Neutrino Driven Streaming Instabilities in a Dense Plasma

L. O. Silva,[1] R. Bingham,[2] J. M. Dawson,[1] J. T. Mendonça,[3] and P. K. Shukla[4]
[1]*Department of Physics, University of California Los Angeles, Los Angeles, California 90095*
[2]*Rutherford Appleton Laboratory, Chilton, Didcot, Oxon OX11 OQX, United Kingdom*
[3]*GoLP/Centro de Física de Plasmas, Instituto Superior Técnico, 1096 Lisboa Codex, Portugal*
[4]*Institut für Theoretische Physik, Ruhr-Universität Bochum, D-44780 Bochum, Germany*
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The nonlinear self-consistent propagation of neutrinos in a dense plasma is described using the relativistic kinetic equations for neutrinos and electrons. The general nonlinear dispersion relation for a neutrino fluid coupled with a plasma is derived. For scenarios close to realistic astrophysical conditions, neutrino driven streaming instabilities can develop, with growth rates which scale with the Fermi constant G_F, thus leading to significant energy transfer from the neutrinos to the plasma.

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The electromagnetic properties of neutrinos and the propagation of neutrinos in a medium are subjects of paramount importance, touching several fundamental problems in physics [1–3]. Until very recently, neutrino propagation in matter has been considered as a non-self-consistent single particle process. All the single particle mechanisms involving the electromagnetic properties of neutrinos in matter have a direct analogy with the processes involving a single electron in a medium (e.g., Cerenkov radiation, transition radiation, bremsstrahlung), and this association can be pushed even further through the concept of the induced charge of neutrinos in a medium [4,5]. However, it is well known that the self-consistent description of a stream of charged particles moving through a medium (for instance, a plasma) leads to the appearance of a new class of collective phenomena (e.g., instabilities, Landau damping) [6], which cannot be accounted for by the single particle description. Previous studies based on a Klein-Gordon description of the neutrino field have pointed out that neutrino driven instabilities can also develop in different astrophysical conditions [7]. In this Letter, we consider the self-consistent propagation of a neutrino distribution in a plasma: the neutrinos (which we assume are electron neutrinos ν_e) interact with the electrons through the weak interaction force. The electrons in the plasma interact with the neutrinos also via the weak interaction force, and between themselves and with the ions through the electromagnetic force. The stream of neutrinos is intense enough to disturb the background medium, generating an electron density modulation. The density modulation will then affect the neutrinos, bunching them in the regions of lower electron density. This, in turn, leads to an increase of the force that the neutrinos exert in the plasma, and thus to a stronger density modulation, closing the feedback loop. This picture is equivalent to the two stream instability of an electron beam in a plasma or to the photon driven forward Raman instabilities found in laser-plasma interactions. In spite of the different type of interactions involved, all these collective processes can be classified as streaming instabilities [6].

