

Interpreting the Neutron's Electric Form Factor: Rest Frame Charge Distribution or Foldy Term?

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The neutron's electric form factor contains vital information on nucleon structure, but its interpretation within many models has been obscured by relativistic effects. I demonstrate that, to leading order in the relativistic expansion of a constituent quark model, the Foldy term cancels exactly against a contribution to the Dirac form factor F_1 to leave intact the naive interpretation of G_E^n as arising from the neutron's rest frame charge distribution.

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In 1962, Sachs showed [1] that the combinations of elastic nucleon form factors ($N = p$ or n)

$$G_E^N = F_1^N - \frac{Q^2}{4m_N^2} F_2^N, \quad (1)$$

$$G_M^N = F_1^N + F_2^N \quad (2)$$

have simple interpretations as the spatial Fourier transforms of the nucleons' charge and magnetization distributions in the Breit frame (where momentum $\vec{p} = -\frac{\vec{Q}}{2}$ is scattered to momentum $\vec{p}' = +\frac{\vec{Q}}{2}$). Here F_1^N and F_2^N are the Dirac and Pauli form factors, respectively, defined by

$$\langle N(\vec{p}', s') | j_{em}^\mu(0) | N(\vec{p}, s) \rangle = \bar{u}(\vec{p}', s') \times \left[F_1^N \gamma^\mu + i \frac{\sigma^{\mu\nu} q_\nu}{2m_N} F_2^N \right] u(\vec{p}, s), \quad (3)$$

where $q_\nu = p'_\nu - p_\nu$ and F_1^N and F_2^N are functions of $Q^2 = -q^2$.

These form factors obviously contain vital information on the internal composition of the nucleons. Although it has proven elusive experimentally, the electric form factor of the neutron G_E^n is particularly fascinating in this respect. In pion-nucleon theory, G_E^n would arise from a π^- cloud, with convection currents producing the anomalous magnetic moments $F_2^p = 1.79 \equiv \mu_p - 1$ and $F_2^n = -1.91 \equiv \mu_n$. In contrast, in a valence quark model the nucleon magnetic moments arise from the underlying charged spin- $\frac{1}{2}$ constituents with the famous SU(6) relation

$$\frac{\mu_p}{\mu_n} = -\frac{3}{2} \quad (4)$$

and with a scale set by

$$\mu_p = \frac{m_N}{m_d} \simeq 3, \quad (5)$$

where $m_d \simeq m_u \simeq \frac{1}{3}m_N$ is a valence quark effective mass. Within this model it has been argued that in the SU(6) limit $G_E^n(Q^2)$ would be identically zero, but that the spin-spin forces which produce the SU(6)-breaking Δ - N splitting create a charge segregation inside the neutron and

lead to a nonzero G_E^n [2–4]. The effect arises because the spin-spin forces push d quarks to the periphery of the neutron and pull the u quark to the center. Thus both the π^- cloud picture and the hyperfine-perturbed quark model predict a negative neutron charge radius, as observed.

Nonrelativistically, the squared charge radius is simply the charge-weighted mean square position of the constituents. More generally

$$G_E^p(Q^2) \equiv 1 - \frac{1}{6} r_{Ep}^2 Q^2 + \dots \quad (6)$$

and

$$G_E^n(Q^2) \equiv -\frac{1}{6} r_{En}^2 Q^2 + \dots \quad (7)$$

define the proton and neutron charge radii, with

$$\frac{G_M^N(Q^2)}{\mu_N} = 1 - \frac{1}{6} r_{MN}^2 Q^2 + \dots \quad (8)$$

defining the corresponding magnetic radii.

I focus here on the belief that the measured r_{En}^2 [5] is explained by the ‘‘Foldy term’’ [6]; i.e., using Eqs. (1) and (7),

$$r_{En}^2 = r_{1n}^2 + \frac{3\mu_n}{2m_N^2} \equiv r_{1n}^2 + r_{\text{Foldy},n}^2, \quad (9)$$

where r_{1n}^2 is the ‘‘charge radius’’ associated with $F_1^n \equiv -\frac{1}{6} r_{1n}^2 Q^2 + \dots$. The second term in Eq. (9), called the Foldy term, appears to arise as a relativistic correction associated with the neutron's magnetic moment and so to have nothing to do with the neutron's rest frame charge distribution. It has the value -0.126 fm^2 , nearly coinciding with the measured value of $-0.113 \pm 0.005 \text{ fm}^2$ [7]. On this basis it has been argued that any ‘‘true’’ charge distribution effect must be very small. In this paper I show that while the Foldy term closely resembles r_{En}^2 numerically, it does not ‘‘explain it.’’ Indeed, I demonstrate that, in the relativistic approximation to the constituent quark model in which the Foldy term first appears, it is canceled exactly by a contribution to the Dirac form factor F_1 leaving r_{En}^2 correctly interpreted as arising entirely from the rest frame internal charge distribution of the neutron.

