Nonlinear Electrodynamics of Randomly Inhomogeneous Superconductors

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We investigate the effect of macroscopic inhomogeneities on nonlinear transport properties of type-II superconductors and develop an effective medium theory to derive general relations between the global and local currents and electric fields. We show that even weak inhomogeneities with $\langle \delta J_c^2 \rangle \ll J_c^2$ can qualitatively change nonlinear transport characteristics (here δJ_c denotes fluctuations of the critical current density J_c), causing a nonmonotonic magnetic field dependence of the global averaged $\bar{J}_c(B)$, even if the local $J_c(B)$ decreases with B. We predict a *superconducting Gunn effect*, for which inhomogeneities can give rise to negative differential conductivity, bistability, and electric field domains.

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One of the central issues of physics of the mixed state in type-II superconductors has been thermally activated vortex creep characterized by the highly nonlinear electric field-current density (E-J) characteristics, $\mathbf{E} = (\mathbf{J}/J)E_c \exp[-U(J)/T]$, below the critical current density $J < J_c$. A description of this state has been offered by vortex glass/collective creep models [1], in which the driven vortex motion is controlled by the diverging activation barriers $U(J) \simeq U_c(J_c/J)^\mu$ at $J \ll J_c$. Numerous experiments have indeed found diverging activation barriers in the flux creep dynamics of high-temperature superconductors (HTS) [1–3]. Yet many important issues remain unresolved, for example, a nonmonotonic magnetic field dependence of $J_c(B)$ ("fishtail" effect) [4], or deviations of the exponent μ and the $J(E,T,B)$ characteristics from the scaling predictions of the vortex glass theory [3], to name a few.

A better understanding of a possible source of these problems was achieved due to recent advances in experimental techniques, especially magneto-optical imaging, which has revealed significant spatial variations of local $\mathbf{J}(\mathbf{r})$ in HTS over macroscopic scales, $L \sim 1\ \mu$m $-$ 1 mm. Even in the best HTS samples (single crystals and films), the fluctuations $\delta J_c(x,y)$ usually exceed 10–30% of the mean $\bar{J}_c$, becoming much stronger, $\delta J_c \sim (1-10)\bar{J}_c$, in practical HTS conductors [5]. This fact poses a fundamental question: to what extent do the observed electromagnetic properties of HTS, obtained by macroscopic measurements, reflect the underlying glassy vortex dynamics, rather than the effects of the nonuniform, often percolative current flow? Glassy properties are formed on the spatial scale of the Larkin length L_c, so *macroscopic* inhomogeneities of superconducting and pinning characteristics on the scales $L \gg L_c$ can strongly affect the global $\bar{J}(E)$ curves. Therefore, many of the past results on the nonlinear vortex transport should be re-examined by taking into account the inevitable macroscopic inhomogeneities characteristic of HTS, especially in the vicinity of the vortex lattice melting [6].

