Excitation of Phonons in a Bose-Einstein Condensate by Light Scattering

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Stimulated small-angle light scattering was used to measure the structure factor of a Bose-Einstein condensate in the phonon regime. The excitation strength for phonons was found to be significantly reduced from that of free particles, revealing the presence of correlated pair excitations and quantum depletion in the condensate. The Bragg resonance line strength and line shift agreed with predictions for the homogeneous Bose gas using a local density approximation.

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Spectroscopic studies have been used to assemble a complete understanding of the structure of atoms and simple molecules. Similarly, neutron and light scattering have long been used to probe the microscopic excitations of liquid helium [1–4], and can be regarded as the spectroscopy of a many-body quantum system. With the realization of gaseous Bose-Einstein condensates, the spectroscopy of this new quantum fluid has begun.

The character of excitations in a weakly interacting Bose-Einstein condensed gas depends on the relation between the wave vector of the excitation q and the inverse healing length $\xi^{-1} = \sqrt{2}\, m c_s/\hbar$, which is the wave vector related to the speed of Bogoliubov sound $c_s = \sqrt{\mu/m}$, where $\mu = 4\pi\hbar^2 a n_0/m$ is the chemical potential, a is the scattering length, n_0 is the condensate density, and m is the atomic mass. For large wave vectors ($q \gg \xi^{-1}$), the excitations are particlelike with a quadratic dispersion relation. Excitations in the free-particle regime have been accessed by near-resonant light scattering [5]. For small wave vectors ($q \ll \xi^{-1}$), the gas responds collectively and density perturbations propagate as phonons at the speed of Bogoliubov sound. Such quasiparticle excitations have been observed at wavelengths comparable to the size of the trapped condensate [6] and thus were strongly influenced by boundary conditions.

In this Letter, we use Bragg spectroscopy to probe excitations in the phonon regime. Two laser beams intersecting at a small angle were used to create excitations in a Bose-Einstein condensate with wave vector $q < \xi^{-1}$, thereby "optically imprinting" phonons into the gas. The momentum imparted to the condensate was measured by a time-of-flight analysis. This study is the first to explore phonons with wavelengths much smaller than the size of the trapped sample, allowing a direct connection to the theory of the homogeneous Bose gas. We show the excitation of phonons to be significantly weaker than that of free particles, providing dramatic evidence for correlated momentum excitations in the many-body condensate wave function.

In optical Bragg spectroscopy, an atomic sample is illuminated by two laser beams with wave vectors $\mathbf{k}_1$ and $\mathbf{k}_2$ and a frequency difference ω which is much smaller than their detuning Δ from an atomic resonance. The intersecting beams create a periodic, traveling intensity modulation $I_{\text{mod}}(\mathbf{r},t) = I\cos(\mathbf{q}\cdot\mathbf{r} - \omega t)$, where $\mathbf{q} = \mathbf{k}_1 - \mathbf{k}_2$. The atoms experience a potential due to the ac Stark effect of strength $V_{\text{mod}} = \hbar\Gamma^2/8\Delta \times I_{\text{mod}}/I_{\text{sat}}$ [7], from which they may scatter. Here, Γ is the linewidth of the atomic resonance, and I_{sat} is the saturation intensity.

The response of an N-particle system to this perturbation can be evaluated using Fermi's golden rule. We express V_{mod} in second-quantized notation $\hat{V}_{\text{mod}} = V/2[\hat\rho^\dagger(\mathbf{q})e^{-i\omega t} + \hat\rho^\dagger(-\mathbf{q})e^{+i\omega t}]$, where $\hat\rho^\dagger(\mathbf{q}) = \sum_k \hat a^\dagger_{k+q}\hat a_k$ is the Fourier transform of the atomic density operator at wave vector $\mathbf{q}$, and $\hat a_k$ ($\hat a^\dagger_k$) is the destruction (creation) operator for an atom with momentum $\hbar\mathbf{k}$. For the ground state $|g\rangle$ with energy E_g, the excitation rate per particle is then

$$\frac{2\pi}{N\hbar}\left(\frac{V}{2}\right)^2 \sum_f |\langle f|\hat\rho^\dagger(\mathbf{q})|g\rangle|^2 \delta(\hbar\omega - (E_f - E_g)) = 2\pi\omega_R^2 S(\mathbf{q},\omega),$$

where excited states $|f\rangle$ have energy E_f, and $\omega_R = V/2\hbar$ is the two-photon Rabi frequency. Thus, light scattering directly measures the dynamical structure factor, $S(\mathbf{q},\omega)$, which is the Fourier transform of density correlations in state $|g\rangle$ [3,8]. Integrating over ω gives the static structure factor $S(\mathbf{q}) = \langle g|\hat\rho(\mathbf{q})\hat\rho^\dagger(\mathbf{q})|g\rangle/N$.

