Critical Behavior of the Conductivity of Si:P at the Metal-Insulator Transition under Uniaxial Stress

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We report new measurements of the electrical conductivity σ of the canonical three-dimensional metal-insulator system Si:P under uniaxial stress S. The zero-temperature extrapolation of $\sigma(S, T \rightarrow 0) \sim |S - S_c|^{\mu}$ shows an unprecedently sharp onset of finite conductivity at S_c with an exponent $\mu = 1$. The value of μ differs significantly from that of earlier stress-tuning results. Our data show dynamic $\sigma(S,T)$ scaling on both metallic and insulating sides, *viz.* $\sigma(S,T) = \sigma_c(T) \cdot \mathcal{F}'(|S - S_c|/T^{y})$ where $\sigma_c(T)$ is the conductivity at the critical stress S_c. We find $y = 1/z\nu = 0.34$ where ν is the correlation-length exponent and z the dynamic critical exponent.

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Quantum phase transitions have become of steadily increasing interest in recent years [1]. These continuous transitions ideally occur at temperature $T = 0$. In particular, certain types of metal-insulator transitions (MITs) such as localization transitions have been studied extensively. Experimentally, the MIT may be driven by an external parameter t such as carrier concentration N, uniaxial stress S, or electric or magnetic fields. Generally, electron localization might arise from disorder (Anderson transition) or from electron-electron (e-e) interactions (Mott-Hubbard transition) [2]. In Nature, these two features go hand in hand. For instance, the disorder-induced MIT, occurring as a function of doping in three-dimensional ($d = 3$) semiconductors where the disorder stems from the statistical distribution of dopant atoms in the crystalline host, bears signatures of e-e interactions as evidenced from the transport properties in both metallic [3] and insulating regimes [4]. This makes a theoretical treatment of the critical behavior of a MIT exceedingly difficult. Even for purely disorder-induced transitions, the critical behavior of the zero-temperature dc conductivity, $\sigma(0) \sim |t - t_c|^{\mu}$, where t_c is the critical value of t, is not well understood. Theoretically, μ is usually inferred from the correlation-length critical exponent ν via Wegner scaling $\mu = \nu(d - 2)$. Numerical values of ν range between 1.3 and 1.6 [5,6].

Experimentally, the critical behavior of $\sigma(0)$ falls into two classes: $\mu \approx 0.5$ for uncompensated semiconductors and $\mu \approx 1$ for compensated semiconductors and amorphous metals [7]. However, there appears to be no clear physical distinction between these materials that would justify different universality classes. While many different materials were reported to show $\mu \approx 1$, the exponent $\mu \approx 0.5$ was largely based on the very elegant experiments by Paalanen and co-workers [8–10], where uniaxial stress S was used to drive an initially insulating uncompensated Si:P sample metallic. This allows one to fine-tune the MIT since S can be changed continuously at low T, thus eliminating geometry errors incurring when different samples are employed in concentration tuning the MIT.

As always when dealing with critical phenomena, the range of critical behavior is a source of controversy. A few years ago we suggested [11] to limit the critical concentration region in doped semiconductors on the metallic side of the MIT to samples where $\sigma(T)$ actually decreases with decreasing T, i.e., the sample becomes less conducting when approaching the MIT. In doped semiconductors, $\sigma(T)$ is nearly independent of T at the crossover concentration N_{cr}, with a value σ_{cr} of a few times 10 $\Omega^{-1}\,\mathrm{cm}^{-1}$, e.g., $\sigma_{cr} \approx 40\ \Omega^{-1}\,\mathrm{cm}^{-1}$ in Si:P. $\sigma(T)$ exhibits a negative temperature coefficient above N_{cr} which is explained in terms of e-e interactions [3]. Typically the critical region $N_c < N < N_{cr}$ is within 10% or less of the critical concentration N_c. This eliminates a large number of studies purporting to show $\mu = 0.5$ where actually only a few samples in the critical regime were investigated. Even the recent study on transmutation-doped Ge:Ga, where $\mu = 0.5$ was suggested, presents only three metallic samples in the critical region below $\sigma_{cr} \approx 10\ \Omega^{-1}\,\mathrm{cm}^{-1}$ [12]. An earlier study of a large number of Si:P samples showed that μ changed from 0.64 for $N > N_{cr} \approx 1.1 N_c$ to 1.3 for $N_c < N < N_{cr}$ [11]. On the other hand, sample inhomogeneities might affect the behavior very close to N_c. For this reason, stress-tuning data for Si:P close to S_c were discarded, leading to $\mu = 0.5$ [8,9,13]. It is therefore necessary to perform additional stress-tuning experiments on Si:P with finely tuned stress values including data on the insulating side to check for the critical behavior.

