Testing Unruh Radiation with Ultraintense Lasers

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We point out that using the state-of-the-art (or soon-to-be) intense electromagnetic pulses, violent accelerations that may be suitable for testing quantum field theory in curved spacetime can be realized through the interaction of a high-intensity laser with an electron. In particular, we demonstrate that the Unruh radiation is detectable, in principle, beyond the conventional radiation (most notably the Larmor radiation) background noise, by taking advantage of its specific dependence on the laser power and distinct character in spectral-angular distributions.

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General relativity (GR) is by birth a classical theory. The celebrated discovery by Hawking [1] of the black hole radiation links the GR to quantum mechanics and thermodynamics in one stroke. While the ultimate theoretical understanding of the Hawking radiation, for example through the superstring theory [2], is still in progress, the fundamental importance of the Hawking radiation is hardly questionable. Subsequent to Hawking's discovery, Unruh [3] established that similar radiation can also occur for a "particle detector" under acceleration. Without resorting to detailed arguments, one can readily appreciate such a notion intuitively based on the equivalence principle. While the celestial observations of GR effects are clearly important, one wonders if by means of extremely violent acceleration in the laboratory setting these effects can be detected or tested by controlled experiments.

There have been proposals for laboratory detection of the Unruh effect [4]. For example, Yablonovich [5] proposed to detect the Unruh radiation using ionization fronts in solids. Darbinyan *et al.* [6] proposed to test it through the crystal channeling phenomena. Since the sought-after effects are typically extremely weak, the most severe problem would be the struggle against paramount background signals. Thus the challenge in general is to find a physical setting which can maximally enhance the signal above its competing backgrounds.

It is known that plasma wakefields excited by either a laser pulse [7] or an intense electron beam [8] can in principle provide an acceleration gradient as high as 100 GeV/cm, or $10^{23} g_{\oplus}$. Such acceleration relies on the collective perturbations of the plasma density excited by the driving pulse and restored by the immobile ions, and therefore is an effect arisen over a plasma period. There is in fact another aspect of laser-driven electron acceleration. Namely, when a laser is ultrarelativistic (i.e., $a_0 \equiv eE_0/mc\omega_0 \gg 1$), an electron under the direct influence of the laser can be instantly accelerated (and decelerated) in every laser cycle (which is typically a much higher frequency than that of the plasma), resulting in an intermittent acceleration that is much more violent than that provided by the plasma wakefields. For the Petawatt-class lasers currently under development [9], 10 TeV/cm, or $10^{25} g_{\oplus}$, will be possible for such intermittent acceleration in the near future.

The outstanding character of our system is that the intermittent laser acceleration is macroscopic and can be described by classical electrodynamics with well-defined trajectory and acceleration, and therefore the semiclassical theory, i.e., the quantum field theory in curved spacetime, where the Unruh effect is based upon, can be readily applied.

According to Unruh [3] and Davies [10], a uniformly accelerated particle finds itself embedded in a thermal heat bath with temperature

$$kT = \frac{\hbar a}{2\pi c}, \tag{1}$$

where a is the constant proper acceleration of the particle. In the standard treatment, an internal degree of freedom of the accelerated particle is invoked as a means to detect the Unruh effect. This can be, for example, a monopole moment (interacting with a scalar field) [11,12], or the spin of an electron (interacting with EM fields) [13]. Since the agency that we rely on for the violent acceleration is electromagnetic and acts only on charged particles, we consider an electron, the lightest charged particle, as our particle detector. As was shown by Bell and Leinaas [13], the manifestation of the Unruh effect through the equilibrium degree of spin polarization would require an unphysically long time in the case of a linear acceleration, yet for such an effect in a circular motion the spin-orbit coupling complicates the issue. In our approach, we do not invoke any internal degree of freedom. Rather, we rely on the quivering motion of the electron under the influence of the nontrivial vacuum fluctuations, and look for the emitted photons so induced as our signals.

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To be sure, the Unruh radiation is not a "new" radiation. Using the standard field theory (in this case quantum electrodynamics), one should in principle be able to arrive at the same result when properly taking particle radiation reaction into account. Treating the problem in the instantaneous proper frame and invoking the particle response to the thermal vacuum fluctuations, however, help to elucidate the phenomenon through a very intuitive picture in the spirit of the fluctuation-dissipation theorem [14] in thermodynamics.

