Leptonic Domains in the Early Universe and Their Implications

Xiangdong Shi and George M. Fuller
Department of Physics, University of California, San Diego, La Jolla, California 92093-0319
(Received 5 April 1999)

We apply causality considerations to active-sterile neutrino transformation-based schemes for lepton number generation in the early Universe. These considerations necessarily lead to the creation of spatial domains of lepton number with opposite signs. Lepton number gradients at domain boundaries can open a new channel for Mikheyev-Smirnov-Wolfenstein resonant production of sterile neutrinos (ν_s). This enhanced ν_s production allows considerable tightening of big bang nucleosynthesis constraints on active-sterile neutrino mixing.

PACS numbers: 98.80.Cq, 14.60.Pq

It is well known that resonant MSW (Mikheyev-Smirnov-Wolfenstein) [1] transitions between active neutrinos and sterile neutrinos in the early Universe could generate large lepton number asymmetries in the neutrino sector [2–5]. Here the lepton number for an active neutrino species ν_α is defined to be $L_{\nu_\alpha} \equiv (n_{\nu_\alpha} - n_{\bar\nu_\alpha})/n_\gamma$, the net asymmetry in ν_α over $\bar\nu_\alpha$ number density normalized by the photon number density n_γ.

Not well noted is a crucial feature involved in the lepton number generation process [3,5,6]: that the lepton number asymmetry is first damped to essentially zero by the active-sterile neutrino mixing, then oscillates chaotically with an progressively larger amplitude as the mixing goes through resonances, until the asymmetry converges to a growing asymptotic value that is either positive or negative.

As a result of this chaotic feature, the sign of the lepton number asymmetry is independent of the initial conditions which obtain before the instability begins, and is exponentially sensitive to the parameters involved during the chaotic oscillatory phase. In turn, the lepton number generated in this process may not have a uniform sign in different causal domains. Obviously, an upper bound to the size of these domains is the particle horizon $H^{-1} \approx (90/8\pi^3)^{1/2} g^{-1/2} m_{\rm pl}/T^2$, where g is the statistical weight in relativistic particles, T is the temperature of the Universe, and $m_{\rm pl} \approx 1.22 \times 10^{28}$ eV.

The typical size of these domains is at least as large as the diffusion length of neutrinos at the time of the lepton number generation. These leptonic domains can persist as long as the resonant neutrino transition is capable of efficient lepton number generation. This is because any reduction of lepton number at domain boundaries due to mixing will be quickly reversed by the generation process so that a boundary region is incorporated into one domain or the other.

The existence of leptonic domains in the early Universe provide a new channel of producing sterile neutrinos, via the resonant MSW conversion of active neutrinos to sterile neutrinos at domain boundaries, where lepton number gradients exist. If during the big bang nucleosynthesis (BBN) epoch the population of sterile neutrinos from this channel becomes comparable to that of an equilibrated active neutrino flavor, the extra neutrino degrees of freedom (an increase in g) can increase the primordial helium abundance significantly [7]. Therefore, such a production mechanism can be constrained by the observationally inferred primordial helium abundance.

The details of the active-sterile neutrino transformation process in the early Universe and the associated generation of lepton number asymmetries can be found in a number of previous works [3–5]. Here we only briefly summarize. In all calculations, we employ the natural units $\hbar = c = k_{\rm B} = 1$. A two-family system $\nu_\alpha \leftrightarrow \nu_s$ ($\alpha = e$, μ, or τ, and ν_s is the sterile neutrino) has a 2×2 evolution Hamiltonian $\mathcal{H}$ with $\mathcal{H}_{\alpha\alpha} = V_z$, $\mathcal{H}_{\alpha s} = \mathcal{H}_{s\alpha}^\dagger = V_x + iV_y$, and $\mathcal{H}_{ss} = 0$. The effective vector potential $\mathbf{V}$ during the BBN epoch (below the QCD phase transition temperature $T \lesssim 100$ MeV) is

$$V_x = -\frac{\delta m^2}{2E}\sin 2\theta, \qquad V_y = 0\,, \qquad V_z = -\frac{\delta m^2}{2E}\cos 2\theta + V_\alpha^L + V_\alpha^T, \tag{1}$$