A systematic description of the many body interaction between neutrinos and background plasma must be based on a kinetic theory for the neutrinos and the plasma. This kinetic theory can be either derived from a particle physics point of view [8] or from more intuitive and easily generalizable statistical and plasma physics considerations. The single neutrino dynamics in matter is determined by a Hamiltonian including the effective potential $V_{\rm eff}$ describing neutrino interaction with matter [9]. For an isotropic plasma, where the ions (i) provide a neutralizing background, and assuming the neutrinos (ν) interact only with the electrons (e) in the plasma through the weak interaction force, the effective Hamiltonian is [5,9] $\mathcal{H}(\mathbf{P}_\nu, \mathbf{r}) = \mathcal{H}_0 + V_{\rm eff}(\mathbf{r}, t) \equiv \sqrt{[\mathbf{P}_\nu c - (\sqrt{2}\, G_F/c)\mathbf{J}_e]^2 + m_\nu^2 c^4} + \sqrt{2}\, G_F n_e(\mathbf{r}, t)$, where G_F is the Fermi constant of weak interaction, $\mathbf{P}_\nu$ is the neutrino canonical momentum, $n_e(\mathbf{r}, t)$ is the local electron number density, $\mathbf{J}_e$ is the electron fluid current, m_ν is the neutrino rest mass, c is the speed of light, and the Weinberg mixing angle θ_W satisfies $\sin\theta_W = 1/2$. Also, we are in the reference frame where the electron fluid is stationary; i.e., the mean electron fluid current is zero. For massless neutrinos, m_ν can be set to zero without loss of generality. In the quasiclassical limit, i.e., as long as the neutrino de Broglie wavelength $\lambda_\nu = 2\pi\hbar/|\mathbf{p}_\nu|$ varies only slightly over the typical length scale of changes in $V_{\rm eff}$, $\lambda_\nu \ll 1/|\nabla \log V_{\rm eff}|$, the single neutrino dynamics can be described by the classical equations of motion derived from $\mathcal{H}$, and $\dot{\mathbf{P}}_\nu = -\partial\mathcal{H}/\partial\mathbf{r}_\nu = -\nabla V_{\rm eff} + (\sqrt{2}\, G_F/\mathcal{H}_0)\nabla\mathbf{p}_\nu \cdot \mathbf{J}_e$, and $\dot{\mathbf{r}}_\nu = \mathbf{v}_\nu = \partial\mathcal{H}/\partial\mathbf{P}_\nu = \mathbf{p}_\nu c^2/\sqrt{p_\nu^2 c^2 + m_\nu^2 c^4}$, where $\mathbf{p}_\nu = \mathbf{P}_\nu - (\sqrt{2}\, G_F/c^2)\mathbf{J}_e$ is the neutrino momentum. The single electrons in the plasma are subject to the electric field $\mathbf{E}$ (due to deviations from the plasma quasineutrality) and to the ponderomotive force due to anisotropies in the neutrino distribution $\mathbf{F}_{\rm pond} = -\sqrt{2}\, G_F \nabla[n_\nu(\mathbf{r}, t) - \frac{\mathbf{v}_e \cdot \mathbf{J}_\nu}{c^2}]$ [10], where $n_\nu(\mathbf{r}, t)$ is the neutrino number density, and $\mathbf{J}_\nu$ is the neutrino fluid current. The collisionless relativistic equations for the neutrinos and electrons can then be written as [5]

$$\frac{\partial f_\nu}{\partial t} + \mathbf{v}_\nu \cdot \frac{\partial f_\nu}{\partial \mathbf{r}} - \sqrt{2}\, G_F\left(\nabla n_e + \frac{1}{c^2}\frac{\partial \mathbf{J}_e}{\partial t} - \frac{\mathbf{v}_\nu}{c^2} \times \nabla \times \mathbf{J}_e\right) \cdot \frac{\partial f_\nu}{\partial \mathbf{p}_\nu} = 0, \tag{1}$$

$$\frac{\partial f_e}{\partial t} + \mathbf{v}_e \cdot \frac{\partial f_e}{\partial \mathbf{r}} - \left[e\mathbf{E} + \sqrt{2}\, G_F\left(\nabla n_\nu + \frac{1}{c^2}\frac{\partial \mathbf{J}_\nu}{\partial t} - \frac{\mathbf{v}_e}{c^2} \times \nabla \times \mathbf{J}_\nu\right)\right] \cdot \frac{\partial f_e}{\partial \mathbf{p}_e} = 0, \tag{2}$$

where f_e (f_ν) is the quasiclassical distribution function for the electrons (neutrinos), satisfying the normalization condition $n_e = 2\int d\mathbf{p}_e\, f_e$ (the factor 2 accounting for spin degeneracy), and $n_\nu = \int d\mathbf{p}_\nu\, f_\nu$, with n_ν the neutrino number density. $\mathbf{J}_{e(\nu)}$ is written as a function of $f_{e(\nu)}$ as $\mathbf{J}_{e(\nu)} = g_{e(\nu)}\int d\mathbf{p}_{e(\nu)}\, \mathbf{v}_{e(\nu)} f_{e(\nu)}$, with $g_{e(\nu)} = 2(1)$. Equations (1) and (2) are equivalent to the covariant relativistic kinetic equations derived by Semikoz using a less intuitive formalism [8]. Our approach here is clearly semiclassical: the interaction of the neutrinos with the electrons is governed by quantum processes (included in $V_{\rm eff}$), and the Fermi statistics of the phase space density of the particle numbers are taken into account, but the neutrino dynamics is determined by the classical Hamiltonian $\mathcal{H}$, and we neglect the spins. By neglecting the collisions $\nu\nu$, νe, and ei, we are assuming that the typical time scale $2\pi/\omega_L$ of the collective processes satisfies $\omega_L \gg \nu_{ei} \gg \nu_{\nu e} \gg \nu_{\nu\nu}$, where ν_{xy} is the collision frequency between species x and y. Inclusion of collisions will be discussed below. Since we are looking only at neutrino driven electrostatic plasma oscillations, the kinetic equations (1) and (2) must be supplemented by Poisson's equation $\nabla \cdot \mathbf{E} = -4\pi e(n_e - n_{e0})$, where n_{e0} is the mean electron density, and e the magnitude of the electron charge. This set of equations provides a self-consistent picture of the neutrino-plasma interaction. The weak interaction between neutrinos could be easily included in our kinetic equations (through an additional potential term acting on the neutrinos), and this would lead to a slight enhancement of the instability discussed below. Inclusion of additional species, such as positrons, neutrons, or other neutrino flavors, is also straightforward, making this formulation a rather general description for collective neutrino-matter interactions.