We assume that in the leading order the accelerated electron is "classical," with well-defined acceleration, velocity, and position. Therefore we can introduce a Lorentz transformation so that the electron is described in its instantaneous proper frame. Also at this level the linearly accelerated electron will execute a classical Larmor radiation. As a response to the Larmor radiation, the electron reacts to the vacuum fluctuations with a quivering motion in its proper frame. This in turn triggers additional radiation. We assume that this quivering motion is nonrelativistic in the proper frame, and the interaction Hamiltonian can be written as

$$\mathcal{H}_I = -\frac{e}{mc}\vec{p}\cdot\vec{A} = -e\vec{x}\cdot\vec{E}\,. \tag{2}$$

The probability of the emission of a photon with energy $\omega = \mathcal{E}' - \mathcal{E}$ is

$$\begin{aligned} N(\omega) &= \frac{1}{\hbar^2}\int d\sigma \int d\tau\, |\langle 1_{\vec{k}}, \mathcal{E}'|\mathcal{H}_I|\mathcal{E}, 0\rangle|^2 \\ &= \frac{e^2}{\hbar^2}\sum_{i,j}^{3}\int d\sigma \int d\tau\, e^{-i\omega\tau}\langle x_i(\sigma)x_j(\sigma)\rangle \\ &\quad \times \langle E_i(\sigma - \tau/2)E_j(\sigma + \tau/2)\rangle\,, \end{aligned} \tag{3}$$

where σ and τ are the absolute and relative proper time, respectively. The τ dependence of the position operator has been extracted to the phase due to a unitary transformation. The last bracket is the autocorrelation function for the fluctuations of the electric field in the vacuum.

For the sake of simplicity, we treat the laser as a plane EM wave. Furthermore, as we would like to work with a quasilinear acceleration, we consider two identical, counterpropagating plane waves that provide a standing wave. Let the lasers be linearly polarized in x direction and propagation in the $\pm z$ direction, with amplitude $E = E_0\cos(\omega_0 t \pm k_0 z)$, where ω_0 and k_0 are the laser angular frequency and wave number, respectively. The Lorentz force equations for the accelerated electron can be written as

$$\begin{aligned} \frac{dp_x}{dt} &= -e(E_x - \beta_z B_y)\,, \\ \frac{dp_z}{dt} &= -e\beta_x B_y\,, \end{aligned} \tag{4}$$

where

$$\begin{aligned} E_x &= E_0[\cos(\omega_0 t - k_0 z) + \cos(\omega_0 t + k_0 z)]\,, \\ B_y &= E_0[\cos(\omega_0 t - k_0 z) - \cos(\omega_0 t + k_0 z)]\,, \end{aligned} \tag{5}$$

Note that at locations where $k_0 z = 0, \pm 2\pi, \ldots,$ $B_y = 0$ identically for all times and E_x takes the maximum value. We will invoke one of these nodal points for the detection of the Unruh signals. Specifically, at $z = 0$ we find

$$\gamma\beta_x = 2a_0\sin\omega_0 t\,, \qquad \gamma = \sqrt{1 + 4a_0^2\sin^2\omega_0 t}\,, \tag{6}$$

where $a_0 \equiv eE_0/mc\omega_0$ is the dimensionless laser strength parameter. The proper acceleration, which is related to that in the laboratory frame by $a = \gamma^3 a_{\mathrm{lab}}$, is thus

$$a = 2ca_0\omega_0\cos\omega_0 t\,. \tag{7}$$

To derive the autocorrelation function, we look for the transformation between laboratory and proper spacetimes. As $d\tau = dt/\gamma$, in the limit $a_0 \gg 1$ we find

$$\begin{aligned} \sin\omega_0 t &= \tanh(2a_0\omega_0\tau)/\sqrt{1 + 4a_0^2\,\mathrm{sech}^2(2a_0\omega_0\tau)}\,, \\ \sin k_0 x &= 2a_0\cos\omega_0 t/\sqrt{1 + 4a_0^2}\,. \end{aligned} \tag{8}$$

As we are dealing with a periodic motion, it is sufficient that we focus on the time interval $-\pi/2 \le \omega_0 t \le \pi/2$. This corresponds to $\pi/a_0 \le \omega_0\tau \le \pi/a_0$. Within the limit where $4a_0^2\,\mathrm{sech}^2(2a_0\omega_0\tau) \gg 1$, the above equation reduces to

$$\begin{aligned} \sin\omega_0 t &\approx \frac{1}{2a_0}\sinh(2a_0\omega_0\tau)\,, \\ \cos k_0 x &\approx \frac{1}{2a_0}\cosh(2a_0\omega_0\tau)\,. \end{aligned} \tag{9}$$