where $\delta m^2 \approx m_{\nu_s}^2 - m_{\nu_\alpha}^2$, θ is the vacuum mixing angle, and E is the neutrino energy. The matter-antimatter asymmetry contribution to $\mathbf{V}$ is [8]

$$V_\alpha^L \approx \pm 0.35 G_F T^3 \left[L_0 + 2L_{\nu_\alpha} + \sum_{\beta\neq\alpha} L_{\nu_\beta} \right] \tag{2}$$

with the "+" sign for ν_α and the "−" sign for $\bar\nu_\alpha$. The quantity L_0 represents the contribution from the baryonic asymmetry and electron-positron asymmetry, i.e., $L_0 \sim 10^{-10}$. The quantity L_{ν_β} is the asymmetry in other active neutrino species ν_β. For simplicity, and with no loss of generality, we will assume $L_{\nu_\beta} = 0$ unless explicitly stated otherwise. The contribution to $\mathbf{V}$ from a thermal neutrino background is [8]

$$V_\alpha^T \approx \begin{cases} -80 G_F^2 E T^4 & \text{for } \alpha = e\,; \\ -22 G_F^2 E T^4 & \text{for } \alpha = \mu, \tau\,. \end{cases} \tag{3}$$

V_α^T is the same for both ν_α and $\bar\nu_\alpha$.

Resonances occur when $\mathcal{H}_{\alpha\alpha} = \mathcal{H}_{ss}$, i.e., $V_z = 0$. This is when ν_α and ν_s become degenerate in effective

 0031-9007/99/83(16)/3120(4)$15.00

mass and maximally mixed in medium. Very simplistically, when V_z evolves through zero, ν_α and ν_s swap their flavors if the resonance is adiabatic ($|dV_z/dt| \ll V_x^2$), and remain essentially unaltered if the resonance is nonadiabatic ($|dV_z/dt| \gg V_x^2$). Since the resonance condition is energy dependent, only neutrinos with $E_{\rm res}$ are resonant at any given temperature.

When $V_\alpha^L = 0$ (or negligibly small compared to $|\delta m^2|/2E$), resonances can occur for the $\nu_\alpha \leftrightarrow \nu_s$ system only if $\delta m^2 < 0$ (i.e., $m_{\nu_\alpha} > m_{\nu_s}$). For $\delta m^2 > 0$, and at high enough temperatures for $\delta m^2 < 0$, it has been shown that active-sterile neutrino transformation can damp $L_0 + 2L_{\nu_\alpha}$ to zero very efficiently. The amplification of $L_0 + 2L_{\nu_\alpha}$ starts for $\delta m^2 < 0$ only as the temperature falls below the critical temperature,

$$T_{\rm c} \approx 22|\delta m^2/1 \text{ eV}^2|^{1/6} \text{ MeV}. \tag{4}$$

Below $T_{\rm c}$, a significant number of ν_α go through MSW resonance. $L_0 + 2L_{\nu_\alpha}$ (or L_{ν_α} once $L_{\nu_\alpha} \gg L_0$) becomes unstable and starts to oscillate chaotically near $T_{\rm c}$, with an exponentially increasing amplitude [3,5]. Eventually, at a temperature slightly below $T_{\rm c}$, the oscillatory behavior abates, and L_{ν_α} settles into one of two fixed points (two T^{-4} power laws) [2–5]:

$$L_{\nu_\alpha}^{(\pm)} \sim \pm \left| \frac{\delta m^2}{10 \text{ eV}^2} \right| \left(\frac{T}{1 \text{ MeV}} \right)^{-4}. \tag{5}$$

The sign of the emergent L_{ν_α}, however, is independent of the initial L_0 and L_{ν_α} above $T_{\rm c}$ and is exponentially sensitive to the evolution of the potential $\mathbf{V}$ during the oscillatory phase. The sign is therefore chaotic and uncorrelated across causal domains.

