

## Elliptic Flow in Heavy-Ion Collisions near the Balance Energy

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The proton elliptic flow in collisions of  $^{48}\text{Ca}$  on  $^{48}\text{Ca}$  at energies from 30 to 100 MeV/nucleon is studied in an isospin-dependent transport model. With increasing incident energy, the elliptic flow shows a transition from positive to negative flow. Its magnitude depends on both the nuclear equation of state (EOS) and the nucleon-nucleon scattering cross section. Different elliptic flows are obtained for a stiff EOS with free nucleon-nucleon cross sections and a soft EOS with reduced nucleon-nucleon cross sections, although both lead to vanishing in-plane transverse flow at the same balance energy.

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Heavy ion collisions provide the possibility to study the properties of nuclear matter in conditions vastly different from that in normal nuclei, such as high density and excitation, as well as the large difference in the proton and neutron numbers [1–5]. Such knowledge is not only of interest in itself but also useful in understanding astrophysical phenomena such as the properties of the core of compact stars, the evolution of the early universe, and the formation of elements in stellar nucleosynthesis. One observable that has been extensively used for extracting such information from heavy ion collisions is the collective flow of various particles [6–21] (for a recent review, see Refs. [22–25]). For example, the proton flow in heavy ion collisions at 200 MeV/nucleon to 1 GeV/nucleon has been found to be consistent with a soft nuclear equation of state [7,8]. From the kaon flow in heavy ion collisions at 1 to 2 GeV/nucleon, the existence of a weak repulsive kaon potential has been obtained [10]. In heavy ion collisions at energies higher than 2 GeV/nucleon, recent studies of proton flow seem to indicate that there is a softening of nuclear equation of state as the nuclear density and excitation increase [18]. There are also suggestions that particle collective flows at ultrarelativistic heavy ion collisions are sensitive to the initial parton dynamics [19] and subsequent phase transitions [17,20,21].

In general, collective flow in heavy ion collisions is affected by both the nuclear mean-field potential and nucleon-nucleon cross sections. In heavy ion collisions at intermediate energies of a few tens MeV/nucleon, the collision dynamics is dominated by the attractive nuclear mean-field potential as nucleon-nucleon scatterings are largely blocked due to the Pauli principle. As a result, the nucleon transverse flow in the reaction plane is negative, i.e., nucleons moving in the projectile direction are deflected to negative angles. With increasing incident energies, the repulsive nucleon-nucleon scattering becomes important and reduces the negative flow caused by the attractive nuclear mean-field potential. At certain incident energy, called the balance energy, in-plane transverse flow vanishes as a result of the cancellation between these two competing effects [26]. The disappearance of trans-

verse collective flow has been experimentally observed in heavy ion collisions [27–29]. The measured balance energy depends strongly on the mass and isospin of colliding nuclei as well as on the impact parameter of collisions [22,27–29]. Studies based on transport models have shown that the same balance energy can be obtained with different nuclear equations of state and in-medium nucleon-nucleon scattering cross sections [30,31]. To extract their information from measured balance energies thus requires the measurement of other observables. One of the present authors [32] has recently shown that different equation of state (EOS) and cross sections which give the same balance energy show different differential transverse flows, i.e., their transverse flows have different dependence on the total transverse momentum. In the present paper, we shall study instead the proton elliptic flow, which measures the anisotropy in their transverse momentum distribution. In particular, using the isospin-dependent Boltzmann-Uehling-Uhlenbeck (IBUU) model [31], we shall consider collisions of  $^{48}\text{Ca} + ^{48}\text{Ca}$  at energies from 30 to 100 MeV/nucleon. As shown below, different EOS and cross sections that give the same balance energy lead to significantly different elliptic flows.

Taking the beam direction along the  $z$  axis and the reaction plane on the  $x$ - $y$  plane, the elliptic flow is then determined from the average difference between the square of the  $x$  and  $y$  components of particle transverse momentum, i.e.,

$$v_2 = \left\langle \frac{p_x^2 - p_y^2}{p_x^2 + p_y^2} \right\rangle. \quad (1)$$

It corresponds to the second Fourier coefficient in the transverse momentum distribution [24,33] and describes the eccentricity of an ellipselike distribution, i.e.,  $v_2 > 0$  indicates in-plane enhancement,  $v_2 < 0$  characterizes the squeeze-out perpendicular to the reaction plane, and  $v_2 = 0$  shows an isotropic distribution in the transverse plane.