These can be readily recognized as conformal transformations of the Rindler transformation for constant acceleration [15], where $t = (c/a)\sinh(a\tau/c)$ and $x = (c^2/a)\cosh(a\tau/c)$. In fact, within our approximation it is consistent to further put $\sin\omega_0 t \approx \omega_0 t$, and $k_0 x \approx \pi/2 - (1/2a_0)\cosh(2a_0\omega_0\tau)$. As the autocorrelation function depends on t and x through $t(\sigma - \tau/2) - t(\sigma + \tau/2)$ and $x(\sigma - \tau/2) - x(\sigma + \tau/2)$, the additive constant phase in x does not contribute to the vacuum fluctuations. We have thus demonstrated that in the $a_0 \gg 1$ regime the laser driven acceleration is quasiconstant, and we recover the well-known expression for autocorrelation function [16] (with the constant proper acceleration replaced by $2ca_0\omega_0$):

$$\begin{aligned} \langle E_i(\sigma - \tau/2)E_j(\sigma + \tau/2)\rangle &= \delta_{ij}\frac{4\hbar}{\pi c^3}(2a_0\omega)^4 \\ &\quad \times \mathrm{csch}^4(a_0\omega_0\tau)\,. \end{aligned} \tag{10}$$

We emphasize that while this derivation is an approximation, it is valid for a good fraction of the laser half-cycle. For example, with $a_0 = 100$, it covers a time interval up to $\omega_0 t \sim \pm 0.4$. The range further expands for even larger a_0. More importantly, it can also be shown that beyond this time range the autocorrelation function diminishes rapidly due to the asymptotic saturation of the hyperbolic tangent function at large arguments [cf. Eq. (8)], i.e., the t and x differences between the proper times $\sigma - \tau/2$ and $\sigma + \tau/2$ are exponentially suppressed when $2a_0\omega_0\sigma \gg 1$. We can therefore safely extend the limits of τ integration in Eq. (3) to $\pm\infty$.

With a change of variable $s = a_0\omega_0\tau$ in Eq. (3), we find

$$\frac{dN(\omega)}{d\sigma} = \frac{1}{2\pi}\frac{e^2}{\hbar c^3}(2a_0\omega_0)^3\langle x^2\rangle \int_{-\infty}^{+\infty} ds e^{-is\omega/a_0\omega_0} \times \operatorname{csch}^4(s - i\epsilon), \tag{11}$$

where $\langle x^2\rangle = \sum_i^3 \langle x_i^2\rangle$. This integral has poles at $s = n\pi i$, and is periodic every $\Delta s = \pi i$. Thus it can be easily performed by returning the contour along the line $\operatorname{Im} s = \pi$, and we get

$$\frac{dN(\omega)}{d\sigma} = \frac{e^2}{3\hbar c^3}(2a_0\omega_0)^2\langle x^2\rangle\left[2\omega + \left(\frac{1}{2a_0\omega_0}\right)^2\omega^3\right] \times (e^{\pi\omega/a_0\omega_0} - 1)^{-1}. \tag{12}$$

The expectation value of x^2 fluctuates due to the random absorption of quanta from the vacuum fluctuations. From the uncertainty principle we have $\langle x_i^2\rangle\langle p_i^2\rangle \gtrsim \hbar^2$. By absorbing a quanta of frequency ω, the corresponding change of momentum is $\langle p_i^2\rangle = \langle p^2\rangle/3 = (2/3)m\hbar\omega$. We shall thus assume that

$$\langle x^2\rangle = \sum_i^3 \langle x_i^2\rangle \sim \frac{9}{2}\frac{\hbar}{m\omega}. \tag{13}$$

Note that this expression is invalid when the quivering motion becomes relativistic, i.e., $\langle p^2\rangle \gtrsim (mc)^2$. Beyond this limit a fully relativistic treatment is necessary. Taking the typical frequency of the vacuum fluctuation spectrum, $\omega \sim kT/\hbar$, the nonrelativistic approximation corresponds to the constraint that $kT \lesssim mc^2$. Accordingly, this means the fluctuations of the electron positron in our case should be larger than the Compton wavelength, i.e., $\langle x^2\rangle \gtrsim \lambda\!\!\!^{-}{}_c^2$, which is consistent with our semiclassical treatment.