In this Letter we develop a theory of nonlinear steady-state transport in superconductors with random macroscopic ($L \gg L_c$) inhomogeneities. We treat HTS as a highly nonlinear conductor with a local $\mathbf{J} = \mathbf{E}J(E,\mathbf{r})/E$ relation and calculate the global $\bar{\mathbf{J}}(\mathbf{E}_0)$ characteristic by averaging the local $\mathbf{J}(\mathbf{E},\mathbf{r})$ over $\mathbf{r}$ and random fluctuations $\delta\mathbf{E}(\mathbf{r})$ around an applied mean electric field $\mathbf{E}_0$, where $\delta\mathbf{E}(\mathbf{r})$ is calculated by solving the Maxwell equations [7]. This universal approach does not assume any particular model of vortex dynamics and can be applied to a wider class of problems related to percolative current transport in nonuniform nonlinear conductors. When discussing new physical effects [nonmonotonic $\bar{J}_c(B)$ or vortex Gunn effect] caused by inhomogeneities, we illustrate our general results using different local $E(J)$ characteristics, such as the power law, $E = E_c(J/J_c)^n$, or vortex glass relation, $E \propto \exp[-(J_c/J)^\mu U_c/T]$, as exemplary models. We show that inhomogeneities that are even weaker than those typically observed in HTS [5] can dramatically change transport characteristics, if $n^{-3/2} \ll \langle \delta J_0^2 \rangle / J_0^2 \ll 1$, where $\langle \delta J_0^2 \rangle$ is the mean-square variance of the fluctuations of the critical current density $J_0 = J(E_0)$ defined at the mean electric field E_0, and $n = \partial \ln E / \partial \ln J \simeq \mu U_c / T \gg 1$ [8]. The difference between $\bar{\mathbf{J}}(\mathbf{E}_0)$ and $J(E)$ becomes especially pronounced for thermally activated vortex motion, since even weak fluctuations of the activation barrier, $\delta U \sim T \ll U$, cause large electric field perturbations of the local $\mathbf{E}(\mathbf{r}) \propto \mathbf{J}\exp[-U(J,\mathbf{r})/T]$. These perturbations change the mean current density $\bar{J} = \langle J\cos\theta \rangle \simeq [1 - \langle \delta E_y^2 \rangle / 2E_0^2] J_0$, where $J_0 = \langle J(E_0,\mathbf{r}) \rangle$, $\theta(\mathbf{r})$ is the angle between $\mathbf{E}(\mathbf{r})$ and $\mathbf{E}_0$, $\langle \delta\mathbf{E} \rangle = 0$, and the brackets $\langle \cdots \rangle$ mean statistical averaging. Therefore, the wandering of the current streamlines results in the difference between the global $\bar{\mathbf{J}}(\mathbf{E}_0, T, B)$ and the local $\mathbf{J}(\mathbf{E}, T, B)$, since $\langle \cos\theta \rangle$ also depends on E_0, T, and B.

We consider an isotropic two-dimensional (2D) current flow in the ab plane of layered HTS in a strong magnetic field, for which the c-axis component of $\mathbf{J}$ and the self-field effects can be neglected. Let $\mathbf{J}[\mathbf{E}, f(\mathbf{r})]$ depend parametrically on a randomly inhomogeneous macroscopic characteristic $f = f_0 + \delta f(x,y), \langle \delta f \rangle = 0$. Here f may stand for any relevant material parameter, such as, J_c, U_c, T_c, etc. Then Maxwell's equations $\mathrm{div}[\mathbf{E}J(E,f)/E] = 0$ and $\mathbf{E} = \mathbf{E}_0 - \nabla\varphi$ reduce to the following equation for the electric potential φ:

$$\frac{\partial}{\partial x}\sigma_x\frac{\partial\varphi}{\partial x} + \frac{\partial}{\partial y}\sigma_y\frac{\partial\varphi}{\partial y} = \frac{J'}{E}\left(E_x\frac{\partial f}{\partial x} + E_y\frac{\partial f}{\partial y}\right), \quad (1)$$

where $J' = \partial J/\partial f$, and the differential conductivities $\sigma_x = \partial J_x/\partial E_x$ and $\sigma_y = \partial J_y/\partial E_y$ are given by

$$\sigma_x = (J/E)\sin^2\theta + \sigma\cos^2\theta\,, \quad (2)$$

$$\sigma_y = (J/E)\cos^2\theta + \sigma\sin^2\theta\,, \quad (3)$$

Here $\sigma = \partial J/\partial E$, $E_0 + \delta E_x = E\cos\theta$, and $\delta E_y = E\sin\theta$. For a uniform state ($\theta = 0$), both $\sigma_x(E)$ and $\sigma_y(E)$ essentially depend on E. Below the crossover electric field E_c separating flux flow and flux creep regimes, the $J - E$ curve can be approximated by the power-law dependence $J = J_c(E/E_c)^{1/n}$ with $n \gg 1$, for which $\sigma_x = J_0/nE_0$, $\sigma_y = J_0/E_0$. For flux flow at $E \gg E_c$, we have $J = J_c + \sigma_f E$, thus $\sigma_x = \sigma_f$ equals the flux flow conductivity σ_f, but $\sigma_y = \sigma_f + J_c/E \gg \sigma_x$. For $E \gg E_c$, the ratio $\sigma_y/\sigma_x = 1 + J_c/\sigma_f E$ increases as E decreases, becoming weakly dependent on E at $E \ll E_c$, where $\sigma_y/\sigma_x \simeq n \gg 1$. The strong conductivity anisotropy, $\sigma_x \ll \sigma_y$, is characteristic of the critical state, for which the longitudinal fluctuations δE_x weakly affect $J \approx J_c$, whereas the transverse fluctuations δE_y cause a much stronger response δJ_y due to the local turn of $\mathbf{J}$. As a result, the current flow past an inhomogeneity of size L is disturbed on the scale $\sim L$ along $\mathbf{E}_0$, while on a much larger scale $\sim L\sqrt{\sigma_y/\sigma_x}$ across $\mathbf{E}_0$ [9].