In the power-law regime, V_α^T is negligible (because it scales as T^5). The growth of L_{ν_α} is driven by the resonant conversion of ν_α to ν_s (if $L_{\nu_\alpha} < 0$) or $\bar\nu_\alpha$ to $\bar\nu_s$ (if $L_{\nu_\alpha} > 0$) in the sector of the neutrino energy spectrum with $E_{\rm res}/T \approx 0.06|\delta m^2/1 \text{ eV}^2||L_{\nu_\alpha}|^{-1}(T/1 \text{ MeV})^{-4}$, which is in general $\ll 1$ [4,5]. The power-law solution Eq. (5) is very stable: a larger $|L_{\nu_\alpha}|$ implies a small $E_{\rm res}/T$, and therefore a less efficient generation of $|L_{\nu_\alpha}|$; a smaller $|L_{\nu_\alpha}|$ implies a larger $E_{\rm res}/T$ and a more efficient generation of $|L_{\nu_\alpha}|$. The rate of this relaxation to Eq. (5) is $d\ln|L_{\nu_\alpha}|/d\ln T \gtrsim 4$.

As $|L_{\nu_\alpha}|$ increases, $E_{\rm res}/T$ will slowly increase and eventually sweep through the entire energy spectrum. This is because keeping the power-law L_{ν_α} growth requires progressively more ν_α or $\bar\nu_\alpha$ to be resonantly converted into sterile neutrinos. Therefore, $|L_{\nu_\alpha}|$ has a physical limit $|L_{\nu_\alpha}| = 3/8$, when the entire ν_α or $\bar\nu_\alpha$ population has been converted into sterile neutrinos. When this limit is approached, the lepton number generation process ceases, and any inhomogeneity of lepton number arising from the process begins to be smoothed out by neutrino diffusion.

Because of the chaotic behavior of the sign of L_{ν_α}, domains of lepton number with opposite signs are expected to form at the epoch when $T \approx T_{\rm c}$. At the domain boundaries, mixing between different domains tends to reduce the asymmetries and increases the thickness of the boundary regions. However, the resonant neutrino mixing tends to maintain the solution in Eq. (5) and so narrows the boundary region. The thickness of the boundaries, very crudely, is thus the diffusion length of active neutrinos within the time in which $|L_{\nu_\alpha}|$ grows by an e-folding, $H^{-1}/4$. Taking the ν_α collision rate $\Gamma_{\nu_\alpha} \sim G_F^2 T^5$, we obtain a boundary thickness relative to the horizon scale ct:

$$\delta_d \sim \frac{c}{\Gamma_{\nu_\alpha}} \left(\frac{\Gamma_{\nu_\alpha}}{4H} \right)^{1/2} \sim ct \left(\frac{T}{1 \text{ MeV}} \right)^{-1.5}, \tag{6}$$

for $T \gtrsim 1$ MeV when active neutrinos are still diffusive. At temperatures $T \lesssim 1$ MeV, neutrinos decouple from the plasma and free-stream at the speed of light. In this case $\delta_d \sim ct$.

The existence of lepton domains results in a gradient of L_{ν_α}, and therefore a gradient in V_z. More importantly, the varying V_z at the domain boundaries satisfies the resonant condition for most ν_α crossing the boundaries. This is because [as Eq. (5) shows] the power-law solutions within each domain result from resonant transitions of ν_α or $\bar\nu_\alpha$ with $E_{\rm res} \ll T$ [4,5]. As a result, at domain boundaries $E_{\rm res}$ becomes larger. This is because $E_{\rm res} \propto |L_{\nu_\alpha}|^{-1}$. Most of ν_α and $\bar\nu_\alpha$ therefore undergo resonant transitions to sterile neutrinos at domain boundaries.

This new channel of sterile neutrino production can populate a significant sterile neutrino sea with a number density comparable to that of an equilibrated active neutrino flavor if (i) the resonant conversion at the boundary region is adiabatic and (ii) this new production channel for sterile neutrinos does not provide negative feedback to the lepton number asymmetry generation process and so does not compromise the domain structure of lepton number. Here we demonstrate that both requirements can be satisfied.