In this work, measurements were performed on both magnetically trapped and freely expanding Bose-Einstein condensates of sodium. Condensates of $\approx 10^7$ atoms were created by laser and evaporative cooling and stored in a cigar-shaped magnetic trap with trapping frequencies of $\omega_r = 2\pi \times 150$ Hz and $\omega_z = 2\pi \times 18$ Hz in the radial and axial directions, respectively [9].

The condensate was then exposed to two laser beams which intersected at an angle of $\approx 14°$ and were aligned symmetrically about the radial direction, so that the difference wave vector $\mathbf{q}$ was directed axially (Fig. 1a).

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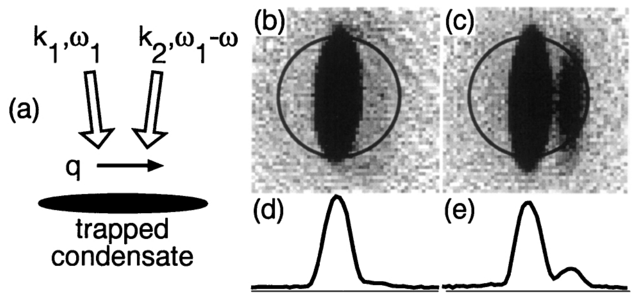

FIG. 1. Observation of momentum transfer by Bragg scattering. (a) Atoms were exposed to laser beams with wave vectors $\mathbf{k}_1$ and $\mathbf{k}_2$ and frequency difference ω, imparting momentum $\hbar\mathbf{q}$ along the axis of the trapped condensate. The Bragg scattering response of trapped condensates [(b) and (d)] was much weaker than that of condensates after a 5 ms free expansion [(c) and (e)]. Absorption images [(b) and (c)] after 70 ms time of flight show scattered atoms distinguished from the denser unscattered cloud by their axial displacement. Curves (d) and (e) show radially averaged (vertically in image) profiles of the optical density after subtraction of the thermal distribution. The Bragg scattering velocity is smaller than the speed of sound in the condensate (position indicated by circle). Images are 3.3×3.3 mm.

Both beams were derived from a common source, and then passed through two acousto-optical modulators operated with the desired frequency difference ω, giving the beams a detuning of 1.6 GHz below the $|F = 1\rangle \rightarrow |F' = 0, 1, 2\rangle$ optical transitions. Thus, at the optical wavelength of 589 nm, the Bragg recoil velocity was $\hbar q/m \simeq 7$ mm/s, giving a predicted Bragg resonance frequency of $\omega_q^0 = \hbar q^2/2m \simeq 2\pi \times 1.5$ kHz for free particles. The beams were pulsed on at an intensity of about 1 mW/cm^2 for a duration of 400 μs. To suppress super-radiant Rayleigh scattering [10], both beams were linearly polarized in the plane defined by the condensate axis and the wave vector of the light.

The Bragg scattering of a trapped condensate was analyzed by switching off the magnetic trap 100 μs after the end of the light pulse, and allowing the cloud to freely evolve for 70 ms. During the free expansion, the density of the atomic cloud dropped and *quasiparticles* in the condensate transformed into *free particles* and were then imaged by resonant absorption imaging (Fig. 1). Bragg scattered atoms were distinguished from the unscattered atoms by their axial displacement. The speed of Bogoliubov sound at the center of the trapped condensate is related to the velocity of radial expansion v_r as $c_s = v_r/\sqrt{2}$ [11] ($c_s = 11$ mm/s at $\mu/h = 6.7$ kHz as shown in Fig. 1). Thus, by comparing the axial displacement of the scattered atoms to the radial extent of the expanded condensate, one sees that the Bragg scattering recoil velocity is smaller than the speed of sound in the trapped condensate, i.e., the excitation in the trapped condensate occurs in the phonon regime.

For comparison, Bragg scattering of free particles was studied by applying a light pulse of equal intensity [12] after allowing the gas to freely expand for 5 ms, during which the atomic density was reduced by a factor of 23 and the speed of sound by a factor of 5 from that of the trapped condensate. Thus, Bragg scattering in the expanded sample occurred in the free-particle regime.