We first consider weak inhomogeneities, $\langle \delta E_x^2 \rangle \ll E_0^2$, for which $\bar{J}(E)$ can be calculated perturbatively in $\langle \delta f^2 \rangle / f_0^2$, expanding $\mathbf{J} = (\mathbf{E}/E)J(E,\mathbf{r})$ in $\delta\mathbf{E}$ and δf,

$$\bar{J} = \left(1 - \frac{\langle \delta E_y^2 \rangle}{2E_0^2}\right)J_0 + \frac{\langle \delta f^2 \rangle}{2}J'' + \frac{\langle \delta E_y^2 \rangle}{2E_0}\frac{\partial J}{\partial E} + \frac{\langle \delta E_x^2 \rangle}{2}\frac{\partial^2 J}{\partial E^2} + \langle \delta f \delta E_x \rangle \frac{\partial J'}{\partial E}\,. \quad (4)$$

Here $\langle \delta E_i \delta E_j \rangle$ are calculated by the Fourier transform of Eq. (1) in the first order in $\langle \delta f^2 \rangle$, taking the uniform $\sigma_{x,y}$ in Eqs. (2) and (3) at $\mathbf{E} = \mathbf{E}_0$ and $\theta = 0$,

$$\langle \delta E_i^2 \rangle = J'^2 \int \frac{d^2k}{(2\pi)^2}\frac{k_x^2 k_i^2 \langle \delta f(\mathbf{k})\delta f(-\mathbf{k})\rangle}{(\sigma_x k_x^2 + \sigma_y k_y^2)^2}\,. \quad (5)$$

For isotropic media, the correlation function $F(\mathbf{k}) = \langle \delta f(\mathbf{k})\delta f(-\mathbf{k})\rangle$ depends only on $|\mathbf{k}|$, allowing for the integration of Eq. (5) in the polar coordinates k, θ. This yields $\langle \delta f \delta E_y \rangle = \langle \delta E_x \delta E_y \rangle = 0$, and

$$\langle \delta E_x^2 \rangle = \frac{J'^2 \langle \delta f^2 \rangle (\sqrt{\sigma_y} + 2\sqrt{\sigma_x})}{2\sigma_x^{3/2}(\sqrt{\sigma_y} + \sqrt{\sigma_x})^2}\,, \quad (6)$$

$$\langle \delta E_y^2 \rangle = \frac{J'^2 \langle \delta f^2 \rangle}{2\sqrt{\sigma_x\sigma_y}(\sqrt{\sigma_x} + \sqrt{\sigma_y})^2}\,, \quad (7)$$

$$\langle \delta E_x \delta f \rangle = -\frac{J' \langle \delta f^2 \rangle}{\sqrt{\sigma_x}(\sqrt{\sigma_x} + \sqrt{\sigma_y})}\,, \quad (8)$$

where $\langle \delta f^2 \rangle = \int_0^\infty kF(k)\,dk/2\pi$, is the mean-square variance. It follows from Eq. (6) that the weak perturbations regime, $\langle \delta E_x^2 \rangle \ll E_0^2$, occurs at $\langle \delta f^2 \rangle / f^2 \ll n^{-3/2}$.

General relations (4)–(8) are independent of the shape of $F(r)$, and thus of the spatial scales of $f(\mathbf{r})$ [10]. In this case inhomogeneities can be quantified by the only dimensionless parameter $\eta = \langle \delta J_0^2 \rangle / J_0^2 = \sum_{km}(\partial J/\partial f_k)(\partial J/\partial f_m)\langle \delta f_k \delta f_m \rangle / J_0^2$, if there are several nonuniform parameters $f_k(\mathbf{r})$. For instance, for local flux flow, $J(E) = J_c + \sigma_f E$, with fluctuating $J_c(\mathbf{r})$, we have $f = J_c, J' = 1$ in Eqs. (4)–(8), which gives $\bar{J}(E_0) = J_c + \sigma_f E_0 - \eta J_c^{3/2}/4\sqrt{\sigma_f E_0}$ [11]. For the flux creep state with $J = J_c(E/E_c)^{1/n}$, Eqs. (4)–(8) yield