The adiabaticity condition at the resonances is $V_x^2 > |dV_z/dt| = |c(\partial V_z/\partial r) - HT(\partial V_z/\partial T)|$, evaluated at $V_z = 0$. The second term, of order HV_α^L, is always small compared to the first term as long as the leptonic domains exist. Since the spatial gradient is expected to be smooth across the boundary (whose thickness is determined by the diffusion/streaming process), we can employ the average $|\partial V_z/\partial r|$ across the domain boundaries, $\sim 0.7 G_F T^3 (L_{\nu_\alpha}^{(+)} - L_{\nu_\alpha}^{(-)})/\delta_d$ [where $L_{\nu_\alpha}^{(+)} \sim -L_{\nu_\alpha}^{(-)}$ satisfying Eq. (5)]. In turn, the adiabaticity condition becomes

$$|\delta m^2|^2 \sin^2 2\theta > 10H \left(\frac{E}{T} \right)^2 G_F T^5 \left(\frac{T}{1 \text{ MeV}} \right)^{1.5} \times \min\left[\left| \frac{\delta m^2}{10 \text{ eV}^2} \right| \left(\frac{T}{10 \text{ eV}^2} \right)^{-4}, \frac{3}{8} \right]. \tag{7}$$

The production of sterile neutrinos via adiabatic conversion of ν_α at domain boundaries will not have a negative impact on the L_{ν_α} generation process and the domain structure of lepton number. Consider a domain boundary with a lepton asymmetry $L_{\nu_\alpha}^{(+)}$ on one side and $L_{\nu_\alpha}^{(-)}$ on the other. The $L_{\nu_\alpha}^{(+)}$ side has fewer $\bar{\nu}_\alpha$ than ν_α, and some $\bar{\nu}_s$ from resonant $\bar{\nu}_\alpha \to \bar{\nu}_s$ conversions. The $L_{\nu_\alpha}^{(-)}$ side has the opposite, with more $\bar{\nu}_\alpha$ than ν_α, and some ν_s from resonant $\nu_\alpha \to \nu_s$ conversions. When neutrinos cross the boundary from the $L_{\nu_\alpha}^{(+)}$ side to the $L_{\nu_\alpha}^{(-)}$ side, the resultant production of sterile neutrinos due to $\nu_\alpha \to \nu_s$ and $\bar{\nu}_\alpha \to \bar{\nu}_s$ has no bearing on the asymmetry of ν_α on the $L_{\nu_\alpha}^{(-)}$ side. The existence of sterile neutrinos does not hinder the L_{ν_α} generation process until the sterile neutrino population is comparable in numbers to the ν_α population [3]. The resonant conversion of $\bar{\nu}_s \to \bar{\nu}_\alpha$ due to the crossing, on the other hand, produces more $\bar{\nu}_\alpha$ in the $L_{\nu_\alpha}^{(-)}$ domain and only reinforces the domain structure.

Therefore, once the adiabaticity condition Eq. (7) is met, this new channel of sterile neutrino production may be potent enough to bring the sterile neutrinos into equilibrium with active neutrinos. (In fact, this condition is conservative because neutrinos may cross multiple domain boundaries within a Hubble time.) On the other hand, the observationally inferred primordial ^{4}He abundance and deuterium abundance constrain the total number of neutrino flavors in equilibrium N_ν to be $\lesssim 3.3$ [5,7,9] (N_ν represents relativistic degrees of freedom in neutral fermions). This constraint implies that the new sterile neutrino production channel cannot be efficient before the decoupling of the $\nu_\alpha \bar{\nu}_\alpha$ pair production process. For $\alpha = \mu$ and τ, this decoupling temperature is $\sim$5 MeV. A constraint on the two-family $\nu_{\mu,\tau} \leftrightarrow \nu_s$ mixing can therefore be obtained by requiring that the adiabaticity condition Eq. (7) is not satisfied at $T \sim \max(5, |\delta m^2/4\ \text{eV}^2|^{1/4})$ MeV (the latter term in the bracket is the temperature at which the growth of L_{ν_α} stops):

$$\begin{aligned} &|\delta m^2| \sin^2 2\theta < 7 \times 10^{-5}\ \text{eV}^2 && \text{for } |\delta m^2| \lesssim 2.5 \times 10^3\ \text{eV}^2; \\ &\sin^2 2\theta < 3 \times 10^{-8} && \text{for } |\delta m^2| \gtrsim 2.5 \times 10^3\ \text{eV}^2. \end{aligned} \tag{8}$$

