Radiation Corrections Increase Tunneling Probability

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We calculate the QED effect of the renormalization of the bare potential for the tunneling of a charged particle due to the interaction with its own radiation field (an analog of the Lamb shift for tunneling). It is demonstrated that radiative corrections increase the tunneling probability.

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The interaction of a tunneling object with other degrees of freedom of the system and the influence of this interaction on the tunneling probability for a long time was a topic of intensive studies initiated by Caldeira and Leggett [1]. Their general conclusion, in agreement with intuitive arguments, was that for a fixed potential barrier any friction-type interaction suppresses the tunneling. At the same time, it was realized that an interaction of the global tunneling coordinate with other degrees of freedom leads to the barrier renormalization which acts in the opposite direction being helpful in endorsing the tunneling [2]; see also the review paper [3]. The simplest effect is associated with the interaction of the tunneling particle with the vibrational modes of the source responsible for the existence of the barrier. This is important for the probabilities of subbarrier nuclear reactions as pointed out by Esbensen [4]. In the last decade, many experimental and theoretical efforts were devoted to the understanding of related aspects of subbarrier reactions, see the recent review [5] and references therein.

For any charged object moving in a static external potential, the interaction with the electromagnetic field always accompanies motion of the object. Below we show that the interaction with the virtual photon field, in agreement with the statement of [2], renormalizes the barrier and increases the tunneling probability. To the best of our knowledge, this QED effect, which can be considered as a tunneling counterpart for the Lamb shift of discrete levels [6], was not calculated earlier. As in the Lamb shift, the result is specific for each original "bare" potential. The effect should not be confused with a number of similar effects considered in the context of condensed matter physics, for example [7–9], which do not depend on the form of the bare barrier. Being independent of spin of the particle, it differs also from the similar Darwin term in the fine structure Hamiltonian which originates from the admixture of small bispinor components. However, the result is not universal and can be different for different boson fields and different interaction vertices.

Formally speaking, we are looking for the effects of radiative corrections on the single-particle tunneling. These effects can be described by the Schrödinger equation with the self-energy operator

$$\hat{H}\Psi(\mathbf{r}) + \int \Sigma(\mathbf{r},\mathbf{r}';E)\Psi(\mathbf{r}')\,d^3r' = E\Psi(\mathbf{r}), \qquad (1)$$

where $\hat{H}$ is the unperturbed particle Hamiltonian, which includes a barrier potential, and $\Sigma = M - i\Gamma/2$ is the complex nonlocal and energy-dependent operator determined by the coupling to virtual photons and by a possibility of real photon emission. In the one-photon approximation the self-energy due to the interaction with the transverse radiation field can be written as

$$\Sigma(\mathbf{r},\mathbf{r}';E) = \sum_{\mathbf{k},\lambda} |g_{\mathbf{k}}|^2 \sum_n \frac{\langle \mathbf{r}|(\hat{\mathbf{p}}\cdot\mathbf{e}_{\mathbf{k}\lambda})e^{i\mathbf{k}\hat{\mathbf{r}}}|n\rangle\langle n|(\hat{\mathbf{p}}\cdot\mathbf{e}^*_{\mathbf{k}\lambda})e^{-i\mathbf{k}\hat{\mathbf{r}}}|\mathbf{r}'\rangle}{E - E_n - \omega_{\mathbf{k}} - i0}. \qquad (2)$$

Here we sum over unperturbed stationary states $|n\rangle$ with energy E_n; $\hat{\mathbf{r}}$ and $\hat{\mathbf{p}}$ are the position and momentum operators, respectively. The states $|n\rangle$ belong to the continuous spectrum in the decay problem (scattering states) or to the discrete spectrum as in a double, or multiple, well potential. The photons are characterized by the momentum $\mathbf{k}$, frequency $\omega_{\mathbf{k}}$, and polarization λ; the polarization vectors $\mathbf{e}_{\mathbf{k}\lambda}$ are perpendicular to $\mathbf{k}$ so that the momentum operators commute with the exponents. The normalization factors are included in $g_{\mathbf{k}} \propto \omega_{\mathbf{k}}^{-1/2}$. We need to emphasize that the only approximation in Eq. (2) is the one-photon truncation of intermediate states which retains the lowest order terms in the fine structure constant. The description (1) using the non-Hermitian and energy-dependent effective Hamiltonian can be rigorously derived by eliminating photon degrees of freedom [10]. A similar approach to one-body continuum channels is widely used in nuclear reaction theory [11,12]. The relativistic generalization of (2) is straightforward.