The relationship (1) between the Sachs form factor G_E and the Dirac and Pauli form factors F_1 and F_2 is relativistic in origin. Unfortunately, relativistic constituent models of the nucleon are notoriously difficult: rest frame models are difficult to boost and infinite-momentum-frame (or light-cone) quark models have trouble constructing states of definite J^P . This could be the reason that the interpretation of G_E^n has not been clarified in the context of such models.

While an *accurate* constituent quark model of nucleon structure must certainly be fully relativistic, the issue at hand can be resolved by using a relativistic expansion around the nonrelativistic limit. This is possible because the Foldy term $r_{\text{Foldy},n}^2$ arises at order Q^2/m^2 and so its character may be exposed by an expansion of G_E^n to order $1/m^2$. I will also exploit symmetries of the problem available in certain limits which will make the discussion independent of the details of models.

I begin with a simple “toy model” in which a “toy neutron” $n_{\bar{S}D}$ is composed of a *scalar* antiquark \bar{S} of mass m_S and charge $-e_D$ and a spin- $\frac{1}{2}$ Dirac particle D of mass m_D and charge e_D bound by flavor-, spin-, and momentum-independent forces into a rest frame nonrelativistic S wave. The calculation begins by noting that, from their definitions,

$$G_E^{n_{\bar{S}D}}(Q^2) = \left\langle n_{\bar{S}D} \left(+\frac{Q\hat{z}}{2}, + \right) \left| \rho_{em} \right| n_{\bar{S}D} \left(-\frac{Q\hat{z}}{2}, + \right) \right\rangle \quad (10)$$

and

$$\langle D(\vec{p} + Q\hat{z}, s') \left| \rho_{em} \right| D(\vec{p}, s) \rangle = e_D \left(1 - \frac{Q^2}{8m_D^2} \right) \langle \tilde{D}(\vec{p} + Q\hat{z}) \left| \rho_{em} \right| \tilde{D}(\vec{p}) \rangle \delta_{ss'} + \rho_{\text{spin-flip}}, \quad (14)$$

where

$$\rho_{\text{spin-flip}} \equiv \frac{e_D Q}{4m_D^2} (p_- \delta_{s'+} \delta_{s-} - p_+ \delta_{s'-} \delta_{s+}) \quad (15)$$

and where \tilde{D} is a fictitious scalar quark with the mass and charge of D . (Here and in the following I use a tilde to denote the fictitious scalar versions \tilde{D} , \tilde{d} , \tilde{u} , \dots of the spin- $\frac{1}{2}$ quarks D , d , u , \dots). This expression is easily obtained by making a nonrelativistic expansion of both the D and \tilde{D} charge density matrix elements.

The spin-flip term $\rho_{\text{spin-flip}}$ could only contribute to $r_{E n_{\bar{S}D}}^2$ via transitions to and from the Wigner-rotated components of the wave function. However, the amplitudes of such components are proportional to Qk/m_D^2 , where k is an internal momentum. Since $\rho_{\text{spin-flip}}$ already carries a factor $1/m_D^2$, such effects may be discarded. Note that nonflip Wigner-rotated contributions are of the same order and may also be neglected.

We conclude that $r_{E n_{\bar{S}D}}^2$ may be computed by replacing D by \tilde{D} provided the additional contribution $-e_D Q^2/8m_D^2$ is added to $G_E^{n_{\bar{S}D}}$. I will denote the

$$G_M^{n_{\bar{S}D}}(Q^2) = \frac{m_{n_{\bar{S}D}}}{Q} \times \left\langle n_{\bar{S}D} \left(+\frac{Q\hat{z}}{2}, + \right) \left| j_{em}^{1+i2} \right| n_{\bar{S}D} \left(-\frac{Q\hat{z}}{2}, - \right) \right\rangle. \quad (11)$$

It is immediately clear that the calculation of these form factors requires boosting the rest frame S -wave bound state to momenta $\pm \frac{Q\hat{z}}{2}$. Doing so can introduce a host of $1/m^2$ effects in the boosted counterpart of the S -wave state and it can also produce new P -wave-like components by Wigner rotation of the D -quark spinors [8]. I show that the latter effect is subleading and deal with the former effect by exploiting an effective charge-conjugation symmetry of the system for $m_D = m_S \equiv m$.