The IBUU transport model used in the present study treats explicitly protons and neutrons. It also includes an asymmetry term in the nuclear mean-field potential and different scattering cross sections for protons and neutrons.

The nuclear mean-field potential is parametrized as

$$U(\rho, \tau_z) = U_0(\rho) + U_{\text{asy}}(\rho, \tau_z), \quad (2)$$

$$U_0(\rho) = a \left( \frac{\rho}{\rho_0} \right) + b \left( \frac{\rho}{\rho_0} \right)^\sigma, \quad (3)$$

$$U_{\text{asy}}(\rho, \tau_z) = C \frac{\rho_p - \rho_n}{\rho_0} \tau_z. \quad (4)$$

In the above,  $\rho_0$  is the normal nuclear matter density;  $\rho$ ,  $\rho_n$ , and  $\rho_p$  are the nucleon, neutron, and proton densities, respectively; and  $\tau_z$  equals 1 for proton and  $-1$  for neutron. For the strength of the asymmetry potential, we take  $C = 32$  MeV. Two different EOS are used in our studies: a stiff EOS with compressibility of 380 MeV ( $a = -124$  MeV,  $b = 70.5$  MeV,  $\sigma = 2$ ) and a soft one with compressibility of 200 MeV ( $a = -356$  MeV,  $b = 303$  MeV,  $\sigma = 7/6$ ). We also include the Coulomb potential for protons. For nucleon-nucleon scatterings, both elastic and inelastic channels are included by using the experimentally measured cross sections with explicit isospin dependence. Details of the IBUU model can be found in Ref. [31].

We first study the flow parameter at midrapidity, which is defined by

$$F = \frac{d\langle p_x \rangle}{dy} \Big|_{y=0}. \quad (5)$$

In Fig. 1, we show the incident energy dependence of the proton flow parameter in  $^{48}\text{Ca} + ^{48}\text{Ca}$  reactions at an impact parameter of 2 fm. Open circles are obtained from the IBUU model using the soft EOS. In this case, the proton in-plane transverse flow below about 45 MeV/nucleon is negative as a result of the dominant effect of attractive nuclear mean-field potential. Above this incident energy, nucleon-nucleon scatterings become more important, and their repulsive effects lead to a positive flow parameter. For the stiff EOS, the effect of nucleon-nucleon scatterings is smaller due to a short total reaction time as a result of a less attractive mean-field potential than that for the soft EOS. This thus leads to a smaller flow parameter as shown by the solid circles in Fig. 1. An exception to this general behavior occurs, however, at very low incident energies below about 40 MeV/nucleon, where the flow parameter increases instead with decreasing incident energy. This is due to the fact that scattering effects at low energies are not strong enough to reverse the effect due to the attractive mean-field potential. We have also shown in Fig. 1 by solid triangles the flow parameter obtained for the soft EOS but with the nucleon-nucleon scattering cross section reduced by 12%. Compared with the case of the soft EOS and free nucleon-nucleon cross section  $\sigma_{NN}$ , the flow parameter is reduced as expected. We note that the same balance energy, about 65.5 MeV/nucleon, is obtained for both the stiff EOS with  $\sigma_{NN}$  and the soft EOS with  $0.88\sigma_{NN}$ .

The excitation function of the proton elliptic flow for the same reaction is shown in Fig. 2. For both the soft

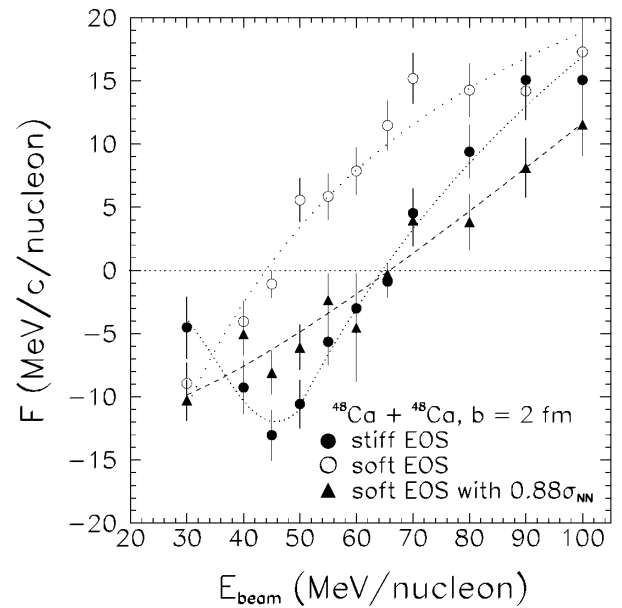


FIG. 1. The excitation function of the proton flow parameter in  $^{48}\text{Ca} + ^{48}\text{Ca}$  collisions at an impact parameter of 2 fm. Open and solid circles correspond to soft and stiff EOS, respectively. Solid triangles are from the soft EOS and a reduced nucleon-nucleon cross section  $0.88\sigma_{NN}$ .