The Hermitian part M of the self-energy operator is given by the principal value integral over photon

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frequencies in (2). The expectation value of M is responsible for the Lamb shift of bound energy levels. It contains also the mass renormalization for a free particle which should be subtracted (for a dissipative system the mass renormalization was discussed in [8]). Our problem is different from the energy shift calculation for bound states since we are interested in those changes of the wave function of the tunneling particle which can influence the exponentially small penetration probability. However, we can exploit some features of the conventional approach. As well known from the Lamb shift calculations, one can use different approximations in the two regions of integration over the photon frequency ω. In the nonrelativistic low-frequency region, $\omega < \beta m$, where the parameter $\beta < 1$ is chosen in such a way that typical excitation energies of a particle in the well δE are smaller than βm (in the hydrogen Lamb shift problem a fine structure constant α can play the role of the borderline scale parameter), it is possible to neglect the exponential factors in (2). The high-frequency contribution to M, where the potential can be considered as a perturbation to free motion, has been calculated, e.g., in Ref. [6]. The two contributions match smoothly at $\omega = \beta m$, which means that the transitional region is not essential for the final result.

It is easy to estimate the mass operator M with logarithmic accuracy. After summation over polarizations and standard regularization [6], the low-frequency part of the operator M can be written as

$$\hat{M}(E) = \frac{2Z^2\alpha}{3\pi m^2}\int d\omega \sum_n \hat{\mathbf{p}}|n\rangle \frac{E - E_n}{E - E_n - \omega}\langle n|\hat{\mathbf{p}}\,, \quad (3)$$

where Ze is the particle charge and m is the mass of the particle (reduced mass in the alpha-decay case). We use the units $\hbar = c = 1$. Substituting the logarithm arising from the frequency integration by its average value $L = \ln(\beta m/\omega_{\min})$, we can use the closure relations and obtain a simple expression

$$\hat{M}(E) = \frac{2Z^2\alpha}{3\pi m^2} L\hat{\mathbf{p}}(\hat{H} - E)\hat{\mathbf{p}} \quad (4)$$

$$= \frac{Z^2\alpha}{3\pi m^2} L\{\nabla^2\hat{U} + [(\hat{H} - E), \hat{\mathbf{p}}^2]_+\}\,. \quad (5)$$

The mean value of the term with the anticommutator $[\ldots,\ldots]_+$ in Eq. (5) is equal to zero since $(\hat{H} - E)\Psi_0 = 0$, where Ψ_0 is the unperturbed wave function. A small correction to the wave function due to this term can be calculated by using perturbation theory and the unperturbed Schrödinger equation,

$$\delta\Psi = \frac{2Z^2\alpha}{3\pi m} L[U - \langle 0|U|0\rangle]\Psi_0\,. \quad (6)$$

This correction is not essential for our purpose since it does not influence the exponent in the tunneling amplitude.

Combining the remaining term in Eq. (5) with the high-frequency contribution which contains $L = \ln(m/\beta m)$ (see Ref. [6]), the result can be presented as an effective local operator proportional to the Laplacian $\nabla^2 U(\mathbf{r})$,

$$M(\mathbf{r},\mathbf{r}';E) \simeq \nabla^2 U(\mathbf{r})\delta(\mathbf{r} - \mathbf{r}')\frac{Z^2\alpha}{3\pi m^2}\ln\frac{m}{U_0} \equiv \delta U(\mathbf{r})\delta(\mathbf{r} - \mathbf{r}')\,. \quad (7)$$

