Observable Isocurvature Fluctuations from the Affleck-Dine Condensate

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In D-term inflation models, Affleck-Dine baryogenesis produces baryonic isocurvature fluctuations. In most cases the Affleck-Dine condensate is unstable with respect to collapse to B-balls, which can transform the baryon number perturbations into perturbations in the number of dark matter neutralinos. The requirement that the deviation of the adiabatic perturbations from scale invariance is not too large imposes a lower bound on the magnitude of the isocurvature fluctuations. In general this is larger than about 10^{-4} times the adiabatic perturbation, and, for the particular case of late decaying B-balls, larger than about 10^{-2} times the adiabatic perturbation, which should be observable by MAP and PLANCK.

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The quantum fluctuations of the inflaton field give rise to fluctuations of the energy density which are adiabatic [1,2]. However, in the minimal supersymmetric standard model (MSSM) or its extensions, the inflaton is not the only fluctuating field. It is well known that the MSSM scalar field potential has many flat directions [3], along which a nonzero expectation value can form during inflation, leading to a condensate after inflation, the so-called Affleck-Dine (AD) condensate [4]. The AD field is a complex field and, in the currently favored D-term inflation models [2,5] on which we focus in this Letter, is effectively massless during inflation. [This is true in the absence of additional fields which are charged under the MSSM gauge group and the Fayet-Illiopoulos U(1). The situation may be more complicated in extensions of the minimal D-term inflation model, such as may arise from string theory [6].] Therefore both its modulus and phase are subject to fluctuations. In D-term inflation models the phase of the AD field receives no order H corrections after inflation and so its fluctuations are unsuppressed [7]. Because the subsequent evolution of the phase of the AD condensate generates the baryon asymmetry [4], the fluctuations of the phase correspond to fluctuations in the local baryon number density, or isocurvature fluctuations, while the fluctuations of the modulus give rise to adiabatic density fluctuations. We will show that the adiabatic fluctuations may in fact dominate over the inflaton fluctuations, with potentially adverse consequences for the scale invariance of the perturbation spectrum, which imposes an upper bound on the amplitude of the AD field. As a consequence, there is a *lower* bound on the isocurvature fluctuation amplitude. The magnitude of this lower bound will depend on the nature of the AD field. D-term inflation models require that $d > 4$, where d is the dimension of the nonrenormalizable superpotential terms stabilizing the potential, in order to avoid thermalizing the AD field too early [7], while R-parity conservation, required to eliminate dangerous renormalizable B and L violating terms from the MSSM, rules out odd values of d. Therefore we will focus on the $d = 6$ direction, in particular the $u^c d^c d^c$ direction, in the following. We will show that the isocurvature fluctuations may well be observable in forthcoming cosmic microwave background (CMB) satellite experiments.

An important point is that the AD condensate is not stable but typically breaks up into nontopological solitons [8,9] which carry baryon (and/or lepton) number [10,11] and are therefore called B-balls (L-balls). This is a generic feature which is not realized only if the fluctuations take the AD field into certain leptonic $d = 4$ ("$H_u L$") directions. The formation of the B-balls takes place with an efficiency f_B (the fraction of the baryon number trapped in the B-balls), likely to be in the range 0.1 to 1 [12]. Hence the AD isocurvature fluctuations are inherited by the B-balls. The properties of the B-balls depend on supersymmetry (SUSY) breaking and on the flat direction along which the AD condensate forms. We will consider SUSY breaking mediated to the observable sector by gravity. In this case the B-balls are unstable but long-lived, decaying well after the electroweak phase transition has taken place [9], with a natural order of magnitude for decay temperature $T_d \lesssim 1\,\mathrm{GeV}$. This assumes a reheating temperature after inflation, T_R, of the order of 1 GeV. Such a low value of T_R is necessary in D-term inflation models because the natural magnitude of the phase of the AD field, δ_{CP}, is of the order of 1 in D-term inflation and AD baryogenesis along the $d = 6$ direction implies that the baryon to entropy ratio is $\eta_B \sim \delta_{\mathrm{CP}} 0.03(T_R/10^9\ \mathrm{GeV})$ [13], requiring $T_R \approx 1\,\mathrm{GeV}$ in order to account for the observed baryon asymmetry. (It is perhaps significant that a low reheating temperature can naturally be achieved in D-term inflation models, as these have discrete symmetries in order to ensure the flatness of the inflaton potential which can simultaneuously lead

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to a suppression of the reheating temperature [13].) Because the B-ball is essentially a squark condensate, in R-parity conserving models its decay produces both baryons and neutralinos (χ), which we assume to be the lightest supersymmetric particles (LSPs), with $n_\chi \simeq 3n_B$ [12,14]. This is particularly interesting, as the simultaneous production of baryons and neutralinos may then explain the remarkable similarity of the baryon and dark matter neutralino number densities [12,14]. With B-ball decay temperatures $T_d \sim \mathcal{O}(1)$ GeV, the neutralinos no longer thermalize completely and, so long as T_d is low enough that they do not annihilate after B-ball decay [14], they retain the form of the original AD isocurvature fluctuation. Therefore in this scenario the cold dark matter particles can have both isocurvature and adiabatic density fluctuations, resulting in an enhancement of the isocurvature contribution relative to the purely baryonic case.