Since $\mu_{n_{\bar{S}D}}$ involves the limit of Eq. (11) as $Q \rightarrow 0$, to the required order in $1/m$ it simply takes on its nonrelativistic value

$$\mu_{n_{\bar{S}D}} = \frac{e_D m_{n_{\bar{S}D}}}{m_D}, \quad (12)$$

where of course $m_{n_{\bar{S}D}} = m_S + m_D$ in this limit. The Foldy term is thus well defined:

$$r_{\text{Foldy},n_{\bar{S}D}}^2 = \frac{3e_D}{2m_D m_{n_{\bar{S}D}}}. \quad (13)$$

We next compute $G_E^{n_{\bar{S}D}}(Q^2)$ directly from Eq. (10). In this model, ρ_{em} remains a one-body current to leading order in $1/m^2$, and the impulse approximation is valid. Within this approximation, we make use of the relation

associated “zitterbewegung” radius $3/4m_D^2$ by $r_{D,\text{zitter}}^2$. (Note that the charge e_D has been removed from this definition, so $r_{D,\text{zitter}}^2$ represents a “matter radius” and not a charge radius.) The effect of $r_{D,\text{zitter}}^2$ is well known in a variety of contexts, including atomic physics [9], nuclear physics [10], hadronic physics [11], and heavy quark physics [12]; at the most elementary and concrete level it appears as the additional factor of $(1 - \frac{Q^2}{8m_D^2})^2$ in the ratio of the Mott cross section to the Rutherford cross section. The problem of computing the remaining contributions to $G_E^{n_{\bar{S}D}}$ to this order from the fictitious $\tilde{S}\tilde{D}$ scalar-scalar bound state would in general be highly nontrivial, since, in addition to the simple nonrelativistic charge distribution, effects of order kQ/M^2 and Q^2/M^2 (where M^2 with dimension GeV^2 is composed of m_S and m_D) can surface in many ways in the boosted wave functions. However, in the limit $m_D = m_S \equiv m$, \tilde{S} acts as though it were the antiparticle of \tilde{D} and the total “scalar charge radius” $r_{E n_{\bar{S}D}}^2$ *vanishes* because every contribution to $G_E^{n_{\bar{S}D}}$ by \tilde{D} will be canceled by one for \tilde{S} ; i.e., this

system has in this limit a pseudo-charge-conjugation invariance under $(\bar{S}, \bar{D}) \rightarrow (S, \bar{D})$ so that $r_{E n_{\bar{S}D}}^2 = 0$ and therefore $r_{E n_{\bar{S}D}}^2 = e_D r_{D, \text{zitter}}^2$. Since in this limit $m_{n_{\bar{S}D}} = 2m$, from Eq. (13) we have

$$r_{E n_{\bar{S}D}}^2 = e_D r_{D, \text{zitter}}^2 = \frac{3e_D}{4m^2} = r_{\text{Foldy}, n_{\bar{S}D}}^2; \quad (16)$$

i.e., in this toy model the “scalar charge distribution” is zero and the Foldy term would indeed account for the full charge radius of the toy neutron. This conclusion is simply interpreted: the two scalar particles \bar{S} and \bar{D} have perfectly overlapping and canceling charge distributions, but the expansion of the \bar{D} distribution by $r_{D, \text{zitter}}^2$ creates a slight excess of \bar{D} at large radii. In terms of its experimental significance, we have concluded that in an $\bar{S}D$ model of the neutron, the observation of an equality of $r_{E n}^2$ and $r_{\text{Foldy}, n}^2$ would indeed indicate the absence of an intrinsic “scalar” charge distribution.

We shall now draw quite another conclusion for the situation in the valence quark model in which the neutron is in the leading approximation made of three mass m_q spin- $\frac{1}{2}$ quarks ddu bound by flavor-, spin-, and momentum-independent forces into flavor-independent nonrelativistic relative S waves. In this case

$$\mu_{n_{ddu}} = -\frac{2m_{n_{ddu}}}{3m_q} \simeq -2 \quad (17)$$

so that

$$r_{\text{Foldy}, n_{ddu}}^2 = -\frac{1}{m_q m_{n_{ddu}}}. \quad (18)$$

In calculating $r_{E n_{ddu}}^2$ via Eq. (10), the transformation of the calculation of the charge radius of ddu to that of three scalar quarks $\bar{d}\bar{d}\bar{u}$ and residual $e_q r_{q, \text{zitter}}^2 = 3e_q/4m_q^2$ terms proceeds as before, as does the neglect of Wigner-rotated components of the boosted state vectors. *However, in this case, since the $r_{q, \text{zitter}}^2$ terms are spin and flavor independent, and since the sum of the three charges is zero, they lead to no net Q^2/m^2 term.* To examine the $\bar{d}\bar{d}\bar{u}$ scalar charge distribution in this model, we use an analog of the pseudo-charge-conjugation invariance that we applied to the scalar part of the $\bar{S}D$ matrix element. Since under our assumptions the three quark wave function belongs to the symmetric representation of the permutation group S_3 and since $\sum_i e_i = 0$, the contributions of the three quarks cancel each other and the scalar part $r_{E, n_{\bar{d}\bar{d}\bar{u}}}^2$ of the charge radius, including boost effects, once again vanishes. Thus in the leading approximation to the valence quark model