Performing the usual perturbative analysis over our set of equations (assuming an isotropic plasma and neglecting ion motion) [6], we obtain the nonlinear dispersion relation for the electron plasma waves (EPWs) driven by the intense neutrino flux

$$1 + \chi_e(\omega_L, \mathbf{k}_L) + \chi_\nu(\omega_L, \mathbf{k}_L) = 0, \tag{3}$$

where $\chi_e(\omega_L, \mathbf{k}_L)$ is the relativistic longitudinal electron susceptibility, given by

$$\chi_e(\omega_L, \mathbf{k}_L) = \frac{\omega_{pe0}^2}{k_L^2}\, m_e \int d\mathbf{p}_e\, \frac{\mathbf{k}_L \cdot (\partial \hat{f}_{e0}/\partial \mathbf{p}_e)}{\omega_L - \mathbf{k}_L \cdot \mathbf{v}_e}, \tag{4}$$

and $\chi_\nu(\omega_L, \mathbf{k}_L)$ is the relativistic longitudinal neutrino susceptibility

$$\chi_\nu(\omega_L, \mathbf{k}_L) = -2G_F^2\, \frac{k_L^2 n_{e0} n_{\nu 0}}{m_e \omega_{pe0}^2}\left(1 - \frac{\omega_L^2}{k_L^2 c^2}\right)^2 \chi_e(\omega_L, \mathbf{k}_L) \times \int d\mathbf{p}_\nu\, \frac{\mathbf{k}_L \cdot (\partial \hat{f}_{\nu 0}/\partial \mathbf{p}_\nu)}{\omega_L - \mathbf{k}_L \cdot \mathbf{v}_\nu}, \tag{5}$$

where ω_L is the EPW frequency, $\mathbf{k}_L$ is the EPW wave vector, $\hat{f}_{e0}$ $(\hat{f}_{\nu 0})$ is the normalized electron (neutrino) distribution function, $\omega_{pe0} = (4\pi n_{e0} e^2/m_e)^{1/2}$ is the electron plasma frequency, and m_e the electron mass.

We now analyze (3) for different physical conditions. We first consider a monochromatic neutrino beam, propagating in a cold plasma ($T_e = 0$). By assuming a cold plasma, dispersion is neglected. Detailed numerical results including the relativistic electron dispersion relation will be presented in a future publication [11]. The most important contribution of an electron thermal distribution is electron Landau damping, and this will be discussed below. The normalized neutrino distribution function is $\hat{f}_{\nu 0} = \delta(\mathbf{p}_\nu - \mathbf{p}_{\nu 0})$, and Eq. (5) can be easily evaluated. The nonlinear dispersion relation then obeys

$$\omega_L^2 = \omega_{pe0}^2 + \frac{\Delta_\nu k_L^4 c^4}{(\omega_L - k_L \cos\theta_p\, p_{\nu 0} c^2/E_{\nu 0})^2}\left(1 - \frac{\omega_L^2}{k_L^2 c^2}\right)^2 \Theta \tag{6}$$

with $\Delta_\nu = 2G_F^2 n_{\nu 0} n_{e0}/(m_e c^2 E_{\nu 0})$, and $\Theta = \left(\frac{m_\nu^2 c^4 \cos^2\theta_p}{E_{\nu 0}^2} + \sin^2\theta_p\right)$, where θ_p is the angle between $\mathbf{k}_L$ and $\mathbf{p}_{\nu 0}$, and $E_{\nu 0}$ is the energy of the neutrinos in the beam. We now proceed by making the same analysis as for the two stream instability [6], putting $\omega_L = \omega_{pe0} + \delta$, assuming that the fastest growing mode verifies $k_L \cos\theta_p\, p_{\nu 0} c^2/E_{\nu 0} \simeq \omega_{pe0}$, and solving for δ. In the weak beam approximation $\delta/\omega_{pe0} \ll 1$, the growth rate is