EOS (open circles) and the stiff EOS (solid circles) the proton elliptic flow changes from positive flow at low energies to negative flow at high energies, i.e., a transition from the dominance of in-plane transverse flow to that of out-of-plane squeeze-out as the beam energy increases. However, the energy at which this transition occurs differs for the two EOS; it is smaller for the stiff EOS than for the soft EOS. To understand this difference, we also show in Fig. 2 the proton elliptic flow in the absence of mean-field potential (open squares), which is negative

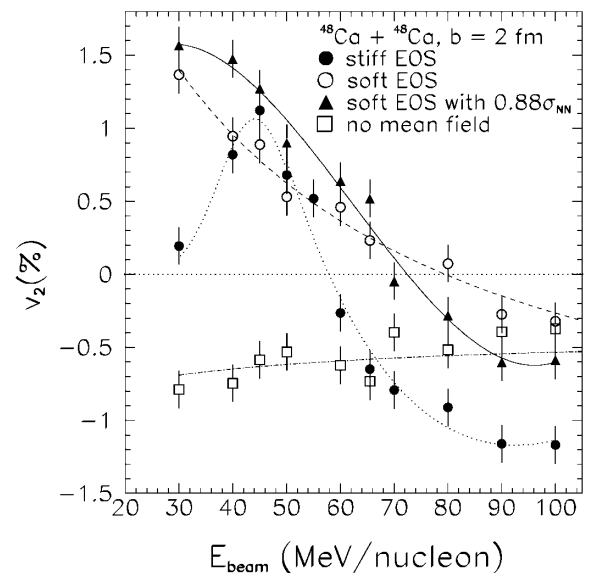


FIG. 2. The excitation function of the proton elliptic flow in  $^{48}\text{Ca} + ^{48}\text{Ca}$  collisions at an impact parameter of 2 fm.

at all energies, i.e., out-of-plane squeeze-out dominates over in-plane transverse flow. Since the soft EOS gives a larger in-plane transverse flow than that due to the stiff EOS, it leads to a higher energy at which the elliptic flow changes sign. The abnormal behavior at energies below 45 MeV/nucleon, where the elliptic flow for the stiff EOS decreases with decreasing energy, reflects its behavior in the flow parameter as shown in Fig. 1.

Also shown in Fig. 2 is the elliptic flow for the soft EOS with  $0.88\sigma_{NN}$  (solid triangles), which gives the same balance energy as the stiff EOS with  $\sigma_{NN}$ . As seen, the two give very different elliptic flows. In particular, at the balance energy (65.5 MeV) it is negative for the stiff EOS with  $\sigma_{NN}$  but is positive for the soft EOS with  $0.88\sigma_{NN}$ . We also note that the elliptic flow is more sensitive to the nuclear EOS than to the in-medium nucleon-nucleon cross section. This is in contrast to the in-plane flow which depends strongly on both these quantities as shown in Fig. 1.

In the present study, we have chosen neutron-rich nuclei. According to Ref. [31], effects due to isospin asymmetry on the proton collective flow are not very strong, and we thus expect that qualitatively similar results would be obtained in collisions between normal nuclei such as  $^{40}\text{Ca} + ^{40}\text{Ca}$ .

In summary, the IBUU model has been used to study the proton elliptic flow in collisions of  $^{48}\text{Ca} + ^{48}\text{Ca}$  at an impact parameter of 2 fm for beam energies from 30 to 100 MeV/nucleon. We find that it shows a transition from positive to negative flow as the incident energy increases. A strong dependence on both the nuclear EOS and the nucleon-nucleon cross section is seen in proton elliptic flow. Although both the stiff EOS with  $\sigma_{NN}$  and the soft EOS with  $0.88\sigma_{NN}$  have the same balance energy, they are found to give very different elliptic flows. The study of both in-plane and elliptic flows at intermediate energies thus allows one to extract simultaneously the information on the nuclear equation of state and the nucleon-nucleon scattering cross section in medium.

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