The dynamics of a quantum phase transition is reflected in the finite-temperature behavior of critical quantities. Concerning the MIT in heavily doped semiconductors, this point has not received much attention. Approximate dynamic scaling of the form

$$\sigma(t,T) = (t - t_c)^{\mu} \mathcal{F}(T/(t - t_c)^{z\nu}), \qquad (1)$$

where z is the dynamic critical exponent, was observed for Si:P on the metallic side of the MIT with $t = N$, yielding $\mu = 1.3$ and $z = 2.4$ [14]. Dynamic scaling had previously been observed at the magnetic-field-driven MIT in semimagnetic semiconductors [15]. On the other hand, the stress-tuning data [9] did not obey scaling [2]. Conductivity data for Si:B under uniaxial stress [16] obey very nicely the dynamic scaling on both metallic and insulating sides, yielding $\mu = 1.6$ and $z = 2$, while concentration tuning of $\sigma(0)$ on the same system had suggested $\mu = 0.63$ [17]. This large difference is not understood at present. In this situation, an examination of possible dynamic scaling of the canonical metal-insulator system Si:P appears of utmost importance in order to resolve the question of critical behavior and to appraise the possibly strongly different roles of S and N.

In this paper, we report on stress tuning of the MIT of Si:P by measuring the electrical conductivity down to 15 mK. We find by extrapolating to $T = 0$ an unprecedently sharp onset of $\sigma(S, 0)$ which allows one to unambiguously extract $\mu \approx 1$. In addition, dynamic scaling yielding $z \approx 3$ is found. The value of μ is in reasonable agreement with that derived from concentration tuning. We further show that stress tuning and concentration tuning lead to very different T dependences of σ.

The samples were taken from the same Si:P crystals which have been employed previously [11]. Here we report on investigations on two crystals with $N = 3.21$ and 3.43×10^{18} cm^{-3}, just below the critical concentration $N_c = 3.52 \times 10^{18}$ cm^{-3} as determined [11] for our samples. Similarly grown samples with an even higher concentration ($N \approx 7 \times 10^{19}$ cm^{-3}) showed no sign of P clustering as investigated with scanning tunneling microscopy [18]. The samples were cut to a size of $\sim 15 \times 0.8 \times 0.9$ mm^3 and contacted with four Au leads by spark welding, with the voltage leads ~6 mm apart. The sample was mounted in a ^{4}He-activated uniaxial pressure cell equipped with a piezoelectric force sensor [19]. The stress was applied along the [100] direction which was the most elongated dimension of the sample. S was determined from the ratio of the area of the cell base plate and the sample cross section. Calibration of the cell at 4.2 K showed a linear increase of force with pressure applied at room temperature to gaseous He, with no hysteresis. The cell, incorporating a thermal shield, was tightly screwed to the mixing chamber of a dilution refrigerator. During one run a thermometer was attached to the sample showing that T deviations to the main thermometer directly mounted at the mixing chamber were less than 0.5 mK at the lowest T of 15 mK. The thermometer was calibrated against a NBS 768 superconducting fixed point standard, with intermediate temperatures determined by a Pd:Fe susceptibility thermometer and a Lake Shore Ge sensor. σ was measured with a LR 700 resistance bridge at 16 Hz with a power $<10^{-13}$ W dissipated in the sample.