Here we used the barrier height U_0, with respect to energy E, as a lower cutoff $\omega_{\min}$ of the integration over frequencies (below we give a semiclassical estimate which leads also to a more accurate evaluation of the logarithmic factor). For the tunneling of an extended object, the mass m in the argument of the logarithm should be replaced by the inverse size of the particle $1/r_0$ which comes from the upper frequency cutoff given in this case by the charge form factor. The obtained result is physically equivalent to the averaging over the position fluctuations due to the coupling to virtual photons. Thus, in the logarithmic approximation the mass operator is reduced to a local correction $\delta U(\mathbf{r})$ to the potential $U(\mathbf{r})$. One may note that the result depends on the properties of the field coupled to the particle. In the case of a Dirac particle coupled to the pseudoscalar meson field, the vertex $(\hat{\mathbf{p}} \cdot \mathbf{e})$ is substituted by $(\boldsymbol{\sigma} \cdot \mathbf{k})$ where $\boldsymbol{\sigma}$ is the fermion spin operator and $\mathbf{k}$ is the meson momentum. Then the main effect reduces to the recoil of the particle because of the meson emission [formally it follows from the identity $\exp(i\mathbf{k} \cdot \mathbf{r})\hat{H}(\mathbf{r},\mathbf{p}) \times \exp(-i\mathbf{k} \cdot \mathbf{r}) = \hat{H}(\mathbf{r},\mathbf{p} - \mathbf{k})$].

The Laplacian of the potential energy $\nabla^2 U(\mathbf{r})$ near the maximum of the barrier is negative (correspondingly, near the bottom of the potential well it is positive). Therefore, we obtained the negative correction $\delta U(\mathbf{r})$ to the potential barrier which leads to a conclusion that jiggling of the photon increases the tunneling amplitude of the particle. The numerical value of the correction to the potential is small, $\sim$1 keV, for the alpha decay, and increases correspondingly for the high-Z fission fragments. Even a small effect may be noticeable in some cases due to the exponential dependence on the height of the barrier (recall the notorious cold fusion problem or atom ionization by the electric field). This effect should be added to other "small effects in astrophysical fusion reactions" considered in [13]. In contrast to the Uehling vacuum polarization potential [14] which exists for a pure Coulomb case, the effective operator (7) vanishes if $\nabla^2 U = 0$, being sensitive to the charge distribution. Through atomic electron screening, it may effect low-energy nuclear reactions of astrophysical interest.

In the double well problem, the accelerated tunneling increases the exponentially small splitting between the stationary states of opposite parity. Also, there exist theories like QCD where the radiation corrections are not small. In many-body systems one can use collective modes, as phonons, to transfer energy. This can influence electron tunneling through quantum dots or insulating surfaces. As we have already mentioned, the interaction between a tunneling particle and vibrational nuclear environment is known to be important in sub-barrier nuclear fission and fusion [5].

An analysis can be performed more in detail by using the semiclassical WKB approximation for the tunneling wave functions. The semiclassical radial Green function of unperturbed motion under the barrier can be written in terms of the classical momentum in the forbidden region, $p(r;E) = \{2m[U(r) - E]\}^{1/2}$, at a given energy E as

$$G(r,r';E) = -m[p(r)p(r')]^{-1/2}\{e^{-\int_{r'}^{r} d\xi\, p(\xi)}\Theta(r - r') + e^{-\int_{r}^{r'} d\xi\, p(\xi)}\Theta(r' - r)\}, \tag{8}$$

where $\Theta(x)$ is the step function. The full three-dimensional Green function $\hat{G}(E) = \sum_n |n\rangle(E - E_n)^{-1}\langle n|$ contains also angular harmonics which could be separated in a routine way, accounting for the fact that in the long wavelength approximation for the s-wave solution the intermediate states are p waves. Indeed, the operator of electric dipole radiation $\hat{\mathbf{p}}$ converts an initial s-wave Ψ in Eq. (1) into an intermediate p-wave state $|n\rangle$. Therefore, it is sufficient to keep the p-wave part of the radial Green function and to use closure in the sum over angular harmonics.