Isocurvature perturbations have been studied previously [15], in particular in the context of axion models [16,17]. The isocurvature perturbations give rise to extra power at large angular scales but are damped at small angular scales. The amplitude of the rms mass fluctuations in an $8h^{-1}$ Mpc^{-1} sphere, denoted as σ_8, is about an order of magnitude lower than in the adiabatic case. Hence COBE normalization alone is sufficient to set a tight limit on the relative strength of the isocurvature amplitude. Small isocurvature fluctuations are, however, beneficial, in that they improve the fit to the power spectrum in $\Omega_0 = 1$ cold dark matter (CDM) models with a cosmological constant [16] (or $\Omega_0 = 1$, $\Lambda = 0$ CDM models with some hot dark matter [17]). For instance, in the context of axion models it has been found [16] that an $\Omega_0 = 1$ mixed fluctuation model with a relative isocurvature perturbation amplitude of 5%, $\Omega_a = 0.4$ and $\Omega_\Lambda = 0.6$ would give a very good fit to the data. However, axionic isocurvature fluctuations seem to require a large axion decay constant, which is already excluded unless there is considerable late entropy production [16]. The Affleck-Dine case we consider here is more economical, in the sense that it requires only the particles of the MSSM.

In D-term inflation models, the AD field $\Phi = \phi e^{i\theta}/\sqrt{2} \equiv (\phi_1 + i\phi_2)/\sqrt{2}$ remains effectively massless during inflation. Therefore the real fields ϕ_i are subject to quantum fluctuations with

$$\delta\phi_i(\mathbf{x}) = \sqrt{V}\int \frac{d^3k}{(2\pi)^3}\, e^{-i\mathbf{k}\cdot\mathbf{x}}\delta_{\mathbf{k}}\,, \tag{1}$$

where V is a normalizing volume and where the power spectrum is the same as for the inflaton field,

$$\frac{k^3|\delta_{\mathbf{k}}|^2}{2\pi^2} = \left(\frac{H_I}{2\pi}\right)^2, \tag{2}$$

where H_I is the value of the Hubble parameter during inflation. Thus, for given background values $\overline{\theta}$ and $\overline{\phi}$ (with $\overline{\theta}$ naturally of the order of 1) one finds

$$\left(\frac{\delta\theta}{\tan(\overline{\theta})}\right)_k = \frac{H_I}{\tan(\overline{\theta})\overline{\phi}} = \frac{H_I k^{-3/2}}{\sqrt{2}\,\tan(\overline{\theta})\overline{\phi}_I}\,, \tag{3}$$

where ϕ_I is the value of ϕ when the perturbation leaves the horizon. After inflation, during the inflaton oscillation dominated period, the mass squared of the magnitude of the AD field will receive an order H^2 correction, which must be negative in order to have a nonzero ϕ and so AD baryogenesis [3], while its phase receives no order H corrections. Therefore, the magnitude of the AD field Φ remains at the nonzero minimum of its potential until $H \simeq m_S$, where $m_S \sim 100$ GeV is the SUSY breaking scalar mass, whence it begins to oscillate and the baryon asymmetry $n_B \propto \sin(\theta)$ forms. Since $\overline{\theta}$ and $\delta\theta$ remain constant until $H \simeq m_S$, we have

$$\left(\frac{\delta n_B}{n_B}\right)_k = \left[\frac{\delta\theta}{\tan(\overline{\theta})}\right]_k, \tag{4}$$

with $\delta\theta/\tan(\overline{\theta})$ given by Eq. (3).

We first consider the case where the adiabatic perturbation is mostly due to the inflaton. The adiabatic perturbation is determined by the invariant $\zeta = \delta\rho/(\rho + p)$ with $\delta\rho = V'\delta\phi$. During inflation, when all the fields are slow rolling, one finds [17]

$$\zeta_{\mathrm{adiab}} = \frac{3}{4}\delta_\gamma^{(a)} = \frac{9}{\sqrt{2}}\frac{H_I^3}{V'}k^{-3/2}, \tag{5}$$

where $\delta_\gamma \equiv \delta\rho_\gamma/\rho_\gamma$.

