Geometrically Constrained Magnetic Wall

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(Received 26 April 1999)

The structure and properties of a geometrically constrained magnetic wall in a constriction separating two wider regions are studied theoretically. They are shown to differ considerably from those of an unconstrained wall, so that the geometrically constrained magnetic wall truly constitutes a new kind of magnetic wall, besides the well known Bloch and Néel walls. In particular, the width of a constrained wall can become very small if the characteristic length of the constriction is small, as is actually the case in an atomic point contact. This provides a simple, natural explanation for the large magnetoresistance observed in ferromagnetic atomic point contacts.

PACS numbers: 75.60.Ch, 75.70.Kw, 75.70.Pa

The investigation of magnetic nanostructures is one of the major current subjects in magnetism. This interest is stimulated, on one hand, by the great progress in nanofabrication techniques and magnetic characterization methods and, on the other hand, by the perspective of technological applications for magnetic storage of information of unprecedented density.

A major question to be addressed in this field of research is: How does the micromagnetic structure (i.e., domains, walls, etc.) respond to geometrical constraints on the nanometer scale?

In an unconstrained system such as in a bulk ferromagnet, as first pointed out by Bloch, the wall structure is determined by a competition between exchange and anisotropy energies [1]. The exact structure of the Bloch wall has been calculated by Landau and Lifshitz [2]. In a ferromagnetic thin film with in-plane easy magnetization axis, as shown by Néel [3], the dipolar interaction leads to a new kind of wall, known as a Néel wall, in which the structure is determined by a competition between exchange, anisotropy, and dipolar energies.

In this Letter, I consider the problem of the structure and energy of a magnetic wall in a constriction separating two regions of wider cross section. This encompasses various situations of great physical interest such as a narrow constriction fabricated in a magnetic ultrathin film by lithographic techniques, or a constriction in a wire.

I point out that when the cross section of the constriction is much smaller than that of the wide region, the structure of the wall becomes almost independent of the material parameters such as magnetization, exchange stiffness, and anisotropy constant and is determined mostly by the geometry of the constriction. The wall energy consists mostly of exchange energy. Thus, geometrically constrained magnetic walls appear as a new kind of magnetic walls, with properties completely different from those of Bloch and Néel walls. In particular, the width of the geometrically constrained magnetic walls is essentially given by the length of the constriction, which can be considerably smaller than the width of a Bloch or Néel wall. In the limit of a point contact of atomic dimensions, the width of the wall would also be of atomic dimensions. This fact has important physical consequences: in particular, the contribution of a geometrically constrained magnetic wall to the electrical resistance of a point contact will be considerably larger than that of a Bloch or Néel wall, so that very large magnetoresistance effects can be anticipated in ferromagnetic point contacts and have actually been reported [4].

Let us consider a homogeneous magnetic system in which the cross section $S(x)$ varies along the x axis and exhibits a minimum at $x = 0$. The easy magnetization axis is along the z axis and the magnetization for $x \to \pm\infty$ is along the $\pm z$ axis, respectively. Obviously, the magnetic wall will tend to localize itself near the constriction, in order to minimize its energy. In order to understand how the wall structure is modified by the constriction, we can make the following reasoning: let us start with an infinitely narrow wall located at the center of the constriction. The exchange energy of such a configuration is too high and can be reduced by allowing the wall to expand. This expansion is counterbalanced by (i) the increase of anisotropy and (ii) by the increase of wall area. If the cross section $S(x)$ increases rapidly with $|x|$, then the second term can be the leading one, so that the wall structure will be controlled essentially by the geometry of the constriction and depend only weakly on the material parameters.

For explicit calculations below, I consider the following models of constrictions:

$$\left.\begin{aligned} S(x) &= S_0 && \text{for } |x| \leq d \\ &= S_1 > S_0 && \text{for } |x| \geq d \end{aligned}\right\} \qquad \text{(model I)}$$

$$S(x) = S_0\left(1 + \frac{x^2}{d^2}\right) \qquad \text{(model II)}$$

$$S(x) = S_0 \cosh(x/d) \qquad \text{(model III)}$$

For the sake of simplification, I make the following assumptions: (i) the magnetization direction depends only on x, i.e., the wall is plane (in reality this assumption is not strictly satisfied and the wall would tend to bend), (ii) the dipolar interactions can be neglected (the validity of this assumption will be discussed later), and (iii) the magnetization remains in the yz plane as in a Bloch wall (this assumption is best suited to the case of a constriction in a film with perpendicular anisotropy; in general, however, the wall structure would deviate from the idealized Bloch-like configuration). One can argue, however, that the above simplications would not modify significantly the underlying physical mechanism, and provide a good approximation of the wall structure and energy. Thus, the wall is described by the angle $\theta(x)$ between the magnetization and the z axis. Let $F(\theta)$ be the anisotropy energy density and A the exchange stiffness. In practice, we assume a uniaxial anisotropy below, i.e., $F(\theta) \equiv K\cos^2\theta$; however, wherever we use the more general form $F(\theta)$, the results are not restricted to this particular case. The total energy of the wall is given by