$$r_{E, n_{ddu}}^2 = r_{E, n_{\bar{d}\bar{d}\bar{u}}}^2 + \sum_i e_i r_{i, \text{zitter}}^2 = 0, \quad (19)$$

which using Eq. (9) implies that

$$r_{1n_{ddu}}^2 = -r_{\text{Foldy}, n_{ddu}}^2. \quad (20)$$

The interpretation of this result is clear: the exactly overlapping and canceling rest frame quark distributions remain exactly overlapping and canceling even after they

are boosted and all equally smeared by $r_{q, \text{zitter}}^2$ [13]. In this case *any net $r_{E n}^2$ must arise in this order from new dynamics which can produce an intrinsic internal charge distribution.* It is just such an internal charge distribution that is posited to arise in quark models [2–4] from spin-dependent forces. Thus the coincidence of the predicted rest frame charge distribution in such models with the experimental value of G_E^n (see, e.g., Fig. 1 of the second entry in Ref. [3]) may be claimed as a success, while the numerical coincidence of $r_{E n}^2$ with the Foldy term may consistently be viewed as a (potentially misleading) accident. Such an accident is possible because while in the nonrelativistic limit $r_{\text{Foldy}, n_{ddu}}^2 \ll r_{E n_{ddu}}^2$, in QCD both constituent masses and hadronic radii are determined by Λ_{QCD} so they are *expected* to be of comparable magnitude.

Before too much is made of this successful prediction of the valence quark model, some other very fundamental questions must still be answered. Perhaps the most fundamental is the possible effect of nonvalence components in the neutron wave function. After all, the classic explanation [14] for $r_{E n}^2$ is that the neutron has a $p\pi^-$ component in its wave function [15] (for a discussion in the more modern context of heavy baryon chiral perturbation theory, see Ref. [16]). Since both hyperfine interactions and $q\bar{q}$ pairs are $1/N_c$ effects, I know of no simple argument for why one should dominate.

Fortunately, there is both theoretical and experimental progress in resolving this old question. Recent theoretical work [17] on “unquenching the quark model” indicates that there are strong cancellations between the hadronic components of the $q\bar{q}$ sea which tend to make it transparent to photons. These studies provide a natural way of understanding the successes of the valence quark model even though the $q\bar{q}$ sea is very strong and, in particular, suggest that the precision of the Okubo-Zweig-Iizuka rule is the result of *both* a factor of $1/N_c$ and strong cancellations within this $1/N_c$ -suppressed meson cloud. Recent and planned experimental work will provide precision measurements of the elastic form factors [18]. At the same time, new parity-violation measurements are delineating the contributions of $s\bar{s}$ pairs to the charge and magnetization distributions of the nucleons [19]. These measurements are beginning to constrain the importance of such effects and, by broken $SU(3)$, their $u\bar{u}$ and $d\bar{d}$ counterparts, and future experiments will either see $s\bar{s}$ effects or very tightly limit them (at the level of contributions of a few percent to $r_{E N}^2$ and μ_N). The resolution of the old question of the origin of μ_n and $r_{E n}^2$ is thus within sight.

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- [15] I have examined the Foldy term in a model of a nonrelativistic PD $L = 1$ bound state, where P is a pseudoscalar particle. In such a system (which represents the most naive version of a $\pi^- p$ model for the neutron), $\rho_{\text{spin-flip}}$ makes a contribution which exactly cancels $r_{D,zitter}^2$. Thus in the pseudo-charge-conjugation symmetry limit $m_P = m_D \equiv m$, $r_{EnPD}^2 = 0$ and, as in the quark model, one has $r_{1nPD}^2 = -r_{\text{Foldy},nPD}^2$ so that the observed r_{En}^2 may be interpreted as arising from an intrinsic internal charge distribution.
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- [19] For an example of recent progress, see The HAPPEX Collaboration, K.A. Aniol *et al.*, Phys. Rev. Lett. **82**, 1096 (1999), and references therein.