$$\bar{J} = \left[1 - \frac{\eta(n+1)}{2(1+\sqrt{n})}\right]J_0\,. \quad (9)$$

Here the nonlinearity of $J(E)$ enhances the effect of inhomogeneities by the factor $\sqrt{n} \gg 1$, and the interference of the magnetic field dependencies of $n(B)$ and $J_c(B)$ can give rise to a nonmonotonic $\bar{J}_c(B)$. Let us consider a characteristic for the HTS case of $J_c(B,\mathbf{r}) = J_c(\mathbf{r})\exp[-B/B_0(\mathbf{r})]$ with independently fluctuating $J_c(\mathbf{r})$ and $B_0(\mathbf{r})$. Then $\eta(B) = \langle \delta J_c^2 \rangle / J_{c0}^2 + B^2 \langle \delta B_0^2 \rangle / B_0^4$ is constant at low B and increases with B above $B \sim B_0$. Accordingly, the flux creep rate $s(B) = 1/(n-1)$ first decreases at low B, reaching a minimum s_m at $B_m \ll B_0$ and then increases approximately linear with B all the way to $B \sim B_0$, where $s(B) \sim n(B) \simeq 1$ [3]. Interpolating $s(B)$ as $s(B) = [s_m^2 + s_1^2(B/B_m - 1)^2]^{1/2}$ with $s_1 \simeq B_m/B_0$, we obtain that $\bar{J}_c(B)$ becomes nonmonotonic, if $\partial s/\partial B > -4s^{3/2}(\partial J_c/\partial B)/\eta J_c$ (Fig. 1). This condition can be written in the form $\eta > \eta_f \sim 4s^{3/2}/B_0(\partial s/\partial B) \sim (4B_m/B_0)(B/B_0)^{3/2}$, which gives $\eta_f \ll 1$ at $B < B_0$. Thus, even weak inhomogeneities ($\eta \ll 1$) can cause the nonmonotonic $\bar{J}_c(B)$ dependence due to a more nonuniform current flow at low B.

To describe *moderate* nonlinear fluctuations at $n^{-3/2} \ll \eta \ll 1$, we develop an effective medium theory, in which the fluctuating $\sigma_x(E)$ and $\sigma_y(E)$ in Eq. (1) are replaced by their self-consistent mean values $\bar{\sigma}_i(E_0)$,

$$\bar{\sigma}_i = \int \sigma_i(f,\mathbf{E})P(f,\mathbf{E},E_0)\,df\,d^2\mathbf{E}\,, \quad (10)$$

$$P = A\exp\left(-\frac{\delta f^2}{a} - \frac{\delta f \delta E_x}{b} - \frac{\delta E_x^2}{c} - \frac{\delta E_y^2}{d}\right). \quad (11)$$

Here $i = x, y$, the distribution function $P(f, \mathbf{E}, E_0)$ is

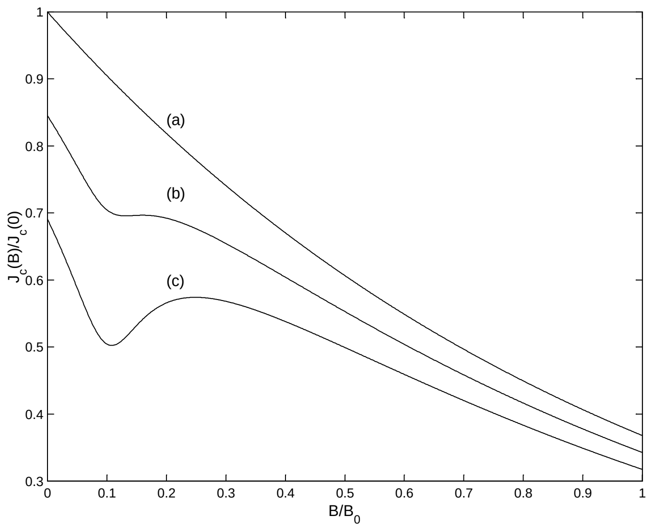

FIG. 1. $\bar{J}_c(B)$ described by Eq. (9) for $1/(n-1) = [s_m^2 + s_1^2(B/B_m - 1)^2]^{1/2}$, $s_m = 0.04$, $s_1 = 0.7(B_m/B_0)$, $B_m = 0.1B_0$, $\langle \delta B_0^2 \rangle = 0$, and $\langle \delta J_c^2 \rangle / J_c^2 = 0$ (a), 0.1 (b), and 0.2 (c).