For superhorizon size isocurvature fluctuations $\delta\rho/\rho = 0$, so that $m_\chi\delta n_\chi + m_B\delta n_B + 4(\rho_\gamma + \rho_\nu)\delta T/T = 0$ (here ρ_γ and $\rho_\nu \simeq 0.68\rho_\gamma$ are, respectively, the photon and the neutrino densities, and we assume for simplicity that there are no massive neutrinos). We then find that in the presence of both adiabatic [$\delta(n_x/s) = 0$] and isocurvature [$\delta(n_x/s) \neq 0$] fluctuations

$$\frac{\delta T}{T} = -\frac{\rho_\chi\delta_\chi^{(i)} + \rho_B\delta_B^{(i)}}{3(\rho_\chi + \rho_B) + 4(\rho_\gamma + \rho_\nu)}\,, \tag{6}$$

where $\delta_x = \delta n_x/n_x$ for nonrelativistic particles x.

In the case where the neutralinos come from B-ball decay, the isocurvature fluctuations of the LSPs are related to the baryonic isocurvature fluctuations by $\delta n_\chi^{(i)} = 3f_B\delta n_B^{(i)}$, with $\delta n_B^{(i)}$ given by Eq. (4). In the linear perturbation theory adiabatic and isocurvature fluctuations evolve independently so that the total perturbation is just the sum of the two. In general, the total LSP number density is the sum of the thermal relic density $n_\chi^{(\mathrm{th})}$ and the density $n_\chi^{(B)} = 3f_Bn_B$ originating from the B-ball decay. $f_B = 0$ for the case of conventional AD baryogenesis with no late-decaying B-balls. Using Eq. (6), the isocurvature fluctuation imposed on the CMB photons is then found to be

$$\begin{aligned}\delta_\gamma^{(i)} = 4\frac{\delta T}{T} &= -\frac{4(1 + \frac{m_B}{3f_Bm_\chi})\rho_\chi^{(B)}\delta_B^{(i)}}{3(\rho_\chi + \rho_B) + 4(\rho_\gamma + \rho_\nu)} \\ &\simeq -\frac{4}{3}\left(1 + \frac{m_B}{3f_Bm_\chi}\right)\left(\frac{\Omega_\chi - \Omega_\chi^{(\mathrm{th})}}{\Omega_m}\right)\delta_B^{(i)} \equiv -\frac{4}{3}\,\omega\,\delta_B^{(i)},\end{aligned} \tag{7}$$

where $\rho_\chi^{(B)}$ is the LSP mass density from the B-ball, $\Omega_m(\Omega_\chi)$ is total matter (LSP) density (in units of the critical density), and $\delta_B^{(i)}$ is given by Eq. (4). To obtain the last line in Eq. (7), we have used the fact that ρ_γ is negligible. In the notation of Ref. [17], and using Eq. (5), we can write

$$\beta \equiv \left(\frac{\delta_\gamma^{(i)}}{\delta_\gamma^{(a)}}\right)^2 = \frac{1}{9}\,\omega^2\left(\frac{M^2V'(S)}{V(S)\tan(\overline{\theta})\overline{\phi}}\right)^2, \tag{8}$$

where S is the inflaton field with a potential $V(S)$ and $M \equiv M_{Pl}/\sqrt{8\pi}$.

In the simplest D-term inflation model, the inflaton is coupled to the matter fields ψ_- and ψ_+ carrying opposite Fayet-Iliopoulos charges through a superpotential term $W = \kappa S\psi_-\psi_+$ [5,7]. At one loop level the inflaton potential reads

$$V(S) = V_0 + \frac{g^4\xi^4}{32\pi^2}\ln\left(\frac{\kappa^2S^2}{Q^2}\right); \qquad V_0 = \frac{g^2\xi^4}{2}, \tag{9}$$

where ξ is the Fayet-Iliopoulos term and g the gauge coupling associated with it. COBE normalization fixes $\xi = 6.6 \times 10^{15}$ GeV [18]. In addition, we must consider the contribution of the AD field to the adiabatic perturbation. During inflation, the potential of the $d = 6$ AD field is simply given by

$$V(\phi) = \frac{\lambda^2}{32M^6}\,\phi^{10}. \tag{10}$$

With $\rho = V(S) + V(\phi)$ and $\rho + p = \dot{S}^2 + \dot{\phi}^2$ one finds, taking both S and ϕ to be slow rolling fields with $\dot{S} = -V'(S)/(3H_I)$ and $\dot{\phi} = -V'(\phi)/(3H_I)$, that the invariant ζ is now

