-velocity of the quark-antiquark fluid. Here the energy density $\mathcal{E}$, pressure P, and scalar density ρ_s are functions of $T(x)$, $\mu(x)$, and $m_q(x)$. They can be expressed in the standard way through the Fermi-Dirac occupation numbers. We solve these equations consistently with the equations of motion for the meson fields which determine the quark effective mass through Eq. (3). This is why we call this approach chiral fluid dynamics.

The evolution of the glueball field is driven by the couplings to the chiral fields and to the thermal bath. The corresponding source $J(x)$ drops with the characteristic hydrodynamical time of order of a few fm/c. Because of some uncertainties in the glueball Lagrangian, particularly, in the derivative terms, below we consider two options of the model, i.e., full dynamics as described above and pure chiral dynamics with a frozen glueball field ($\chi \equiv \chi_0$).

Numerical results.—For numerical simulations we have used the RHLLE algorithm described and tested in [13] for fluid dynamics and the staggered leapfrog method for the field equations. Below we present results for the real-time evolution of spherical droplets of radii $R = 2$ and $R = 4$ fm. In the initial state we take a baryon-free ($\mu = 0$) fluid with a Woods-Saxon temperature profile and a linear profile of the collective momentum density. Initially the system is assumed to be in the chiral-symmetric phase at temperature $T \approx 280$ MeV. The initial conditions for the fields are chosen uniformly within the droplet and smoothly interpolated to their vacuum values outside the droplet. We assume that σ and χ fields are initially close to their equilibrium values at this high temperature.

The results of the calculations for $R = 4$ fm are presented in Figs. 1 and 2. It is seen that within a few fm/c the energy density of the fluid drops from the initial value of about 5.0 GeV/fm^3 to below 0.1 GeV/fm^3. A

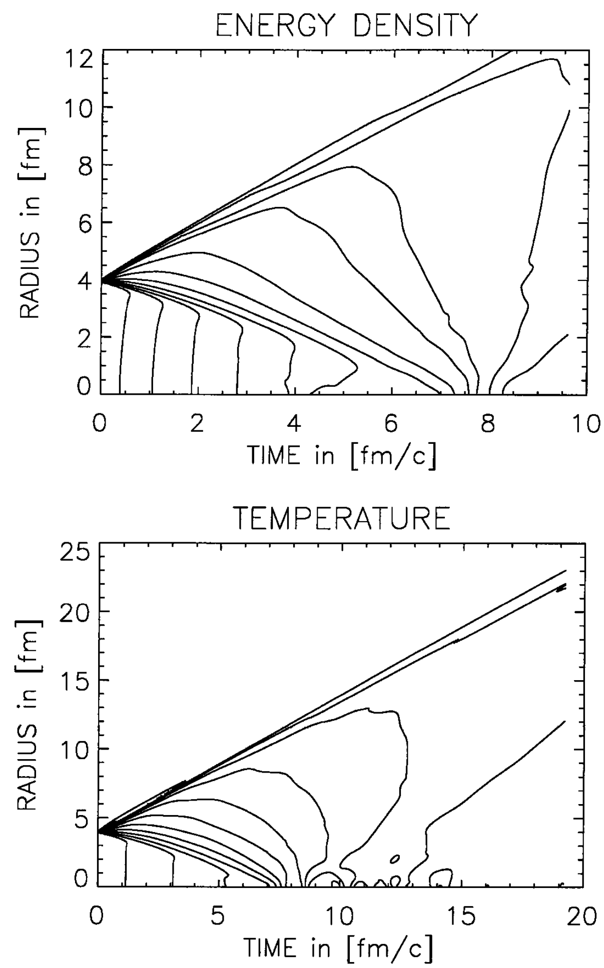

FIG. 1. Contour plots in the r-t plane (case I) of the energy density in GeV/fm^3 [(top) from left the subsequent decreasing levels are 5.0, 4.55, 4.05, 3.55, 3.05, 1.55, 2.05, 2.55, 1.05, 0.55, 0.35, 0.15, and 0.05 GeV/fm^3] and temperatures in MeV [(bottom) from left the subsequent decreasing levels are 280, 260, 240, 220, 200, 180, 160, 140, 120, 100, 80, 60, and 40 MeV] in the expanding spherical quark-antiquark droplet of initial radius $R = 4$ fm. The initial collective velocity is about $0.2c$ at the surface.