Figure 1 shows the electrical conductivity $\sigma(T)$ of sample 1 ($N = 3.21 \times 10^{18}$ cm^{-3}) for S between 1 and 3.05 kbar. The data are plotted vs $\sqrt{T}$ which is the T dependence expected due to e-e interactions and, indeed, observed well above the MIT, $\sigma(T) = \sigma_0 + m\sqrt{T}$ with $m < 0$ [3]. The smooth curves are, in fact, polygons connecting adjacent data points (see Fig. 2a for a set of actual data points). For S between 1 and 2.57 kbar the $\sigma(T)$ curves evolve smoothly from insulating to metallic behavior with $m > 0$, and $\sigma(T)$ becomes nearly independent of T with a value $\sigma_{cr} \approx 12\ \Omega^{-1}\,\mathrm{cm}^{-1}$ at ~2.7 kbar. For larger stress $\sigma(T)$ passes over a shallow maximum signaling the crossover to $m < 0$, as observed with N tuning [20]. It is interesting to note that $\sigma_{cr}(S) \approx 0.3\sigma_{cr}(N)$, thus severely limiting the critical region. Our data do not exhibit the precipitous drop of $\sigma(T)$ below ~40 mK for S close to the MIT, in distinction to the earlier stress-tuning work on Si:P extending to 3 mK [8,9]. Instead, our $\sigma(T)$ data exhibit a T dependence that varies only gently with stress.

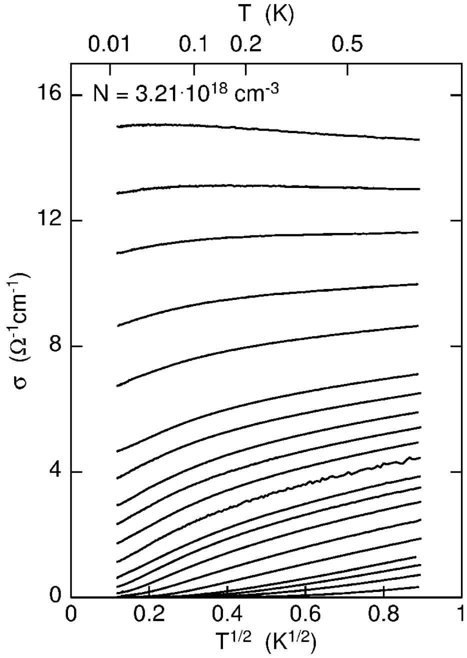

FIG. 1. Conductivity σ of a Si:P sample with P concentration $N = 3.21 \times 10^{18}$ cm^{-3} versus $\sqrt{T}$ for several values of uniaxial stress. From top to bottom: $S = 3.05$, 2.78, 2.57, 2.34, 2.17, 2.00, 1.94, 1.87, 1.82, 1.77, 1.72, 1.66, 1.61, 1.56, 1.50, 1.41, 1.33, 1.26, 1.18, and 1.00 kbar. Solid lines are connecting the very finely spaced individual data points.

Closer inspection shows that the data near the MIT are actually better described by a $T^{1/3}$ dependence for low T as can be seen from Fig. 2a. $\sigma(0)$ obtained from the $T^{1/3}$ extrapolation to $T = 0$ is shown in Fig. 2b, together with data for sample 2 ($N = 3.43 \times 10^{18}$ cm^{-3}). $\sigma(0)$ is plotted linearly vs S, yielding $S_c = 1.75$ kbar for sample 1 and 1.54 kbar for sample 2. Note that the critical stress S_c is quite well defined, as $\sigma(0)$ breaks away roughly linearly from zero within less than 0.1 kbar. Applying our criterion for the critical region, the analysis should be limited to data with $\sigma < \sigma_{cr} \approx 12\ \Omega^{-1}\,\mathrm{cm}^{-1}$. In this range the critical exponent μ is 0.96 and 1.09 for samples 1 and 2, respectively. $\mu \approx 1$ is found also when the more conventional $\sqrt{T}$ extrapolation is employed. This behavior contrasts with the earlier stress-tuning data [8] reproduced in the inset of Fig. 2b, where