taken in the Gaussian form, for which $A = \sqrt{4b^2 - ac}/2\pi|b|\sqrt{\pi adc}$, $d = 2\langle \delta E_y^2 \rangle$, $a = 2\langle \delta f^2 \rangle - 2\langle \delta f \delta E_x \rangle^2/\langle \delta E_x^2 \rangle$, $b = \langle \delta f \delta E_x \rangle - \langle \delta f^2 \rangle \langle \delta E_x^2 \rangle / \langle \delta f \delta E_x \rangle$, and $c = 2\langle \delta E_x^2 \rangle - 2\langle \delta f \delta E_x \rangle^2 / \langle \delta f^2 \rangle$. Here $\langle \delta f \delta E_x \rangle$ and $\langle \delta E_x^2 \rangle$ are calculated from Eqs. (6)–(8), in which σ_i are replaced by $\bar{\sigma}_i$. In this approach the linearized Eq. (1) describes perturbations $\delta \mathbf{E}$ in an effective medium with the conductivity $\bar{\sigma}_{ik}$, which depends on the variance $\langle \delta E_i^2 \rangle$. Having solved the self-consistency equations (10) for $\bar{\sigma}_x$ and $\bar{\sigma}_x$, we can calculate $\bar{J}(E_0)$ for any nonlinear $J(f, E)$,

$$\bar{J} = \int df \int_0^\infty E\, dE \int_0^{2\pi} JP \cos\theta\, d\theta\,. \tag{12}$$

For weak fluctuations, $c \ll E_0^2$, Eq. (12) reduces to Eq. (4). For $n^{-3/2} \ll \eta \ll 1$, the longitudinal fluctuations, $\langle \delta E_x^2 \rangle \sim E_0^2$, become nonlinear, but the transverse perturbations, $\langle \delta E_y^2 \rangle / E_0^2 \sim \langle \theta^2 \rangle \ll 1$ remain weak, enabling us to solve Eqs. (10)–(12) analytically. In this case weak inhomogeneities mostly modify the longitudinal conductivity $\bar{\sigma}_x \simeq (\langle \theta^2 \rangle + 1/n) J_0/E_0$, and the small ratio $\bar{\sigma}_x/\bar{\sigma}_y \simeq \langle \theta^2 \rangle$ is determined by the inhomogeneity parameter $\eta \ll 1$, if $n^{-1} \ll \langle \theta^2 \rangle \ll 1$. Writing σ_{ik} as $\bar{\sigma}_x = \alpha J_0/E_0$, $\bar{\sigma}_y = \beta J_0/E_0$, we reduce Eq. (10) to the following self-consistent equations for α and β [12]:

$$\frac{\sqrt{\pi}\,\eta}{2\beta} = \int_0^\infty \frac{s^2 ds}{(p + s^2)^{1/2}(q + s^2)^{3/2}} \exp\left(-\frac{ps^2}{p + s^2}\right), \tag{13}$$

$$\frac{\sqrt{\pi}\,\eta}{2\alpha} = \int_0^\infty \frac{q ds}{(p + s^2)^{1/2}(q + s^2)^{3/2}} \exp\left(-\frac{ps^2}{p + s^2}\right). \tag{14}$$

Here $p = E_0^2/2\langle \delta E_x^2 \rangle = \alpha^{3/2}\beta^{1/2}/\eta$, $q = E_0^2/2\langle \delta E_y^2 \rangle = \alpha^{1/2}\beta^{3/2}/\eta$, and Eq. (12) becomes

$$\bar{J} = J_0 \operatorname{erf}(\sqrt{p})\,, \tag{15}$$

where $\operatorname{erf}(x)$ is the error function. For $\beta \gg \alpha$, that is, $\eta \ll 1$ and $1/n\sqrt{p} \ll e^{-p} \ll 1$, the evaluation of the integrals (13) and (14) yields the following equation for p:

$$p = \frac{2}{3} \ln \frac{k}{\eta} - \frac{1}{6} \ln p\,, \tag{16}$$

giving $p \simeq (2/3)\ln(k/\eta)$, $\alpha \simeq (p\eta)^{2/3}$, $\beta \simeq 1 + 2/3p$, and $k = (2/\sqrt{\pi})^{3/2} = 1.2$. Since $e^{-p} \sim \eta^{2/3} \ll 1$, the asymptotic expansion of Eq. (15) gives the final result

$$\bar{J} \simeq \left(1 - \frac{\eta^{2/3}}{2p^{1/3}}\right) J_0\,. \tag{17}$$

In general, η also depends on E_0, thus changing the shape of $\bar{J}(E_0)$ as compared to $J(E_0)$. For instance, if in $J = J_c(E_0/E_c)^{1/n}$ both J_c and n independently fluctuate, then $\eta = \langle \delta J_0^2 \rangle / J_0^2 = \eta_c + \eta_n \ln^2(E_c/E_0)$, where $\eta_c = \langle \delta J_c^2 \rangle / J_c^2$, and $\eta_n = \langle \delta n^2 \rangle / n^2$. Moreover, the dependence of η on E_0 causes a negative global differential conductivity $G = \partial \bar{J} / \partial E_0$, if $J_0 \partial \eta / \partial E_0 > 3(p\eta)^{1/3} \partial J_0 / \partial E_0$. As an important illustration, we consider the vortex glass $J - E$ curve, $J = J_c/[1 - (T/U)\ln(E/E_c)]^{1/\mu}$, where E_c is a fixed electric field criterion for J_c. In this case fluctuations of δJ_c are coupled with fluctuations of the potential barrier δU due to the general relation $U \propto J_c^{-\gamma}$ of the collective creep theory [1]. This gives $\delta U/U = -\gamma \delta J_c/J_c$, where γ ranges from 1/2 for the 3D weak collective pinning to 3 for the 2D weak pinning of pancake vortices, depending on the relevant region of the B-T phase diagram [1]. As a result, $\eta = \langle \delta J_0^2 \rangle / J_0^2$ takes the form

$$\eta = \eta_c \left[1 + \frac{\gamma (T/U) \ln(E_0/E_c)}{\mu [1 - (T/U) \ln(E_0/E_c)]}\right]^2. \tag{18}$$

Equation (18) describes $\eta(E_0)$ at $E_0 < E_c$, where $\eta(E_0)$ increases with E_0 because of the coupling of fluctuations of δU and δJ_c. Generally, $\eta(E_0)$ reaches a maximum at $E_0 \sim E_c$ and then decreases with E_0, becoming independent of E_0 in the flux flow state, $E_0 \gg E_c$, where $\eta \to \eta_c$ is determined by only δJ_c fluctuations. The evolution of $\bar{J}(E_0)$ with η_c is shown in Fig. 2. Clearly both the slope $\partial \bar{J} / \partial \ln E$ and the curvature of $\bar{J}(E_0)$ essentially change as η increases. This fact can account for the substantial deviations of the observed $\bar{J}(E_0)$ and μ extracted from flux creep or resistive measurements [3] from predictions of the vortex glass theory. For $T \ll U$, $\bar{J}(E_0)$ becomes nonmonotonic, if $\eta_c > \sqrt{p}\,[1 + (2\gamma/3)]^{-3/2}$. Therefore, the dependence of $\eta(T, B, E_0)$ on B and E_0 can cause both the fish-tail effect and the negative differential conductivity $\bar{G}(E_0)$. In turn, the negative $G(E_0)$ qualitatively changes macroscopic electrodynamics of HTS, resulting in the Gunn instability of uniform current flow and the appearance of macroscopic electric field domains [13]. Notice that the strong coupling of fluctuations of δU and δJ_c for pancake vortices ($\gamma = 3$) makes layered HTS particularly susceptible to such instability, which can be induced by rather weak inhomogeneities, $\eta_c \ll 1$. The