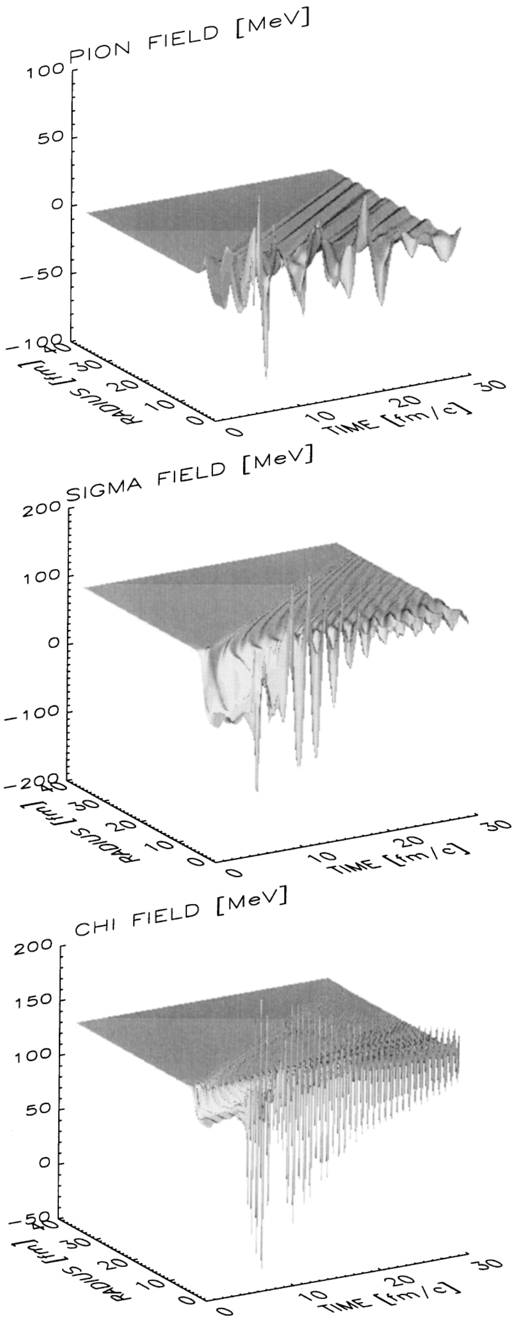

FIG. 2. The evolution of pion, sigma, and glueball fields in the t-r plane for an expanding droplet of initial radius $R = 4$ fm. The initial values (case I) of the fields are $\sigma = -0.05 f_\pi$, $\dot{\sigma} = 0$, $\pi_3 = 0.2 f_\pi$, $\dot{\pi}_3 = 0$, $\chi = 0.79\chi_0$, and $\dot{\chi} = 0$.

shell-like structure of the matter distribution is clearly seen later [14,15]. The fluid is cooled down to $T = 130$ MeV already at $t \approx 5$ fm$/c$. As Fig. 2 shows, at this time the σ field changes rapidly from its initial value, $\sigma \approx 0$, towards the new asymptotic value, $\sigma = f_\pi$. This transition is accompanied by strong nonlinear oscillations of the all coupled fields.

The pion field oscillations are especially strong and spread over the whole space within the light cone. In accordance with previous studies [16–20], our calculations show that soft pion modes are indeed strongly amplified (by a factor of 10) in the course of the chiral transition even in a finite expanding droplet. As suggested earlier (see, e.g., [21–23]), a perfect isospin alignment of the classical pion field should lead to a nonstatistical distribution of the ratio of neutral to charged pions. One should however bear in mind that these coherent pions will be accompanied by a large number of genuine pions (in the considered example, about 1000) produced at the hadronization of plasma.

It is interesting to note that the heavy σ and χ fields have quite different dynamics compared to the pion field. Initially they evolve almost adiabatically following the instantaneous temperature. Instead of expanding they first shrink and then rebound at about the time of the chiral transition, when strong oscillations start. One can understand this behavior from the following consideration. The regions where σ and χ significantly deviate from their vacuum values, f_π and χ_0, have an extra energy density $\Delta\mathscr{E}_{\rm vac} \sim B$, compared to the normal vacuum. This vacuum energy excess generates a negative pressure, $P_{\rm vac} \equiv -\Delta\mathscr{E}_{\rm vac}$. Such regions can survive only until the internal pressure of matter (in our case, quark-antiquark fluid), $P_{\rm mat}$, is large enough to counterbalance the external vacuum pressure, i.e., when $P_{\rm mat} + P_{\rm vac} \geq 0$. This condition is always fulfilled in an equilibrated system. However, in the course of a rapid expansion the opposite condition, $P_{\rm mat} + P_{\rm vac} < 0$, can eventually be reached in a certain region of space which we call a vacuum bubble. Then the outside vacuum propagates into this bubble trying to minimize its size. This process looks like a collapse of an air bubble in a liquid. In our case the role of a liquid is played by the vacuum quark and gluon condensates.