To find the radiation power, we insert Eq. (13) into Eq. (12) and formally integrate over $\hbar d\omega$ with an infrared cutoff set by the laser frequency ω_0. (Note that for nonperiodic accelerations, the Unruh radiation power would be exponentially suppressed if the acceleration proper time is much shorter than a critical value, $\tau_c = 2\pi ca$. But this is not the case here.) We obtain

$$\frac{dI_U}{d\sigma} = \int_{\omega_0}^{\infty} \hbar d\omega \frac{dN}{d\sigma} \approx \frac{12}{\pi}\frac{r_e\hbar}{c}(a_0\omega_0)^3\log(a_0/\pi). \tag{14}$$

The above result applies to an accelerated electron located exactly at $z = 0$. At the vicinity of this point, e.g., $k_0 z = \epsilon \ll 1$, there is a nonvanishing magnetic field $|B_y| = \epsilon|E_x|$, which induces a $\beta_z \sim \mathcal{O}(\epsilon)$ in addition to the dominant β_x. But the proper acceleration is affected only to the order ϵ^2: $a \approx 2c\omega_0 a_0[1 - \mathcal{O}(\epsilon^2)]$. For electrons farther away where the decrease of acceleration becomes more significant, both the Unruh and the background Larmor radiations will decrease much more rapidly due to their strong dependences on acceleration. The experiment should therefore focus tightly on the signals from the origin and its immediate surroundings.

At the classical level, the same linear acceleration induces a Larmor radiation. In our theory, the Unruh radiation is induced by the reaction to the Larmor radiation and is a minute perturbation of it. In addition the photon $\vec{k}$ space that we are interested in detecting is along the direction of acceleration where the Larmor radiation is the weakest. Therefore the two radiations can be treated as independent processes without interference. The total Larmor radiation power is

$$\frac{dI_L}{dt} = \frac{2}{3}\frac{e^2}{m^2c^3}\left(\frac{dp_\mu}{d\tau}\frac{dp^\mu}{d\tau}\right) = \frac{8}{3} r_e mc a_0^2\omega_0^2\cos^2(\omega_0 t), \tag{15}$$

and the total energy radiated during each laser half-cycle is $\Delta I_L = (4\pi/3)r_e mc a_0^2\omega_0$. On the other hand, the Unruh radiation is significant over a reduce proper time period $\omega_0\Delta\sigma \gtrsim \mathcal{O}(1/a_0)$. Nevertheless, within this time the electron has become relativistic, with $\gamma \sim a_0$. As a result, the total energy radiated in the lab frame, i.e., $\Delta I_U \sim (dI_U/d\sigma)\gamma\Delta\sigma$, is $\Delta I_U \sim (12/\pi) \times (r_e\hbar/c)a_0^3\omega_0^2\log(a_0/\pi)$. Thus the relative yield is

$$\frac{\Delta I_U}{\Delta I_L} \sim \frac{9}{\pi^2}\frac{\hbar\omega_0}{mc^2} a_0\log(a_0/\pi). \tag{16}$$

Since $a_0 \propto 1/\omega_0$, the relative yield is not sensitive to the laser frequency.

Consider a Petawatt-class laser currently under development [9], where, let us assume, $\omega_0 \sim 2 \times 10^{15}\ \mathrm{sec}^{-1}$ and $a_0 \sim 100$. This gives $(\Delta I_U/\Delta I_L) \sim 3 \times 10^{-4}$. Alternatively, if one invokes a free-electron-laser-driven coherent x-ray source [17], it is conceivable to have $\hbar\omega_0 \sim 10$ keV and $a_0 \sim 10$. This would raise the signal-to-noise ratio to order unity. Even though this ratio is at best of order unity, the time structure of these radiations and their different characters in spectral-angular distributions and polarizations help to much relax the demand on acceleration for detectability.

In our treatment the thermal fluctuation is isotropic [cf. Eq. (10)] [18] in the electron's proper frame. The radiation induced is therefore also isotropic in the electron's proper frame. But since at each half cycle the electron rapidly becomes relativistic, with $\gamma \sim a_0$, the Unruh radiation is boosted along the direction of polarization (x axis) in the lab frame. Furthermore, as we have discussed above, the autocorrelation function, and therefore the Unruh signals, tend to diminish more rapidly than that from Larmor within the laser half cycle. This should induce a sharper time structure for the former.