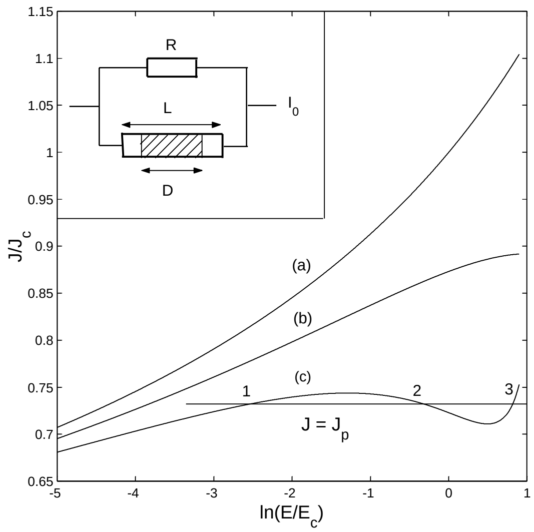

FIG. 2. $\bar{J}(E_0)$ calculated from Eqs. (15)–(18) for $\gamma = 3$, $\mu = 2$, $T/U = 0.2$, and $\langle \delta J_c^2 \rangle / J_c^2 = 0$ (a), 0.2 (b), and 0.6 (c). The inset shows the circuit, which provides a stable electric field domain (hatched).

electric field domains are also likely to appear in the vicinity of the second peak in $J_c(B)$, where the effect of inhomogeneities is amplified by the premelting conditions of the vortex lattice, which breaks up into a mixture of the fluidlike puddles with $J_c = 0$ and solid crystalline domains with finite $J_c > 0$. This two-phase vortex structure gives rise to a highly nonuniform current distribution, facilitating the characteristic manifestations of the Gunn instability, such as electric field domains, hysteresis of $\bar{J}(E_0)$, and temporal voltage oscillations in the fixed current mode [13,14].

The electric bistability at the fixed current density, $J = J_p$, is illustrated in Fig. 2, where the intersection points 1 and 3 correspond to stable "phases" with $E = E_1$ and $E = E_3$, respectively. A macroscopic domain of $E = E_3$ and length D expands if $J > J_p$ and contracts if $J < J_p$, where J_p satisfies the "equal area theorem" in Fig. 2 [13],

$$\int_{E_1}^{E_3} [J(E) - J_p] \, dE = 0 \, . \tag{19}$$

The electric field domain can be stabilized by a feedback circuit shown in the inset of Fig. 2. In this case the equilibrium domain length D is calculated by equating the voltages on the shunt R and the superconductor of length L provided that the current in the sample equals I_p. Thus, $(I_0 - I_p)R = LE_1 + (E_3 - E_1)D$, and D linearly increases with the applied dc current I_0,

$$D = [(I_0 - I_p)R - LE_1]/(E_3 - E_1) \, . \tag{20}$$

In conclusion, we have calculated the global current $\bar{J}(E_0, T, B)$ in randomly inhomogeneous superconductors and have shown that weak macroscopic nonuniformities can result in fish-tail and vortex Gunn effects. Our approach can also be applied to a wide class of transport phenomena controlled by percolative current flow in nonuniform nonlinear conductors.

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[12] For $\bar{\sigma}_x \ll \bar{\sigma}_y$, we obtain from Eqs. (6)–(8), that $4b^2/ac \sim (\bar{\sigma}_y/\bar{\sigma}_x)^{1/2} \gg 1$, thus $A = \bar{\sigma}_x\bar{\sigma}_y/\langle \delta J_0^2 \rangle^{3/2}\pi\sqrt{2\pi}$, $a = 2\langle \delta J_0^2 \rangle$, $c = \langle \delta J_0^2 \rangle / \bar{\sigma}_x^{3/2}\bar{\sigma}_y^{1/2}$, and $d = \langle \delta J_0^2 \rangle / \bar{\sigma}_y^{3/2}\bar{\sigma}_x^{1/2}$, where $\delta J_0 = J' \delta f$. For $n \gg 1$, we can also replace $J(E)$ in Eqs. (10)–(12) by J_0, since $J_c(E/E_c)^{1/n} \simeq J_0[1 - \langle \delta E_x^2 \rangle / 2nE_0^2]$, where the last term in the brackets is negligible, even if $\langle \delta E_x^2 \rangle \sim E_0^2$. To calculate the integrals (10) and (12), we use the identity $\sqrt{\pi}/2E = \int_0^\infty \exp(-s^2E^2)\, ds$, which allows us to perform all Gaussian integrations over $\delta\mathbf{E}$, and δf, if $J(E) \approx J_0$.
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