The momentum transferred to the atomic sample was determined by the average axial position in time-of-flight images. To extract small momentum transfers, the images were first fitted (in regions where the Bragg scattered atoms were absent) to a bimodal distribution which correctly describes the free expansion of a condensate in the Thomas-Fermi regime, and of a thermal component [13]. The chemical potential μ of the trapped condensate was determined from the radial width of the condensate distribution [11]. The noncondensate distribution (typically less than 20% of the total population) was subtracted from the images before evaluating the momentum transfer.

By varying the frequency difference ω, the Bragg scattering spectrum was obtained for trapped and for freely expanding condensates (Fig. 2). The momentum transfer per atom, shown in units of the recoil momentum $\hbar q$, is antisymmetric about $\omega = 0$ as condensate atoms are Bragg scattered in either the forward or the backward direction, depending on the sign of ω [14].

From these spectra, we determined the total line strength and the center frequency (Fig. 3) by fitting the momentum transfer to the difference of two Gaussian line shapes, representing excitation in the forward and the backward direction. Since $S(\mathbf{q}) = 1$ for free particles, we obtain the static structure factor as the ratio of the line strengths for the trapped and the expanded atomic samples. Spectra were taken for trapped condensates at three different densities by compressing or decompressing the condensates in the magnetic trap prior to the optical excitation.

The Bragg resonance for the expanded cloud was centered at 1.54(15) kHz with an rms width of 900 Hz consistent with Doppler broadening [15]. This frequency includes an expected 160 Hz residual mean-field shift,

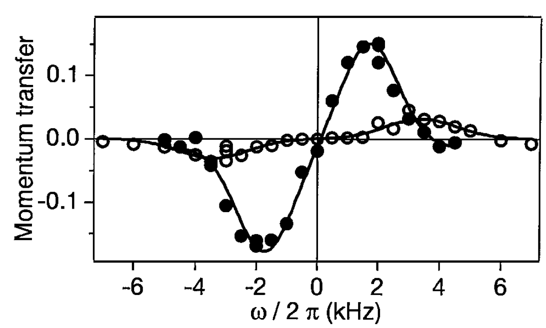

FIG. 2. Bragg scattering of phonons and of free particles. Momentum transfer per particle, in units of $\hbar q$, is shown vs the frequency difference $\omega/2\pi$ between the two Bragg beams. Open symbols represent the phonon excitation spectrum for a trapped condensate at a chemical potential $\mu/h = 9.2$ kHz. Closed symbols show the free-particle response of an expanded cloud. Lines are fits to the difference of two Gaussian line shapes representing excitation in the forward and backward directions.

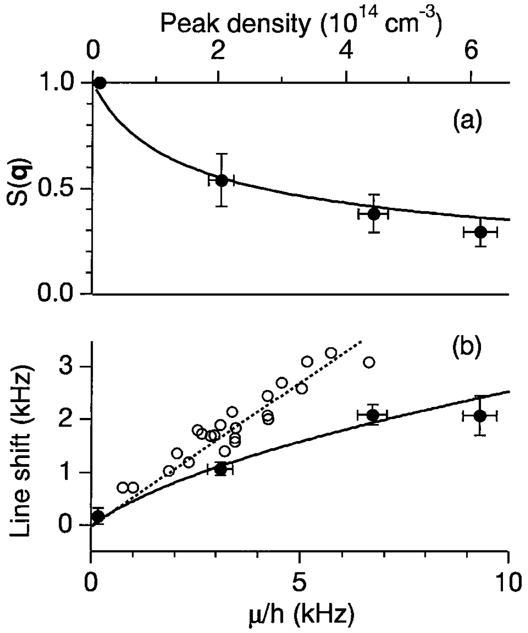

FIG. 3. (a) Static structure factor $S(\mathbf{q})$ and (b) shift of the line center from the free-particle resonance. $S(\mathbf{q})$ is the ratio of the line strength at a given chemical potential μ to that observed for free particles. As μ increases, the structure factor decreases, and the Bragg resonance frequency increases. Solid lines are predictions of a local density approximation [Eq. (5)] using $\omega_q^0 = 2\pi \times 1.38$ kHz. Dotted line indicates the mean-field shift of $4\mu/7h$ in the free-particle regime, with data from [5] shown in open symbols.

giving a measured free-particle resonance frequency of 1.38 kHz. The response of trapped condensates was strikingly different. As the density of the trapped condensates was increased, the Bragg scattering resonance was significantly weakened in strength and shifted upwards in frequency. This reflects the changing character of the excitations created by Bragg scattering as the speed of sound was increased: at a fixed Bragg scattering momentum, the excitations passed from the free-particle to the phonon regime.