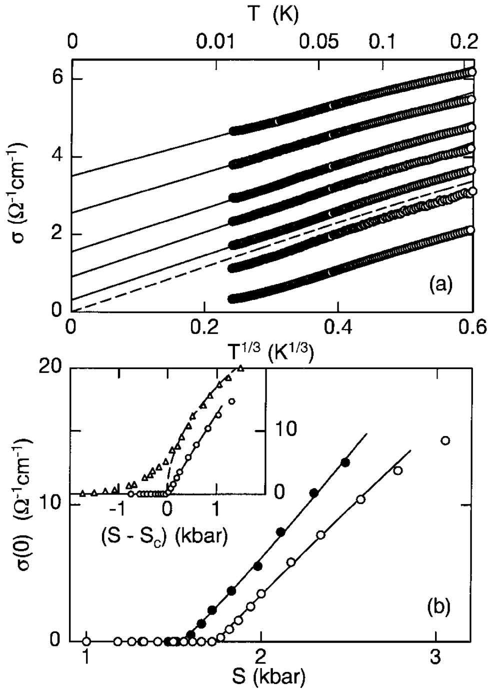

FIG. 2. (a) Low-temperature data of σ of Fig. 1 in the immediate vicinity of the metal-insulator transition plotted against $T^{1/3}$. Dashed line indicates the conductivity at the critical stress (see text). (b) Extrapolated conductivity $\sigma(0)$ for $T \rightarrow 0$ versus uniaxial stress S for two P concentrations $N = 3.21$ and 3.43×10^{18} cm^{-3} (open and closed circles, respectively). The inset shows earlier $\sigma(0)$ versus $S - S_c$ data (triangles) from Ref. [8] in comparison to our data for sample 1 (circles).

appreciable rounding close to N_c is visible when plotted against $S - S_c$ (see also [13]). However, those $\sigma(0)$ data between 4 and 16 Ω^{-1} cm^{-1} are compatible with a linear dependence on uniaxial stress.

Figure 3 shows $\sigma(T)$ of sample 2 for a range of selected S, again applied along [100]. The overall behavior is very similar to that of sample 1, with the same σ_{cr} for both samples. It has been suggested that tuning with S or N should yield the same critical exponents [8–10,21]. The decrease of N_c with S is attributed to the admixture of the more extended $1s(E)$ and $1s(T_2)$ excited states to the $1s(A_1)$ ground state of the valley-orbit split sixfold donor $1s$ multiplet [21]. Comparison of $\sigma(T)$ for various S and N (Fig. 3) reveals that stress and concentration tuning lead to strikingly different T dependences of σ in the vicinity of the MIT. As the exact origin of the $\sigma(T)$ behavior close to the MIT is unknown, we cannot offer an explanation for the different behavior which, of course, must arise from the change of donor wave functions under uniaxial stress. In this respect, experiments on similar samples for S applied along different directions leading to different types of mixing among the states of the $1s$ multiplet will be helpful. The fact that S was applied to different directions in the previous and present studies, i.e., $[12\bar{3}]$ and [100], respectively, may well be one reason for the different behavior of $\sigma(T)$.

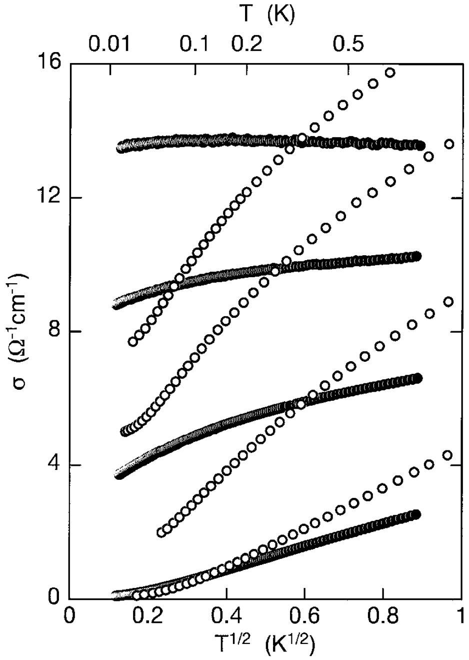

FIG. 3. Comparison of the concentration dependence of $\sigma(T)$ of Si:P (open symbols, from top to bottom: $N = 3.60, 3.56, 3.50, 3.38 \times 10^{18}$ cm^{-3}, from Ref. [11]) and stress dependence of $\sigma(T)$ (closed symbols, $N = 3.43 \times 10^{18}$ cm^{-3}, from top to bottom: $S = 2.48$, 2.11, 1.72, 1.32 kbar).