$$\gamma_{\rm weak} = \frac{\sqrt{3}}{2}\, \omega_{pe0}\left(\frac{\tilde{\Theta}}{\cos^4\theta_p}\Delta_\nu\right)^{1/3} \propto G_F^{2/3}, \tag{7}$$

where $\tilde{\Theta} = \Theta\left(\sin^2\theta_p \frac{p_{\nu 0}^2 c^2}{E_{\nu 0}^2} + \frac{m_\nu^2 c^4}{E_{\nu 0}^2}\right)^2$. We observe that direct forward scattering is strongly suppressed due to the small neutrino mass, since $m_\nu c^2/E_{\nu 0} \ll 1$ thus leading to $\tilde{\Theta} \simeq \sin^6\theta_p$. Also, the maximum growth rate increases with θ_p. A simplistic analysis would lead us to conclude that maximum growth would occur for $\theta_p = \pi/2$. However, and since the wave number of the fastest growing mode decreases with increasing θ_p, Landau damping plays a significant role when $\omega_L/k_L \simeq v_{\rm th}$ only allowing the instability to grow for angles $\theta_p \leq \theta_L = \arccos(v_{\rm th}/c)$, where $v_{\rm th} = \sqrt{k_B T_e/m_e}$ is the electron thermal velocity. Therefore, neutrinos propagate across high temperature regions without significant energy transfer to the plasma waves, and only when they reach sufficiently low electron temperature regions (in the outer layers of the star) the instability starts to go. Furthermore, and most important, the growth rate in Eq. (7) scales with $G_F^{2/3}$, thus much stronger than the single scattering processes, for instance, $\nu - e$ scattering, which are proportional to G_F^2. The previous model for neutrino emission is still a crude one. In general, we expect neutrino emission with some momentum spread $\sigma_{\mathbf{p}\nu}$. Assuming a neutrino distribution function obeying $\hat{f}_{\nu 0} = \hat{f}_{\nu 0x}(p_{\nu x})\delta(p_{\nu y})\delta(p_{\nu z})$, where $\hat{f}_{\nu 0x}(p_{\nu x})$ describes the energy distribution of the neutrinos, the neutrino susceptibility (5) can still be evaluated as before as long as neutrino Landau damping [12] can be neglected; neutrino kinetic effects can be discarded whenever the phase velocity of the unstable plasma modes does not overlap the neutrino velocity distribution function; i.e., $|\omega_L/k_L - v_{\nu 0}| \gg \sigma_{v\nu}$, where $v_{\nu 0} = p_{\nu 0}c^2/E_{\nu 0}$ is the neutrino distribution central velocity, and $\sigma_{v\nu} = \sigma_{\mathbf{p}\nu} m_\nu^2 c^6/E_{\nu 0}^3$ is the neutrino velocity spread. In the hydrodynamic regime of the neutrino-plasma instability, and considering that $m_\nu c^2 \ll E_\nu$, the neutrino susceptibility satisfies $\chi_\nu(\omega_L, \mathbf{k}_L) \simeq \chi_\nu^{\text{m-beam}}\langle\lambda_\nu\rangle E_{\nu 0}/(2\pi\hbar c)$, where $\langle\lambda_\nu\rangle$ is the average of the neutrino de Broglie wavelength over the neutrino distribution function, and $\chi_\nu^{\text{m-beam}}$ is the neutrino susceptibility for the monoenergetic neutrino beam with energy $E_{\nu 0}$. In the hydrodynamic regime, the growth rates are identical, and independent of the neutrino distribution function details, being a function only of the neutrino density $n_{\nu 0}$ and the averaged de Broglie wavelength $\langle\lambda_\nu\rangle$. During a supernova explosion, neutrinos are emitted from the surface of the neutrinosphere, and for distances R much longer than the radius of the neutrinosphere $r_{\rm sphere}$, a neutrino beam with angular spread $\theta_{\max} \approx r_{\rm sphere}/R$ is present, with a normalized distribution function $\hat{f}_{\nu 0} = \delta(p_\nu - p_{\nu 0})/[2\pi p_{\nu 0}^2(1 - \cos\theta_{\max})]$ for $\theta \le \theta_{\max}$, where θ denotes the polar angle in spherical coordinates, and $p_{\nu 0}$ is the neutrinos' momentum. The angular spread will contribute to a decrease of the instability growth rate. Even for this simple neutrino distribution model, the neutrino susceptibility can only be determined analytically for EPWs propagating radially. We point out that this is the worst case scenario since the growth rate is then proportional to $m_\nu c^2/E_{\nu 0}$. In this scenario, to be compared with slab geometry for $\theta_p = 0$, the instability growth rate for the fastest growing mode $\omega_L \simeq k_L v_{\nu 0}$, with $v_{\nu 0} = p_{\nu 0}c^2/E_{\nu 0}$, is