For $\alpha = e$, the constraint from BBN is more severe. The sterile neutrino production cannot be efficient above the $\nu_e \bar{\nu}_e$ pair production decoupling temperature $T \sim$ 3 MeV and also at the weak freeze-out temperature of $T \sim 1$ MeV. At this temperature, a significant $\nu_e \to \nu_s$ and $\bar{\nu}_e \to \bar{\nu}_s$ transition would cause a deficit in the $\nu_e \bar{\nu}_e$ number density, which cannot be replenished by pair production. A significant deficit in the $\nu_e \bar{\nu}_e$ number density causes the neutron-to-proton ratio to freeze out too early and results in a primordial ^{4}He abundance that is too large. (For example, a 10% deficit in the $\nu_e \bar{\nu}_e$ number density has roughly the same effect on the ^{4}He abundance as $N_\nu \approx 3.5$.) Therefore the BBN constraint on the two-family $\nu_e \leftrightarrow \nu_s$ mixing is obtained at $T \sim \max(1, |\delta m^2/4\ \text{eV}^2|^{1/4})$ MeV:

$$\begin{aligned} &|\delta m^2| \sin^2 2\theta < 5 \times 10^{-8}\ \text{eV}^2 && \text{for } |\delta m^2| \lesssim 4\ \text{eV}^2; \\ &\sin^2 2\theta < 10^{-8} && \text{for } |\delta m^2| \gtrsim 4\ \text{eV}^2. \end{aligned} \tag{9}$$

These bounds are summarized in Fig. 1. They apply in addition to the previous bounds based on a universe with homogeneous lepton numbers, and together they offer much tighter constraints on the two-family active-sterile neutrino mixing. Note that our constraints do not depend on the geometric shape of the domains.

Intriguing results can also be obtained if there is active-sterile neutrino mixing involving three or more families. One example is the proposal that a $\nu_\mu \leftrightarrow \nu_s$ and $\nu_\tau \leftrightarrow \nu_{s'}$ (in principle $\nu_{s'}$ and ν_s can be the same flavor) mixing might be able to simultaneously explain the Super Kamiokande atmospheric neutrino data and satisfy the BBN bound [4,5]. A stand-alone $\nu_\mu \leftrightarrow \nu_s$ oscillation solution to the Super Kamiokande data would violate the BBN bound by bringing ν_s into equilibrium during BBN [10,11]. The double active-sterile neutrino oscillation proposal avoids the violation of the BBN bound by having a resonant $\nu_\tau \leftrightarrow \nu_{s'}$ transformation in the early Universe generate a L_{ν_τ} that hinders the ν_s production from the $\nu_\mu \leftrightarrow \nu_s$ mixing [by creating a L_{ν_β} term in Eq. (2)]. This argument is no longer valid once we consider the existence of the L_{ν_τ} domains as a result of the resonant $\nu_\mu \leftrightarrow \nu_{s'}$ transformation. Rather the contrary is true. The L_{ν_τ} domains *facilitate* the production of ν_s via resonant $\nu_\mu \to \nu_s$