To account for this behavior, we use the zero-temperature Bogoliubov description of a weakly interacting homogeneous Bose-Einstein condensate [16]. The Hamiltonian,

$$\mathcal{H} = \sum_k \hbar\omega_k^0 \hat{a}_k^\dagger \hat{a}_k + \sum_{k,l,m} \frac{2\pi\hbar^2 a}{mV} \hat{a}_k^\dagger \hat{a}_l^\dagger \hat{a}_m \hat{a}_{k+l-m}\,, \tag{1}$$

for a gas in volume V, where $\hbar\omega_k^0 = \hbar^2 k^2/2m$, is approximated by replacing the zero-momentum operators with c-numbers $\hat{a}_0^\dagger = \hat{a}_0 = \sqrt{N_0}$, where N_0 is the number of atoms with zero momentum. Neglecting terms of order $N^{-1/2}$, the Hamiltonian is diagonalized by a canonical transformation to operators defined by $\hat{a}_k = u_k \hat{b}_k - v_k \hat{b}_{-k}^\dagger$, where $u_k = \cosh\phi_k$, $v_k = \sinh\phi_k$, and $\tanh 2\phi_k = \mu/(\hbar\omega_k^0 + \mu)$. The energy of the Bogoliubov excitation created by $\hat{b}_k^\dagger$ is $\hbar\omega_k^B = \sqrt{\hbar\omega_k^0(\hbar\omega_k^0 + 2\mu)}$.

Neglecting small contributions representing multiparticle excitations [3,4], the single quasiparticle contribution to the static structure factor is

$$S(\mathbf{q}) = \frac{N_0}{N} \langle g| (\hat{a}_q \hat{a}_q^\dagger + \hat{a}_{-q}^\dagger \hat{a}_{-q} + \hat{a}_{-q}^\dagger \hat{a}_q^\dagger + \hat{a}_q \hat{a}_{-q}) |g\rangle\,. \tag{2}$$

Substituting the Bogoliubov operators, one obtains [17]

$$S(\mathbf{q}) \simeq (u_q^2 + v_q^2 - 2u_q v_q) = \omega_q^0/\omega_q^B\,. \tag{3}$$

In the limit $\hbar\omega_q^0 \gg \mu$, the Bogoliubov excitations become identical to free-particle excitations ($u_q \to 1$, $v_q \to 0$), and $S(\mathbf{q}) \to 1$. For phonons ($\hbar\omega_q^0 \ll \mu$), $S(\mathbf{q}) \to \hbar q/2mc_s$, and the line strength diminishes linearly with q.

To the same order of approximation, the quasiparticle resonance is undamped, and the dynamical structure factor is $S(\mathbf{q}, \omega) = S(\mathbf{q})\delta(\omega - \omega_q^B)$ (satisfying the f sum rule: $\int \omega S(\mathbf{q}, \omega)\, d\omega = \omega_q^0$ [3]). Thus, accompanying the diminished line strength, the Bragg resonance is shifted upward from the free particle resonance by $\omega_q^B - \omega_q^0$.

Equivalently, the suppression of the Bragg resonance in the phonon regime can be understood in terms of the many-body condensate wave function. The static structure factor is the magnitude of the state vector $|e\rangle = \sum_k \hat{a}_{k+q}^\dagger \hat{a}_k |g\rangle/\sqrt{N}$. The macroscopic population of the zero-momentum state picks out two relevant terms in the summation:

$$|e\rangle \simeq (\hat{a}_q^\dagger \hat{a}_0 |g\rangle + \hat{a}_0^\dagger \hat{a}_{-q} |g\rangle)/\sqrt{N} = |e^+\rangle + |e^-\rangle\,. \tag{4}$$

These represent two means by which momentum is imparted to the condensate: either by promoting a zero-momentum particle to momentum $\hbar\mathbf{q}$, or else by demoting a particle from momentum $-\hbar\mathbf{q}$ to zero momentum.

If correlations could be neglected, the total rate of excitation would simply be the sum of the independent rates for these two processes, proportional to $\langle e^+ | e^+\rangle = \langle N_q^0\rangle + 1 = u_q^2$ and $\langle e^- | e^-\rangle = \langle N_{-q}^0\rangle = v_q^2$, where $\langle N_k^0\rangle$ is the expected number of atoms of momentum $\hbar\mathbf{k}$ in the condensate. This would apply, for example, to a condensate in a pure number state, or to an ideal gas condensate with a thermal admixture of atoms with momenta $\pm\hbar\mathbf{q}$, and would always lead to $S(\mathbf{q}) > 1$.