$$\gamma_{\rm sph} \simeq \omega_{pe0}\left(\frac{\Delta_\nu}{1 - \cos\theta_{\max}}\frac{m_\nu^6 c^{12}}{E_{\nu 0}^6}\right)^{1/2} \propto G_F\,. \qquad (8)$$

Therefore, $\gamma_{\rm sph}/\gamma_{\rm weak}(\theta_p = 0) \simeq (\Delta_\nu m_\nu^6 c^{12}/E_{\nu 0}^6)^{1/6}/(1 - \cos\theta_{\max})^{1/2}$, thus indicating that for $\theta_p \neq 0$, $\gamma_{\rm sph}/\gamma_{\rm weak}$ should scale approximately with $(\Delta_\nu\tilde{\Theta})^{1/6}/(1 - \cos\theta_{\max})^{1/2}$. For small angles, $\theta_p \le \theta_{\max}$, the instability regime is still hydrodynamic.

Collisions can also be included in our kinetic model. Collisions involving neutrinos are not taken into account, since the collisional cross section is $\sigma_{\nu x} \approx (G_F k_B/2\pi\hbar c)^2 T_\nu T_x$, with T_ν denoting the neutrino temperature and T_x the temperature of the species x (electrons, neutrons), and the collision frequency is much smaller than ω_{pe0}. However, electron-ion collisions should be considered since the electron-ion collision frequency ν_{ei} can be comparable to ω_{pe0}. We assume the Bhatnagar-Gross-Krook e-i collision model [13]. Hence, for a cold plasma the electron susceptibility satisfies $\chi_e(\omega_L, \mathbf{k}_L) = -\omega_{pe0}^2/(\omega_L^2 - i\nu_{ei}\omega_L)$, and the new dispersion relation is obtained from Eq. (6) by replacing ω_L^2 by $\omega_L(\omega_L + i\nu_{ei})$. Performing the same instability analysis as before, and assuming $\delta/\nu_{ei} \ll 1$, we obtain the growth rate $\gamma_{\rm coll}$ for the fastest growing mode ($k_L\cos\theta_p p_{\nu 0}c^2/E_{\nu 0} \simeq \omega_{pe0}$):

$$\gamma_{\rm coll} = \frac{\sqrt{2}}{2}\,\omega_{pe0}\left(\Delta_\nu\frac{\tilde{\Theta}}{\cos^4\theta_p}\frac{\omega_{pe0}}{\nu_{ei}}\right)^{1/2} \propto G_F\,. \qquad (9)$$

Even though $\nu_{ei} \gg \gamma_{\rm coll}$, the inclusion of collisions does not shut down the instability, it only slows down growth, $\gamma_{\rm coll} < \gamma_{\rm weak}$. As for photon driven instabilities [14], the threshold of the instability is proportional to the product of the electron damping ($\propto \nu_{ei}$) with the neutrino damping ($\propto \nu_{\nu x} \propto G_F^2$), giving a negligible threshold for typical parameters.