We finally turn to the scaling behavior of σ at finite temperatures using the data of sample 1. We employ the scaling relation [16]

$$\sigma(t,T) = \sigma_c(T)\mathcal{F}'((t - t_c)/T^y), \qquad (2)$$

where $\sigma_c(T) = \sigma(t_c,T)$ is the conductivity at the critical value t_c of the parameter t driving the MIT. This scaling relation is equivalent to Eq. (1); both are derived from the general scaling relation

$$\sigma(t,T) = b^{-(d-2)}\mathcal{F}''((t - t_c)b^{1/\nu}, b^z T), \qquad (3)$$

where b is a scaling parameter. If the leading term to $\sigma_c(T)$ is proportional to T^x, one obtains $x = \mu/\nu z$ and $y = 1/\nu z$ from a scaling plot. Figures 1 and 2a show that σ for S close to S_c does not exhibit a simple power-law T dependence over the whole T range investigated. We therefore determine $\sigma_c(T)$ with $S_c = 1.75$ kbar by interpolating linearly between the two $\sigma(T)$ curves for $S = 1.72$ and 1.77 kbar. The resultant $\sigma_c(T)$ is then fitted by the function $\sigma_c(T) = aT^x(1 + dT^w)$ with $a = 6.01\ \Omega^{-1}$ cm^{-1}, $x = 0.34$, $d = -0.202$, $w = 0.863$, and T is expressed in K. Here the dT^w term presents a correction to the critical dynamics. This $\sigma_c(T)$ curve is shown as the dashed line in Fig. 2a. All $\sigma(S,T)$ curves with 1.00 kbar $< S <$ 2.34 kbar up to 800 mK are then used for the scaling analysis according to Eq. (2). The same procedure was repeated for other choices of $\sigma_c(T)$ between the two measured $\sigma(T)$ curves embracing the critical stress, with clearly less satisfactory results.

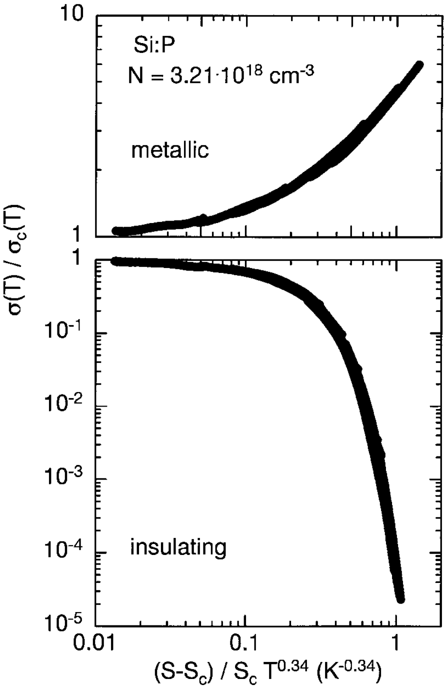

FIG. 4. Scaling plot of σ/σ_c vs $|S - S_c|/S_c T^y$ for sample 1, with $S_c = 1.75$ kbar and $y = 0.34$.

Figure 4 shows the resulting scaling plot of $\sigma(S,T)/\sigma_c(T)$ vs $|S - S_c|/S_c T^y$. The data are seen to collapse on a single branch each for the metallic and insulating sides, respectively. The best scaling, as shown, is achieved for $y = 1/z\nu = 0.34$. Together with $\mu = 1.0$ as obtained from Fig. 2 and assuming Wegner scaling $\nu = \mu$ for $d = 3$, we find $z = 2.94$, which is indeed consistent with $\sigma_c \sim T^{1/z} \sim T^{1/3}$ for $T \to 0$ (see Fig. 2a). Alternatively, we may use Eq. (1) plotting $\sigma(S,T)/|S - S_c|^\mu$ vs $T/|S - S_c|^{z\nu}$ (not shown) with the three parameters S_c, $\mu = \nu$, and z. The best data collapse is found for $\mu = 1.0 \pm 0.1$ and $z = 2.94 \pm 0.3$, in very good agreement with the values obtained from Fig. 4. Additionally, we note the broad consistence with the earlier concentration tuning data where $\mu = 1.3$ and $z = 2.4$ was inferred [14]. We estimate the error of our combined analysis of the present stress-tuned data to 10% for μ and z. The critical stress is determined with a relative accuracy to better than 0.1 kbar. It is important to note that either $\sigma(0)$ scaling (Fig. 2b) or dynamic scaling (Fig. 4), when taken by itself, may lead to a rather large error in μ and/or z, just because of the ambiguity of determining the critical region. However, the consistent determination of exponents from the combined scaling lends confidence to the values reported here.