Yet, for the many-body ground state of the interacting Bose gas, the behavior is dramatically different. Collisions of zero-momentum atoms admix into the condensate pairs of atoms at momenta $\pm\hbar\mathbf{q}$, the population of which comprises the quantum depletion [18]. As a result, the two momentum transfer mechanisms described above produce indistinguishable states, and the rate of momentum transfer is given by the interference of two amplitudes, not by the sum of two rates. Pair excitations in the condensate are correlated so as to minimize the total energy, and thereby give destructive interference between the two momentum transfer processes, i.e., $S(\mathbf{q}) = (u_q - v_q)^2 < 1$. For high momentum, $\langle N_q^0\rangle \ll 1$ and the interference plays a minor role. In the phonon regime, while the independent rates u_q^2 and v_q^2 (and, hence, $\langle N_{\pm q}^0\rangle$) diverge as $1/q$, the correlated quantum depletion extinguishes the rate of Bragg excitation.

These results for the homogeneous Bose gas can be applied to trapped, inhomogeneous condensates by a local density approximation since the reduced phonon wavelength q^{-1} (0.4 μm) is much smaller than the condensate size ($r > 20$ μm) and since the zero-point Doppler width is smaller than the mean-field shift ($\hbar q/mr \ll \mu/\hbar$) [19,20]. In the Thomas-Fermi regime, the condensate has a normalized density distribution $f(n) = 15n/4n_0\sqrt{1 - n/n_0}$, where n_0 is the maximum condensate density. The Bragg excitation line shape is then

$$I(\omega)d\omega = \frac{15}{8}\frac{\omega^2 - \omega_q^{02}}{\omega_q^0(\mu/\hbar)^2}\sqrt{1 - \frac{\omega^2 - \omega_q^{02}}{2\omega_q^0\mu/\hbar}}\,d\omega\,, \quad (5)$$

from which one can obtain the line strength $S(\mathbf{q})$ and center frequency. The line strength has the limiting values of $S(\mathbf{q}) \rightarrow 15\pi/32(\hbar\omega_q^0/2\mu)^{1/2}$ in the phonon regime and $S(\mathbf{q}) \rightarrow 1 - 4\mu/7\hbar\omega_q^0$ in the free-particle regime [21]. In accordance with the f-sum rule, the center frequency $\bar{\omega}$ is given as $\omega_q^0/S(\mathbf{q})$.

These predictions are shown in Fig. 3 using $\omega_0^q = 2\pi \times 1.38$ kHz. Both the line strength and the shift of the Bragg resonance are well described by our treatment. For comparison, previous measurements [5] of the mean-field shift of the Bragg resonance ($4\mu/7\hbar$) in the free-particle regime are also shown, clearly indicating the many-body character of low energy excitations.

Finally, let us discuss finite temperature effects. The structure factor at nonzero temperature is increased as $S(\mathbf{q}) = (u_q - v_q)^2 \times (1 + N_q^B + N_{-q}^B)$ due to the populations $N_{\pm q}^B$ of thermally excited Bogoliubov quasiparticles at wave vectors $\pm\mathbf{q}$. However, in our measurements using stimulated scattering from two laser beams, the contribution of the thermal excitations cancels out [14], and thus we extract the zero-temperature structure factor. In contrast, by measuring light scattering from a single beam one could determine the temperature-dependent structure factor. Such a measurement could detect low-momentum thermal excitations, and thus could serve as a thermometer for a low-temperature gas.

In conclusion, stimulated light scattering was used to excite phonons in trapped Bose-Einstein condensates with wavelengths much smaller than the size of the trapped sample. The static structure factor was shown to be substantially reduced in the phonon regime. This modification of light-atom interactions arises from the presence of a correlated admixture of momentum excitations in the condensate. The observed reduction of $S(\mathbf{q})$ also implies a reduction of inelastic Rayleigh scattering of light with wave vector k by a condensate when $\hbar\omega_k^0 < \mu$ [22]. This effect may reduce heating in optical dipole traps and reduce the optical density probed in absorption imaging. For example, the absorption of near-resonant light by a homogeneous sodium condensate at a density of 3×10^{15} cm^{-3} [23] should be reduced by a factor of 2.

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