$$E(\theta) = \int_{-\infty}^{\infty} dx\,[A\dot{\theta}^2 + F(\theta)]S(x)\,, \tag{1}$$

where $\dot{\theta} \equiv d\theta/dx$. The structure of the wall is obtained by solving the corresponding Euler equation

$$\ddot{\theta} + \dot{\theta}\,\frac{\dot{S}}{S} - \frac{F'(\theta)}{2A} = 0\,, \tag{2}$$

where $F'(\theta) \equiv dF/d\theta$, subject to the boundary conditions, $\theta(\pm\infty) = \pm\pi/2$, respectively, and $\dot{\theta} = 0$ for $\theta = \pm\pi/2$. The new term $\dot{\theta}\dot{S}/S$, which is absent in the case of an unconstrained Bloch wall considered by Landau and Lifshitz [2], expresses the influence of the geometry of the constriction on the wall structure.

We are interested, in particular, in the width and energy of the wall. Various definitions of the wall width have been proposed in the literature [5]. Here, since we have in mind the electrical transport properties of the wall, we need an appropriate definition of the wall width. As the electrical resistance of a magnetic wall is determined by $\dot{\theta}(x)$ [6], this naturally leads one to use for the wall width the following new definition:

$$w \equiv 4\left[\int_{-\infty}^{\infty} \dot{\theta}^2(x)\,dx\right]^{-1} = 4\left[\int_{-\pi/2}^{\pi/2} \dot{\theta}\,d\theta\right]^{-1}, \tag{3}$$

where the prefactor has been chosen so that this definition yields $w_0 = 2\sqrt{A/K}$ for the unconstrained Bloch wall.

Let us neglect the term $-F'(\theta)/A$ in Eq. (2) and call $\theta^\star(x)$ the corresponding solution, of width $w^\star$ and energy $E^\star$. This provides a good approximation of the true solution if $(\dot{S}/S)^2$ is large as compared to $|F'(\theta)|/A$, and in any case yields an upper limit for the wall width. The solution then takes the general form

$$\theta^\star(x) = \pi\left[\frac{\int_{-\infty}^{x} S^{-1}(x')\,dx'}{\int_{-\infty}^{\infty} S^{-1}(x')\,dx'} - \frac{1}{2}\right]. \tag{4}$$

The width is given by

$$w^\star = \frac{4}{\pi^2}\,\frac{[\int_{-\infty}^{\infty} S^{-1}(x)\,dx]^2}{\int_{-\infty}^{\infty} S^{-2}(x)\,dx} \tag{5}$$

and the wall energy is

$$E^\star = \frac{\pi^2 A}{\int_{-\infty}^{\infty} S^{-1}(x)\,dx}\,. \tag{6}$$

In fact, we can argue that the above approximation is justified whenever $w^\star$ is small as compared to the width $w_0 = 2\sqrt{A/K}$ of the unconstrained Bloch wall, which is the only relevant parameter characterizing the material. Obviously, the usefulness of this approximation depends on whether the integral $\int_{-\infty}^{+\infty} S^{-1}(x)\,dx$ converges or not (for brevity, we shall term the constriction, respectively, "integrable" and "nonintegrable"). If $S(x)$ diverges more rapidly than $|x|$ for $|x| \to \pm\infty$, the constriction is integrable; this is the case for model I with $S_1 = \infty$, as well as for models II and III. In the opposite case of a nonintegrable constriction (e.g., model I with S_1 finite), $w^\star = \infty$, so that the term $-F'(\theta)/A$ can never be neglected.

Let us first discuss the case of an integrable constriction with $w^\star \ll w_0$. In this case, the above discussion shows that the wall structure is *independent* of the material parameter w_0 and is determined only by the geometry of constriction; furthermore, the wall profile $\theta^\star(x)$ and the wall width $w^\star$ are independent of the constriction cross section S_0. The area of the constriction is irrelevant; the only relevant parameter is the length d on which $S(x)$ varies significantly. Thus, we expect that the width will be of the order of $w^\star \sim d$ and the wall energy of the order of AS_0/d. Performing explicitly the calculation, this yields, for model I with $S_1 = \infty$,

$$\theta^\star(x) = \frac{\pi x}{2d}\,, \tag{7a}$$

$$w^\star = \frac{8d}{\pi^2}\,, \tag{7b}$$

$$E^\star = \frac{\pi^2 A S_0}{2d}\,, \tag{7c}$$

whereas for model II one gets

$$\theta^\star(x) = \arctan(x/d)\,, \tag{8a}$$

$$w^\star = \frac{8d}{\pi}\,, \tag{8b}$$

$$E^\star = \frac{\pi A S_0}{d}\,, \tag{8c}$$

and for model III

$$\theta^\star(x) = \arcsin[\tanh(x/d)], \tag{9a}$$

$$w^\star = 2d, \tag{9b}$$

$$E^\star = \frac{\pi A S_0}{d}, \tag{9c}$$

which confirms the above qualitative discussion. Interestingly, we remark that, for model III, the wall profile has the same form as for the unconstrained Bloch wall, with d replacing $L \equiv \sqrt{A/K}$.