$$\zeta_{\rm adiab} \propto \frac{V'(\phi) + V'(S)}{V'(\phi)^2 + V'(S)^2}\,\delta\phi\,, \tag{11}$$

where we have used the fact that both fields are effectively massless during inflation [$V''(\phi) \ll H$], so that $\delta S = \delta\phi$. Thus the field which dominates the spectral index of the perturbation will be that with the largest value of V' and V''. The index of the power spectrum is given by $n = 1 + 2\eta - 6\epsilon$, where ϵ and η are defined as

$$\epsilon = \frac{1}{2}M^2\left(\frac{V'}{V}\right)^2, \quad \eta = M^2\,\frac{V''}{V}\,. \tag{12}$$

The present lower bounds imply $|\Delta n| \lesssim 0.2$. (This bound will be much improved by future satellite missions.) In the case where the derivatives with respect to the inflaton dominate (for which the potential is dominated by V_0 for all $\xi < M$), $|\Delta n| = 1/N \approx 0.02$ for $N \sim 50$. Once the derivatives with respect to the AD field dominate, the spectral index increases rapidly with ϕ; from η (ϵ), $|\Delta n|$ is proportional to ϕ^8 (ϕ^{18}). The condition for the AD field to dominate the spectral index is that $\phi > \max(\phi_{c_1}, \phi_{c_2})$, where

$$\phi_{c_1} \simeq 0.64(g^3\lambda^{-2}\xi^4M^5)^{1/9} \tag{13}$$

and

$$\phi_{c_2} \simeq 0.48\left(\frac{g}{\lambda}\right)^{1/4}(M\xi)^{1/2}, \tag{14}$$

and where we have used the fact that during the slow rollover $S_N^2 \simeq g^2M^2N/(4\pi^2)$. The inflaton derivatives will dominate once $\phi < \min(\phi_{c_1}, \phi_{c_2})$. (In practice ϕ_{c_1} and ϕ_{c_2} only differ by a factor of less than 2.) As a result of the rapid increase of the spectral index once the AD derivatives dominate, the condition that the spectral index is acceptably close to scale invariance essentially reduces to the condition that it is dominated by the inflaton. The lower bounds on β corresponding to ϕ_{c_1} and ϕ_{c_2} are then

$$\beta > \beta_{c_1} \simeq 6.5 \times 10^{-3}g^{4/3}\lambda^{4/9}\omega^2\tan(\overline{\theta})^{-2} \tag{15}$$

and

$$\beta > \beta_{c_2} = 2.5 \times 10^{-2}g^{3/2}\lambda^{1/2}\omega^2\tan(\overline{\theta})^{-2}. \tag{16}$$

(For most values of the couplings the latter leads to a slightly more stringent lower bound.)

There are two limiting cases: $f_B \gg m_B/3f_Bm_\chi$, for which $\omega = 1$, and $f_B \ll m_B/3f_Bm_\chi$, for which $\omega = \Omega_B/\Omega_m$. The latter corresponds to the case where the B-balls form very inefficiently or where the neutralino contribution to the isocurvature perturbation is erased by annihilations [12,14]. The case of conventional AD baryogenesis corresponds to the limit $f_B = 0$, in which case only the baryonic isocurvature perturbation remains.

The actual lower limit on β depends on the unknown couplings g and λ, as well as on $\overline{\theta}$. To obtain an estimate for β_{c_2}, let us adopt the following values: $g \simeq g_{\rm GUT} \simeq 0.7$, $\lambda \simeq 1/5! = 0.008$ (corresponding to a nonrenormalizible interaction with physical strength set by M [7,13]) and $\tan(\overline{\theta})^2 \simeq 1$ (corresponding to $\overline{\theta} \approx \pi/4$). For the case with late-decaying B-balls (assuming f_B is not too small) $\omega \simeq 1$ and we find that $\beta > \beta_{c_2} \simeq 1.3 \times 10^{-3}$, with the conservative lower bound perhaps an order of magnitude smaller, $\beta \sim 10^{-4}$. For the case of conventional AD baryogenesis with purely baryonic isocurvature fluctuations, the value of ω depends on Ω_B and Ω_m. Nucleosynthesis combined with the current best estimate of the expansion rate ($0.6 \lesssim h \lesssim 0.87$ [19]) implies that $0.006 \lesssim \Omega_B \lesssim 0.036$. Thus with $\Omega_m = 1(0.4)$ we obtain that ω for the purely baryonic case is 30–150 (10–60) times smaller than in the late decaying B-ball case. Therefore in the baryonic case the corresponding value of β is 2 to 4 orders of magnitude smaller. Although there is a significant enhancement of the isocurvature perturbation in the case where the dark matter neutralinos come from B-ball decay, it should be emphasized that there is no physical reason to expect ϕ to be close to its upper bound, so β (which is proportional to ϕ^{-2}) may be expected to be much larger than these lower bounds. If ϕ is 1 to 2 orders of magnitude smaller than its upper bound, then $\beta \gtrsim 10^{-4}$ can occur in the purely baryonic case also. Thus even without the neutralino enhancement, the purely baryonic isocuvature fluctuation, corresponding to the case of conventional Affleck-Dine baryogenesis, can still be important.