The collapse starts from the surface of the quark-antiquark droplet. As the energy density of the fluid decreases the speed of the ingoing wave increases. Finally the true vacuum penetrates to the center. Because of the inertial forces the condensates overshoot their equilibrium vacuum values. This is why very strong oscillations are developed at the center of the bubble. This violent dynamics may lead to very interesting phenomena like particle production by the bremsstrahlung mechanism, reheating of the fluid, or trapping of some quarks and antiquarks (especially heavy ones s, $\bar{s}$, c, $\bar{c}$) in the bubble. In this paper we consider only the first process.

Particle production.—In general, a time-dependent meson field $\phi(\mathbf{r}, t)$ can be represented asymptotically as an ensemble of quanta of this field. Using the coherent state formalism one can write the explicit expression for the momentum distribution of the mesons produced (see, e.g., Refs. [10,16])

$$2\omega_{\mathbf{k}} \frac{dN_\phi}{d^3k} = \frac{1}{(2\pi)^3}\left[|\dot{\phi}(\mathbf{k}, t)|^2 + \omega_{\mathbf{k}}^2 |\phi(\mathbf{k}, t)|^2\right], \quad (6)$$

where $\omega_{\mathbf{k}} = \sqrt{\mathbf{k}^2 + m_\phi^2}$ is the single-particle energy of a meson with the vacuum mass m_ϕ. In this formula $\phi(\mathbf{k},t)$ and $\dot{\phi}(\mathbf{k},t)$ are the three-dimensional Fourier transforms of the meson field $\phi(\mathbf{r},t)$ and its time derivative $\dot{\phi}(\mathbf{r},t)$. They are obtained from the dynamical simulations described above. The right-hand side should be calculated at sufficiently late times, when nonlinearities in the field equations are negligible.

The numbers of produced particles depend sensitively on the initial conditions for the fields, droplet size, and the expansion dynamics. In Table I we present the particle numbers (calculated at $t = 30$ fm$/c$) for the illustrative example of Fig. 2, $R = 4$ fm, with the initial glueball field $0.79\chi_0$ (case I) and $0.1\chi_0$ (case II), as well as for a smaller droplet, $R = 2$ fm. For comparison, the results for the frozen glueball field, $\chi \equiv \chi_0$, are also shown. The inclusion of a dynamical glueball field leads to an increase in the produced particle number by a factor of 2 or more, mainly due to the splitting of soft χ modes into pion and sigma modes with momenta of about $m_\chi/2$. But a larger fraction of energy, associated with the initially suppressed gluon condensate, goes into the bremsstrahlung of the χ field. One can see that much more particles, especially glueballs, are produced when the system is initially trapped in a metastable state with $\chi \approx 0$ (case II). On the other hand, significantly fewer particles are produced from a smaller droplet with $R = 2$ fm.

Conclusions.—It is demonstrated that the chiral transition in an expanding finite droplet is accompanied by strong space-time oscillations of the background fields. The long wavelength modes of the pion field are strongly amplified, by a factor of 10, in the course of transition. The gluon condensate brings into play a new scale of energy density, $B = 0.5$ GeV$/$fm^3, which significantly changes the dynamics.

The simulations reveal a novel phenomenon, the formation and collapse of the vacuum bubbles, associated with the regions of out-of-equilibrium quark and gluon condensates and low matter pressure. The additional energy released in the collapse goes partly into the coherent pions, but to a larger extent, to the production of σ mesons and glueballs. Because of a very large width of the σ meson its direct observation in heavy ion collisions is practically impossible. But the glueballs can be detected by the characteristic decay channels $G \to \pi\pi, \bar{K}K$ with widths of about 100 MeV, as well as by the electromagnetic decay ($G \to \gamma\gamma$) with a width of a few keV.

TABLE I. The numbers of neutral pions (N_{π_0}), σ mesons (N_σ), and glueballs (N_G) produced by the bremsstrahlung mechanism in the course of a quark-antiquark droplet expansion. Results are presented for initial droplet radii 2 and 4 fm, as well as for full (cases I and II) and frozen χ dynamics.

R	χ dynamics	N_{π_0}	N_σ	N_G
2 fm	Frozen	0.4	0.4	0
	Full (I)	0.4	0.7	0.5
	Full (II)	3	2	0.7
4 fm	Frozen	5	2	0
	Full (I)	4	6	3
	Full (II)	13	10	17

In the future we are planning to improve the model in two directions. First, a more realistic treatment of the gluon condensate and its coupling to the partonic plasma should be introduced. Second, friction terms due to the interaction of meson fields with the thermal bath should be included.

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