We now evaluate the growth rate of the neutrino driven streaming instabilities for supernova IIa conditions. The relevant parameter for comparison is the mean free path for neutrino-electron single scattering, which is roughly $l_{\nu e} \approx 10^{15}$ cm, with a neutrino luminosity of $L_\nu = 10^{52}$ erg/s, $n_{e0} = 10^{29}$ cm^{-3}, $T_\nu = 10$ MeV, and T_e in the hundreds of keV range, at $R = 300$ km from the core of the star, and a neutrinosphere radius $r_{\rm sphere} = 3$ km. We also assume the electron-neutrino mass is $m_\nu = 0.1$ eV. With a finite neutrino mass, direct forward scattering is present, but it is too small to be of any effect for electron neutrinos. For scattering angles $\theta_p \neq 0$, the growth rate is much stronger. In Fig. 1(a), we present the maximum growth rate for slab geometry neglecting collisions, in the weak beam approximation, Eq. (7), plotted as a function of θ_p. The vertical lines denote the angle for which Landau damping becomes significant (for a well determined electron temperature), and a mechanism equivalent to forward stimulated Compton scattering starts to play a dominant role [15]. We observe growth rates of the order of 10^9 s^{-1}, corresponding to growth distances of 1 m. In the more realistic scenario where electron-ion collisions are included [Fig. 1(b)], the growth distance for 20 e-foldings is 6 km.

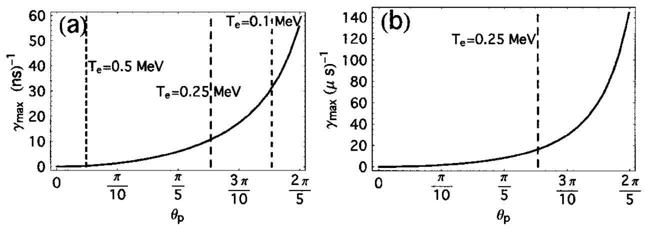

FIG. 1. Growth rate of the neutrino driven streaming instability (a) without *e-i* collisions, Eq. (7), (b) including *e-i* collisions, Eq. (9), for typical supernovae parameters: $\langle E_\nu \rangle =$ 10 MeV , $n_{e0} = 10^{29}$ cm^{-3}, and $L_\nu = 10^{52}$ erg/s at 300 km from the core. The vertical lines indicate the maximum angle the instability can grow without significant influence from electron Landau damping, for a given temperature T_e.

It is then clear that neutrino driven streaming instabilities can play an important role in astrophysical conditions where intense fluxes of neutrinos are present. In supernovae, the role of the neutrino driven instabilities can contribute to the evolution of the explosion [7]. Through this collective process, energy is transferred from the neutrinos to plasma waves and subsequently to the electrons via collisional and Landau damping. Once the plasma heats up sufficiently ($T_e \approx 500$ keV), all the plasma modes become heavily Landau damped (and hence become quasimodes), and the instability turns off limiting the energy deposited in the stellar envelope. This plasma heating leads to a pressure increase behind the stalled shock, and to a strong ejection of the outer layers of the star. In the early universe, a neutrino driven filamentation instability ($\theta_p \approx \pi/2$, with ion motion) can also lead to the formation of nonlinear structures (filaments), contributing to the formation of the large-scale structure of the Universe [16]. This instability will be extremely important whenever there is structure in the beam, such as neutrino hot spots in neutron stars [17].

In this Letter, we have presented a self-consistent picture for neutrino-plasma interactions, based on a relativistic kinetic theory for electron neutrinos and electrons. This formalism can be easily extended to include other neutrino flavors and background species, and provides the foundations for the numerical simulation of collective neutrino-plasma processes. We have shown that for conditions where intense neutrino fluxes are present, neutrino driven streaming instabilities can develop, with growth distances scaling with G_F^{-1}, as compared with the single neutrino-electron scattering mean free path which is proportional to G_F^{-2}. Even direct forward scattering, corresponding to the least favorable situation, gives an effect that must be considered when copious amounts of neutrinos are present. Our results indicate that collective neutrino-plasma instabilities can play a significant role for extreme astrophysical conditions like those present in supernovae explosions, gamma-ray bursts, or the early universe.

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[11] Inclusion of χ_e valid for all temperatures and densities can be easily achieved by writing $\chi_e = -\Omega^2/\omega_L^2$, where $\Omega^2 \equiv \Omega^2(\omega_L, \mathbf{k}_L, n_e, T_e)$ can be determined explicitly in different limits [see, for instance, G. G. Raffelt, *Stars as Laboratories for Fundamental Physics* (University of Chicago Press, Chicago, 1996)]. The perturbative analysis of the dispersion relation is performed in a similar way, leading to an implicit expression for the growth rate of the instability.
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