In Fig. 1 we display the difference between the purely adiabatic power spectrum and the spectra with $\beta \neq 0$. We also plot the expected error for the Planck Surveyor Mission, following the estimates in Ref. [20]. (A similar error is expected for MAP for $l \lesssim 500$). The standard error reads $(\Delta C_l)^2 = 2(C_l + \delta)^2/[(2l + 1)f_{\rm sky}]$, where $f_{\rm sky}$ is the fraction of the sky sampled (we take $f_{\rm sky} = 0.65$) and δ is from the beam, the angular resolution, and the sensitivity, as discussed in [20]; δ becomes non-negligible only for $l \gtrsim 1000$ for PLANCK and $l \gtrsim 500$ for MAP. One should bear in mind that, in principle, each multipole provides an independent measurement of the spectrum. As can be seen, detecting isocurvature fluctuations at the level of $\beta \sim 10^{-4}$ should be quite realistic by averaging over a sufficient number of multipole measurements. However, setting an actual lower limit on β will require a much more careful analysis. Nevertheless, on the basis of Fig. 1, it seems likely that the forthcoming CMB experiments will definitely be able to see isocurvature perturbations in the case where the baryons and neutralinos come directly from the decay of unstable B-balls in the context of D-term inflation models, hence offering a test not only of the inflationary Universe but also of the B-ball variant of AD baryogenesis.

In conclusion, AD baryogenesis in the context of D-term inflation generally implies the existence of isocurvature density fluctuations. In the case of conventional AD baryogenesis, the observability of the isocurvature fluctuations depends on the how close the AD field is to the upper bound from adiabatic fluctuations during inflation. If it is 1 to 2 orders of magnitude less than the upper bound, then the isocurvature fluctuations should be observable. In the case where B-balls, which are usually expected to form in AD baryogenesis, decay late enough to directly produce the observed dark matter in the form of neutralinos, the isocurvature flucteuctions are enhanced and should be observable by MAP and PLANCK. Thus the observation of isocurvature fluctuations would generally provide support for the idea of D-term inflation combined with Affleck-Dine baryogenesis. In particular, for the case where the neutralino dark matter comes directly from B-ball decays, so allowing for an understanding of the remarkable similarity of the baryon and dark matter number densities [12,14], the observation of isocurvature perturbations together with a nonthermal dark matter neutralino density (testable by observation of the sparticle spectrum [14]) would strongly support the D-term inflation/late decaying B-ball scenario.

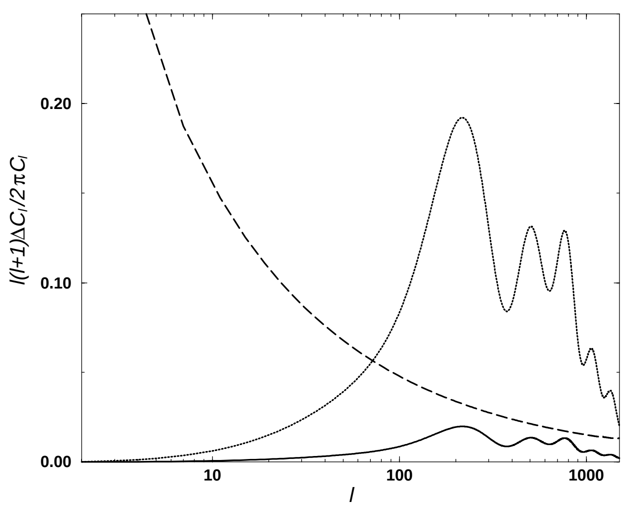

FIG. 1. The relative difference $\Delta C_l/C_l$ between the purely adiabatic and a mixture of adiabatic and isocurvature angular power spectra with $\beta = 0.001$ (dotted line) and $\beta = 0.0001$ (solid line) for a purely CDM $\Omega = 1$ model (with $\Omega_B = 0.05$, $h = 0.5$ and the spectral index $n = 1$). Shown is also the projected PLANCK error level, averaged over ten multipoles (dashed line).

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