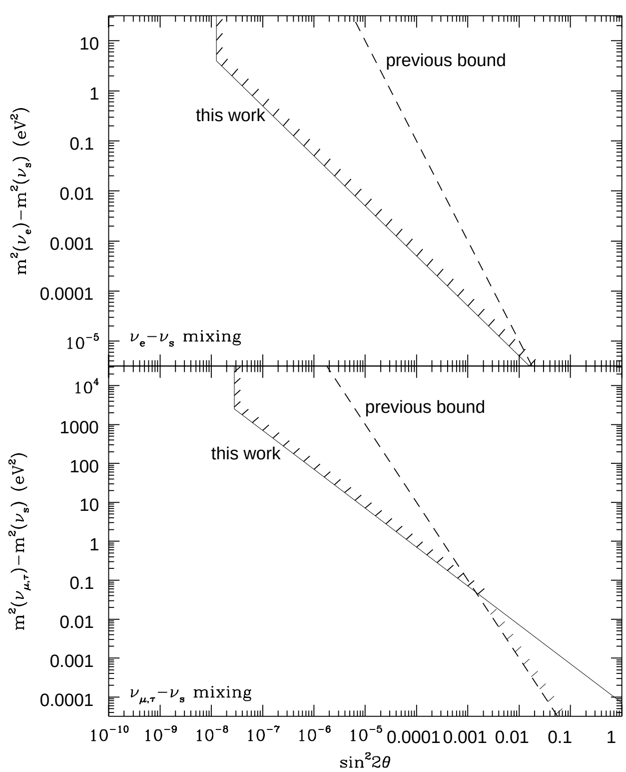

FIG. 1. Parameter spaces to the right of the hatched lines are excluded by BBN. The solid lines indicate bounds obtained in this work, and the dashed lines are previous bounds assuming a universe with a homogeneous lepton number [5,10].

transformation at domain boundaries. The adiabaticity condition, Eq. (7), is modified in this double mixing situation to be

$$|\delta m_1^2|^2 \sin^2 2\theta_1 > 5H\left(\frac{E}{T}\right)^2 G_F T^5 \left(\frac{T}{1\ \mathrm{MeV}}\right)^{1.5} \times \min\left[\left|\frac{\delta m_2^2}{10\ \mathrm{eV}^2}\right|\left(\frac{T}{1\ \mathrm{MeV}}\right)^{-4}, \frac{3}{8}\right], \tag{10}$$

where $\delta m_1^2 \approx |m_{\nu_\mu}^2 - m_{\nu_s}^2|$, θ_1 is the $\nu_\mu \leftrightarrow \nu_s$ vacuum mixing angle, and $\delta m_2^2 \approx m_{\nu_\tau}^2 - m_{\nu_{s'}}^2$. To be consistent with BBN, the adiabaticity condition cannot be satisfied from the onset of the L_{ν_τ} generation $T_c \approx 22|\delta m_2^2/1\ \mathrm{eV}^2|^{1/6}$ MeV to the $\nu_{\mu,\tau}$ decoupling temperature $T \sim 5$ MeV. However, for the parameters required to explain the Super Kamiokande data ($\delta m_1^2 \sim 10^{-3}$ to 10^{-2} eV^2 and $\sin^2 2\theta_1 \sim 1$), the adiabaticity condition is always satisfied in this double mixing proposal for any reasonable choices of the tau neutrino mass. Therefore, *BBN unambiguously rules out an active-sterile neutrino oscillation explanation to the Super Kamiokande data.*

Another interesting situation involving multifamily active-sterile neutrino mixing arises from neutrino mixing schemes proposed to explain simultaneously the Los Alamos Liquid Scintillator Neutrino Detector (LSND) $\bar{\nu}_e$ signal, the Super Kamiokande atmospheric ν_μ deficit, and the solar neutrino deficit [12]. In these models, $\nu_\mu \leftrightarrow \nu_e$ mixing (with $m_{\nu_\mu}^2 - m_{\nu_e}^2 \sim 0.2$ to 10 eV^2 and $\sin^2 2\theta_{\mu e} \sim 10^{-3}$) is employed to explain the LSND result and $\nu_e \leftrightarrow \nu_s$ mixing (with $m_{\nu_s}^2 - m_{\nu_e}^2 \sim 10^{-5}$ eV^2 and $\sin^2 2\theta_{es} \sim 10^{-3}$ for the MSW solution, and $|m_{\nu_s}^2 - m_{\nu_e}^2| \sim 10^{-10}$ eV^2 and $\sin^2 2\theta_{es} \sim 1$ for the vacuum solution) is invoked to explain the solar neutrino data. If there is mixing between ν_μ and ν_s as well, however, with $\sin^2 2\theta_{\mu s} \gtrsim 10^{-11}$ [3,4], the L_{ν_μ} domains generated by the mixing in the early Universe would imply that the $\nu_e \leftrightarrow \nu_s$ mixing would not populate enough ν_s to violate BBN constraints only if

$$|m_{\nu_s}^2 - m_{\nu_e}^2|^2 \sin^2 2\theta_{es} < 2 \times 10^{-14} \times \left|\frac{m_{\nu_\mu}^2 - m_{\nu_s}^2}{1\ \mathrm{eV}^2}\right|^{1/4} \mathrm{eV}^2 \sim 2 \times 10^{-14}\ \mathrm{eV}^2. \tag{11}$$