The kernel of the integral term in the Schrödinger equation (1) contains

$$K(r,r';E) = \int d\omega\, G(r,r';E - \omega). \tag{9}$$

The integrand consists of terms falling exponentially as $|r - r'|$ increases. The potential $U(r)$ is assumed to be a smooth function. Therefore, we can put $p(r') \approx p(r)$. Now it is easy to perform the integration over ω in Eq. (9) which leads to

$$K(r,r';E) = -\frac{1}{|r - r'|}\{e^{-p_{\min}|r-r'|} - e^{-p_{\max}|r-r'|}\}, \tag{10}$$

where $p_{\min} = \{2m[U_p(r) - E]\}^{1/2}$, $p_{\max} = [2\beta]^{1/2}m$, and $U_p(r)$ is the effective p-wave radial potential which includes the centrifugal part. This expression has a very narrow maximum near $r = r'$ with the width $|r - r'| \sim 1/p_{\max}$. This is a measure of nonlocality of the self-energy operator $M(r,r';E)$. In any nonrelativistic application the kernel can be treated as proportional to the delta function. The proportionality coefficient can be found by the integration over r. Thus, we obtain the local behavior of the kernel,

$$K(r,r';E) \approx -L(r)\delta(r - r'), \tag{11}$$

where now we determine the lower limit of the logarithm which has appeared in our previous derivation (7) as related to the local value of the potential,

$$L(r) = \ln\frac{m}{|U_p(r) - E|}. \tag{12}$$

The substitution into Eq. (7) gives

$$\delta U(\mathbf{r}) = \frac{Z^2\alpha}{3\pi m^2}\ln\frac{m}{|U_p(r) - E|}\nabla^2 U(\mathbf{r}). \tag{13}$$

As usual, this semiclassical expression is not valid near the turning points where $U_p(r) = E$. However, a very weak logarithmic singularity does not produce any practical limitations on the applicability of Eq. (13).

The conclusion of enhancement of the tunneling probability seems to contradict one's common sense: radiation should cause energy losses and reduce the tunneling amplitude of the charged particle. However, such an argument may be valid only for the real photon emission. This emission is described by the anti-Hermitian part of the self-energy operator which is originated from the delta function corresponding to on-shell processes,

$$\Gamma(\mathbf{r},\mathbf{r}';E) = \frac{4Z^2\alpha}{3m^2}\int d\omega \sum_n \langle \mathbf{r}|\hat{\mathbf{p}}e^{i\mathbf{k}\hat{\mathbf{r}}}|n\rangle\langle n|\hat{\mathbf{p}}e^{-i\mathbf{k}\hat{\mathbf{r}}}|\mathbf{r}'\rangle\omega\,\delta(E - E_n - \omega). \tag{14}$$

Because of the energy conservation the sum here includes only states $|n\rangle$ with energy E_n below E. Consider, for example, the tunneling from the ground s state. A dipole transition transfers the particle from the s state to a p state. However, there are no quasidiscrete p states $|n\rangle$ below the ground state in the potential well. Scattering p waves can penetrate the potential barrier from the continuum with an exponentially small amplitude. This means that $\Gamma(\mathbf{r},\mathbf{r}';E)$ is again exponentially small if one or both arguments $\mathbf{r}$ and $\mathbf{r}'$ are under the barrier or inside the potential well. Therefore, $\Gamma(\mathbf{r},\mathbf{r}';E)$ does not considerably influence the tunneling amplitude. The reason for that can be easily understood. A real radiation would be impossible if there were no tunneling; whence the radiation width must vanish together with the tunneling width. On the contrary, the real part of Σ under the barrier would be present even if the tunneling probability would vanish.