Transforming the Unruh radiation power back to the lab frame with $\gamma \sim a_0$, the angular distribution in the small-angle expansion becomes

$$\frac{dI_U}{dtd\Omega} \simeq \frac{4}{\pi^2}\frac{r_e\hbar}{c}\frac{\omega_0^3 a_0^3}{[1 + a_0^2\theta^2]^3}. \qquad (17)$$

The Larmor radiation is polarized and its angular distribution in the small (θ, ϕ) polar-angle expansion is [19]

$$\frac{d^2I_L}{dtd\Omega} \simeq \frac{8r_e mca_0^2\omega_0^2}{[1 + a_0^2\theta^2]^3}\left[1 - \frac{4a_0^2\theta^2(1 - \phi^2)}{[1 + a_0^2\theta^2]^2}\right]. \qquad (18)$$

It is clear that the Larmor radiation power is minimum at $(\theta, \phi) = (1/a_0\pi, 0)$, where $d^2I_L/dtd\Omega = 0$. Consider a detector which covers an azimuthal angle $\Delta\phi = 10^{-3}$ around this "blind spot," and an opening polar angle, $\Delta\theta \ll 1/a_0$. Then the partial radiation power for the Unruh signal would dominate over that for the Larmor within this solid angle.

In our discussion, we did not specify the source for the electrons. One possibility is to create low energy photoelectrons near the surface of a solid material. There should also be other means, for example through laser trapping and cooling, in producing ultralow energy electrons. In these approaches the laser-electron interaction occurs in vacuum and there should be minimal additional backgrounds induced. If it is found more desirable to invoke a low temperature plasma, then the accelerated electrons will interact with the plasma ions and trigger the conventional bremsstrahlung. Even in this case we find that as long as the plasma density is low enough, the Unruh signal wins over that from bremsstrahlung. The cross section of bremsstrahlung for an unscreened hydrogen nucleus per unit photon energy is well known: $d\chi/d\hbar\omega \sim (16/3)\alpha r_e^2 ln(EE'/mc^2\omega)$. However, it is important to note that the bremsstrahlung yield depends quadratically, whereas the Unruh signals linearly on the plasma density. The break-even point between the signal and the noise is $n_p \lesssim 10^{18}$ cm^{-3}. Therefore as long as one chooses a plasma density below this value, the backgrounds from bremsstrahlung can be minimized.

We have investigated the Unruh effect associated with a sinusoidally time-varying linear accelerating field, such as that provided by a laser standing wave. We demonstrate that the Unruh radiation can in principle be detectable against the backgrounds from the conventional radiations using the frontier laser technology and the various experimental techniques. The violent, macroscopic acceleration available from ultrarelativistic lasers can also be a useful tool to test other salient features of general relativity in the laboratory setting.

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[1] S.W. Hawking, Nature (London) **248**, 30 (1974); Commun. Math. Phys. **43**, 199 (1975).
[2] A. Strominger and C. Vafa, Phys. Lett. B **379**, 99 (1996).
[3] W. Unruh, Phys. Rev. D **14**, 870 (1976).
[4] For a review, see, for example, H.C. Rosu, Int. J. Mod. Phys. D **3**, 545 (1994).
[5] E. Yablonovich, Phys. Rev. Lett. **62**, 1742 (1989).
[6] S.M. Darbinyan, K.A. Ispiryan, M.K. Ispiryan, and A.T. Margaryan, JETP Lett. **51**, 110 (1990).
[7] T. Tajima and J.M. Dawson, Phys. Rev. Lett. **43**, 267 (1979).
[8] P. Chen, J.M. Dawson, R. Huff, and T. Katsouleas, Phys. Rev. Lett. **54**, 693 (1985).
[9] M. Perry and G. Mourou, Science **264**, 917 (1994).
[10] P.C.W. Davies, J. Phys. A **8**, 609 (1975).
[11] B.S. DeWitt, in *General Relativity,* edited by S.W. Hawking and W. Israel (Cambridge University Press, Cambridge, England, 1979).
[12] N.D. Birrell and P.C.W. Davies, *Quantum Fields in Curved Space* (Cambridge University Press, Cambridge, England, 1982).
[13] J.S. Bell and J.M. Leinaas, Nucl. Phys. **B212**, 131 (1983).
[14] R. Kubo, J. Phys. Soc. Jpn. **12**, 570 (1957).
[15] W. Rindler, *Essential Relativity* (Van Nostrand, New York, 1969).
[16] T.H. Boyer, Phys. Rev. D **21**, 2137 (1980).
[17] M. Cornacchia, in *Proceedings of the 19th International Free Electron Laser Conference, Beijing, 1997* (Elsevier, Amsterdam, 1998).
[18] It has been argued that the vacuum fluctuations in this case are not entirely isotropic [see, for example, K. Hinton, P.C.W. Davies, and J. Pfautsch, Phys. Lett. **120B**, 88 (1983)], but we shall ignore it in our discussion.
[19] J.D. Jackson, *Classical Electrodynamics* (John Wiley & Sons, New York, 1975), 2nd ed.