[This is in analogy to the previous example if we take $\delta m_1^2 \approx |m_{\nu_s}^2 - m_{\nu_e}^2|$ and $\delta m_2^2 \approx |m_{\nu_\mu}^2 - m_{\nu_s}^2|$ in Eq. (10).] This requirement is not satisfied by the MSW $\nu_e \leftrightarrow \nu_s$ solution to the solar neutrino problem. The vacuum solution to the solar neutrino problem, however, satisfies the requirement and is consistent with BBN when $\sin^2 2\theta_{\mu s} \gtrsim 10^{-11}$.

Therefore, in light of LSND, Super Kamiokande and solar neutrino experiments, the neutrino oscillation explanations of the LSND data and the MSW solution to solar neutrino data is inconsistent with BBN unless the $\nu_\mu \leftrightarrow \nu_s$ mixing is extremely small, $\sin^2 2\theta_{\mu s} \lesssim 10^{-11}$. This result holds even though the $\nu_\mu \leftrightarrow \nu_e$ mixing amplitude and the $\nu_e \leftrightarrow \nu_s$ mixing amplitude differ by a factor $\gtrsim 10^8$. This restriction severely constrains the ν_τ-ν_μ-ν_e-ν_s mixing matrix required to fit the current neutrino experiment results.

In summary, we have discussed the existence of leptonic domains as an inevitable consequence of proposed resonant active-sterile neutrino oscillation mechanisms for generation of lepton number. Resonant MSW conversion due to the lepton number gradients at domain boundaries therefore provides a new channel for sterile neutrino production. As a result, the big bang nucleosynthesis constraint on active-sterile neutrino mixing becomes much more stringent. Likewise, we expect more stringent constraints on multifamily neutrino mixing schemes involving sterile neutrinos. We have found that an active-sterile neutrino mixing explanation for the Super Kamiokande data is inconsistent with big bang nucleosynthesis in spite of the lepton number generation. We have also found that together the $\nu_\mu \leftrightarrow \nu_e$ explanation of the LSND result and the MSW $\nu_e \leftrightarrow \nu_s$ solution to the solar neutrino problem are incompatible with big bang nucleosynthesis considerations unless the amplitude of the mixing between ν_μ and ν_s is $\gtrsim 10^8$ smaller than that between ν_μ and ν_e and that between ν_e and ν_s.

X. S. and G. M. F. are supported in part by NSF Grant No. PHY98-00980 at UCSD.

[1] L. Wolfenstein, Phys. Rev. D **17**, 2369 (1978); **20**, 2634 (1979); S. P. Mikheyev and A. Yu. Smirnov, Sov. Phys. JETP **64**, 4 (1986).
[2] R. Foot, M. J. Thomson, and R. R. Volkas, Phys. Rev. D **53**, 5349 (1996).
[3] X. Shi, Phys. Rev. D **54**, 2753 (1996).
[4] R. Foot and R. R. Volkas, Phys. Rev. D **55**, 5147 (1997).
[5] X. Shi, G. M. Fuller, and K. Abazajian, Phys. Rev. D **59**, 063006 (1999).
[6] K. Enqvist, K. Kainulainen, and A. Sorri (unpublished).
[7] G. Steigman, D. N. Schramm, and J. E. Gunn, Phys. Lett. **66B**, 262 (1977); for a recent update, see C. Y. Cardall and G. M. Fuller, Astrophys. J. **472**, 435 (1996).
[8] D. Nötzold and G. Raffelt, Nucl. Phys. **B307**, 924 (1988).
[9] S. Burles, K. M. Nollett, J. N. Truran, and M. S. Turner (to be published).
[10] X. Shi, D. N. Schramm, and B. D. Fields, Phys. Rev. D **48**, 2563 (1993), and references therein.
[11] K. Enqvist, K. Kainulainen, and M. Thomson, Nucl. Phys. **B373**, 498 (1992).
[12] D. O. Caldwell, Int. J. Mod. Phys. A **13**, 4409 (1998), and references therein.