To avoid misunderstanding we need to stress that the contribution to the radiation intensity from the barrier area and the potential well, which was, in application to the nuclear alpha decay, the subject of recent experimental [15] and theoretical [16–19] studies, still may be important. The radiation amplitude with $E_s - E_p = \omega$ contains the matrix element

$$\langle s|\hat{\mathbf{p}}|p\rangle = \frac{1}{\omega}\langle s|[\hat{H},\hat{\mathbf{p}}]|p\rangle = \frac{i}{\omega}\langle s|\nabla U|p\rangle. \tag{15}$$

When one moves inside the barrier from the outer turning point inwards, the resonance s-wave function exponentially increases while the nonresonance p-wave function exponentially decreases. As a result, the product $\psi_s(r)\psi_p(r)$ does not change considerably. This means that the contribution to the real radiation from the inner area may be comparable to that from the area outside the barrier. The gradient ∇U changes its sign near the maximum of the potential which implies a destructive interference between the radiation from the different areas [since $|s\rangle$ is the nonoscillating ground state wave function, the product $\psi_s(r)\psi_p(r)$ does not change sign inside the barrier]. The resulting complicated pattern was discussed in detail in [18].

The QED effect considered above reminds one of the feat of the famous baron von Münchhausen who saved himself from the swamp by pulling his hair by his own hand [20]. The advanced part of the wave function of a charged tunneling particle can send a photon to the rear part which absorbs this photon and penetrates the barrier with enhanced probability. The "photon hand" here connects two points $\mathbf{r}$ and $\mathbf{r}'$ of the same wave function, and, as we have seen, these points are close to each other effectively renormalizing the local potential. This Münchhausen mechanism with a photon feedback may be particularly helpful for a composite system. In the tunneling of a two-body object, the first particle, while continuing to be accelerated by a bell shaped potential after the tunneling, can emit a (virtual) photon that increases energy of the second particle and its tunneling probability. As compared to phonon assisted tunneling this mechanism does not require any special device, being automatically provided by the interaction of a charged particle with the radiation field.

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[1] A. O. Caldeira and A. J. Leggett, Ann. Phys. (N.Y.) **149**, 374 (1983).
[2] A. Widom and T. D. Clark, Phys. Rev. Lett. **48**, 63 (1982); see also discussion in A. O. Caldeira and A. J. Leggett, Phys. Rev. Lett. **48**, 1571 (1982); A. Widom and T. D. Clark, Phys. Rev. Lett. **48**, 1572 (1982).
[3] P. Hänggi, P. Talkner, and M. Borkovec, Rev. Mod. Phys. **62**, 251 (1990).
[4] H. Esbensen, Nucl. Phys. **A352**, 147 (1981).
[5] A. B. Balantekin and N. Takigawa, Rev. Mod. Phys. **70**, 77 (1998).
[6] A. I. Akhiezer and V. B. Berestetskii, *Quantum Electrodynamics* (Interscience Publishers, New York, 1965).
[7] W. Zwerger, Z. Phys. B **47**, 129 (1982).
[8] G. W. Ford, J. T. Lewis, and R. F. O'Connell, Phys. Lett. A **158**, 367 (1991).
[9] C. L. Foden and D. M. Whittaker, Phys. Rev. B **58**, 12 617 (1998).
[10] W. Pauli and M. Fierz, Nuovo Cimento **15**, 167 (1938).
[11] C. Mahaux and H. A. Weidenmüller, *Shell-Model Approach to Nuclear Reactions* (North-Holland, Amsterdam, 1969).
[12] V. V. Sokolov and V. G. Zelevinsky, Nucl. Phys. **A504**, 562 (1989).
[13] A. B. Balantekin, C. A. Bertulani, and M. S. Hussein, Nucl. Phys. **A627**, 324 (1997).
[14] E. A. Uehling, Phys. Rev. **48**, 55 (1935).
[15] J. Kasagi *et al.*, Phys. Rev. Lett. **79**, 371 (1997).
[16] M. I. Dyakonov and I. V. Gornyi, Phys. Rev. Lett. **76**, 3542 (1996).
[17] T. Papenbrock and G. F. Bertsch, Phys. Rev. Lett. **80**, 4141 (1998).
[18] N. Takigawa, Y. Nozawa, K. Hagino, A. Ono, and D. M. Brink, Phys. Rev. C **59**, R593 (1999).
[19] C. A. Bertulani, D. T. de Paula, and V. G. Zelevinsky, Phys. Rev. C **60**, 031602 (1999).
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