Comment on "Two Scaling Regimes for Rotating Rayleigh-Bénard Convection"

Recently Canuto and Dubovikov (CD) [1] used a turbulence model to argue for two scaling regimes in rotating Rayleigh-Bénard convection which they state qualitatively agree with experiments [2,3]. They proposed that for large Ta and high Ra, Nu scales as $Ra^{1/3}$ and at low Ra, the scaling is $Nu = ax^3(1 + b/x)^4$ where a and b are determined constants for infinite aspect ratio and large Prandtl number ($a = 1/256, b = 3$) and where $x = Ra/Ta^{2/3} \propto Ra/Ra_c$ [4]. Experiments [3,5] qualitatively confirm the high-Ra scaling but experimental, numerical, and theoretical results are inconsistent with the scaling proposed for smaller Ra. From the perspective of weakly nonlinear theory, the proposed scaling is startling and completely unexpected. This discrepancy arises, I believe, from the inappropriate application of a turbulent scaling model based on a broad range of spatial scales to a regime where the scaling is well approximated by weakly nonlinear theory describing a small range of spatial scales.

The onset of convection with or without rotation is continuous and, based on theory, experiments, and numerical simulations, Nu should scale [6] like $Nu - 1 \propto \epsilon$ where $\epsilon = x - 1$. This result comes from an expansion of the fluid equations near onset, is a laterally infinite theory, and is valid for small ϵ with higher-order corrections. Without rotation, these are the Newell-Whitehead-Segel equations which describe a continuum of modes in a narrow bandwidth around the critical wave number. The simple linear scaling with ϵ is contrasted with the complicated function of x suggested by CD. The proper scaling should be resolvable from experimental data but the first bifurcation in rotating convection at sufficiently large Ta is to a sidewall mode [3]. Here we are interested in just the bulk contribution. By considering data for low, intermediate, and large Ta, it is possible to subtract out the sidewall-state contribution to Nu and scale ϵ from the onset of bulk convection Ra_b.

The experimental data [3,5], numerical simulations of Nu for rotating convection [7], and the proposed CD scaling are compared in Fig. 1. The first two sets of data for $Nu\text{-}Nu(Ra_b)$ have a radius-to-height ratio $A = 5$, whereas the highest value of Ta is for a cell with $A \approx 1$. The numerical results [7] for Nu assumed a laterally infinite fluid layer with $Ta = 10^4$ and 10^5. Finally, the scaling proposed by CD is shown for both the laterally infinite case and for a best fit to the experimental data with $Ta = 3 \times 10^5$. The data and numerics are beautifully consistent with linear ϵ scaling over two decades in ϵ. Further, the downward curvature of Nu at larger ϵ is readily explained by a higher-order ϵ^2 term with a negative coefficient [6]. Lateral size plays no apparent role in the scaling as both the numerics (large A based on taking enough Fourier modes) and experiments ($A = 5$) agree quite well. Further, although the simulations are two dimensional, whereas the real patterns near onset are not, several other experiments [8] ($A = 2.5$, $Ta \approx 2 \times 10^7$; $A = 40$, $Ta \approx 4 \times 10^3$) demonstrate that linear ϵ scaling of Nu is preserved for three-dimensional flows and for large A. On the other hand, the CD function does not describe the data well even when the two parameters are varied to obtain a best fit. For example, the curvature of the data is of opposite sign to that of the fit for *any* value of b and for realistic values of x, i.e., $x > 1$. In summary, the CD scaling model cannot be made to match the data at small Ra, whereas the weakly nonlinear theory fits beautifully.

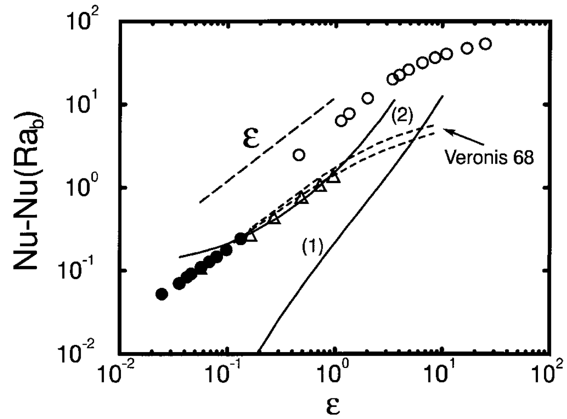

FIG. 1. Log-log plot of $Nu - Nu(Ra_*)$ vs ϵ. Data [5] for Ta: 2×10^4 (●), 3×10^5 (△), and 5×10^9 (○). Numerical calculations [7] (short-dashed curves) Ta: 10^4 (lower curve), 10^5 (higher curve). ϵ power-law scaling (long-dashed curve). CD scaling (solid curves): (1) $A = \infty$, $a = 3.9 \times 10^{-3}$, $b = 3$; (2) fit using $a = 5.2 \times 10^{-2}$, $b = 1.15$.

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