The above procedure to determine $\sigma_c(T)$ is necessary because $4\sigma_c$ does not obey a simple power-law T dependence over the whole T range. Above 100 mK the correction term dT^w (with $d < 0$) comes into play. On the other hand, a $T^{1/3}$ dependence of σ in the vicinity of the MIT has been reported for transmutation-doped Ge:Ga over a large T range [12]. However, those data do not exhibit dynamic scaling with the exponents [12] $\mu = 0.5$ and $z = 3$. We remark that a simple algebraic T dependence $\sigma_c = aT^x$, which with $x = 1/2$ yields good dynamic scaling for Si:B up to 800 mK [16], clearly leads to less satisfactory scaling in Si:P for any choice of x.

In conclusion, we have demonstrated dynamic scaling of stress-tuned Si:P at the MIT. The conductivity exponent $\mu \approx 1$ is close to the exponents derived earlier from concentration tuning. However, upon application of stress, the critical range is narrowed from 40 $\Omega^{-1}\,\mathrm{cm}^{-1}$ to conductivities below 12 $\Omega^{-1}\,\mathrm{cm}^{-1}$. Therefore, it is the absence of appreciable rounding effects in our samples close to the MIT that allows us to determine $\mu \approx 1$ reliably, thus resolving the conductivity exponent puzzle. The T dependence of σ starting from the same $\sigma(0)$ value is distinctly different for samples under zero stress and under stress. In view of these differences away from the quantum critical point, the similarity of asymptotic dynamic scaling behavior is particularly noteworthy. This shows that while ν and z are universal in Si:P the scaling function itself must be derived from a microscopic model. Hence a more detailed theoretical treatment which may eventually also account for the effective exponent $\mu \approx 0.5$ for samples above the crossover conductivity σ_{cr} is highly desirable.

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[1] S.L. Sondhi *et al.,* Rev. Mod. Phys. **69**, 315 (1997).
[2] D. Belitz and T.R. Kirkpatrick, Rev. Mod. Phys. **66**, 261 (1994).
[3] T.F. Rosenbaum *et al.,* Phys. Rev. Lett. **46**, 568 (1981).
[4] X. Liu *et al.,* Phys. Rev. Lett. **77**, 3395 (1996).
[5] B. Kramer *et al.,* Physica (Amsterdam) **163A**, 167 (1990).
[6] K. Slevin and T. Ohtsuki, Phys. Rev. Lett. **82**, 382 (1999).
[7] G.A. Thomas, Philos. Mag. B **52**, 479 (1985).
[8] M.A. Paalanen *et al.,* Phys. Rev. Lett. **48**, 1284 (1982).
[9] G.A. Thomas, M. Paalanen, and T.F. Rosenbaum, Phys. Rev. B **27**, 3897 (1983).
[10] T.F. Rosenbaum *et al.,* Phys. Rev. B **27**, 7509 (1983).
[11] H. Stupp *et al.,* Phys. Rev. Lett. **71**, 2634 (1993).
[12] K.M. Itoh *et al.,* Phys. Rev. Lett. **77**, 4058 (1996); M. Watanabe *et al.,* Phys. Rev. B **58**, 9851 (1998).
[13] T.F. Rosenbaum, G.A. Thomas, and M.A. Paalanen, Phys. Rev. Lett. **72**, 2121 (1994).
[14] H. Stupp *et al.,* Phys. Rev. Lett. **72**, 2122 (1994).
[15] T. Wojtowicz *et al.,* Phys. Rev. Lett. **56**, 2419 (1986).
[16] S. Bogdanovich, M.P. Sarachik, and R.N. Bhatt, Phys. Rev. Lett. **82**, 137 (1999).
[17] P. Dai, Y. Zhang, and M.P. Sarachik, Phys. Rev. Lett. **66**, 1914 (1991).
[18] T. Trappmann, C. Sürgers, and H.v. Löhneysen, Europhys. Lett. **38**, 177 (1997).
[19] C. Pfleiderer *et al.,* Rev. Sci. Instrum. **68**, 3120 (1997).
[20] A. Blaschette *et al.,* Europhys. Lett. **36**, 527 (1996).
[21] R.N. Bhatt, Phys. Rev. B **26**, 1082 (1982).