On the other hand, if the constriction is nonintegrable, or if $w^\star$ is not small as compared to w_0, the term $-F'(\theta)/A$ cannot be neglected *a priori;* thus, this case deserves a more careful study. We now specify to the case of uniaxial anisotropy. The Euler equation becomes

$$\ddot{\theta} + \dot{\theta}\,\frac{\dot{S}}{S} + \frac{\sin\theta\cos\theta}{L^2} = 0. \tag{10}$$

In order to make this equation easily soluble, we perform the following approximation:

$$\cos^2\theta \approx \alpha(\pi - 2|\theta|), \tag{11a}$$

$$\sin\theta\cos\theta \approx \alpha\,\mathrm{sgn}(\theta), \tag{11b}$$

for $|\theta| \leq \pi/2$, and where the parameter α is determined variationally by minimizing the energy with respect to α for the unconstrained Bloch wall, which yields $\alpha = 0.298\,901\ldots$. In spite of its simplicity and its apparent crudeness, this approximation is an excellent one and yields a wall profile which is almost identical to the exact one, as can be seen in Fig. 1, while the errors on the wall width and energy are smaller than 1.5%.

With the help of this approximation, it is straightforward (although tedious) to solve the Euler equation almost completely analytically. Since the resulting expressions are rather cumbersome [7], I shall give below only approximate expressions valid in a restricted range of parameters, from which the physical meaning appears more clearly; the figures, however, display results obtained from the full expressions.

The wall profile calculated for model I is shown in Fig. 2. The wall consists of a core region of width $2d$ in which most of the magnetization rotation takes place and tails of width of the order of w_0 in which the magnetization rotates only weakly. Thus, if $d \ll w_0$, the constrained wall is much narrower than an unconstrained Bloch wall.

The wall width (normalized to d) and energy (normalized to the energy in the absence of constriction, $E_0 = \gamma_0 S_1$, where $\gamma_0 \equiv 4\sqrt{AK}$ is the energy per unit area of the unconstrained Bloch wall) as a function of w_0/d are displayed in Figs. 3 and 4, respectively, for various values of the ratio S_1/S_0.

We can distinguish here three different regimes, clearly visible in Figs. 3 and 4, depending on the values of the parameters w_0/d and S_1/S_0. In the first regime, i.e., for $w_0/d \leq 1$, one has

$$w \approx w_0, \tag{12a}$$

$$E \approx \gamma_0 S_0. \tag{12b}$$

This is easily understood: since the unconstrained wall width w_0 is smaller than the characteristic length of the constriction, the wall is entirely confined in the constriction and is therefore not significantly influenced by it.

The situation is completely different in the regime characterized by $1 \leq w_0/d \leq S_1/S_0$, for which one gets

$$w \approx \frac{8d}{\pi^2}, \tag{13a}$$

$$E \approx \frac{\pi^2 A S_0}{2d}, \tag{13b}$$

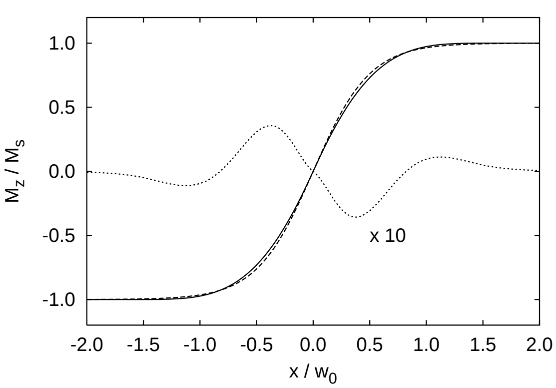

FIG. 1. Magnetization profile of the unconstrained Bloch wall (long-dashed line), as compared with the one calculated using an approximation [(11a) and (11b)] (solid line); the difference between the approximate and exact solutions (magnified by a factor of 10) is also shown (short-dashed line).

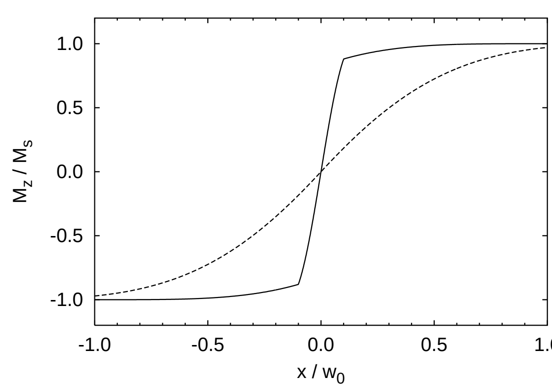

FIG. 2. Magnetization profile of a geometrically constrained magnetic wall calculated for model I with $d/w_0 = 0.1$ and $S_1/S_0 = 10$ (solid line), as compared with to the unconstrained Bloch wall (dashed line).

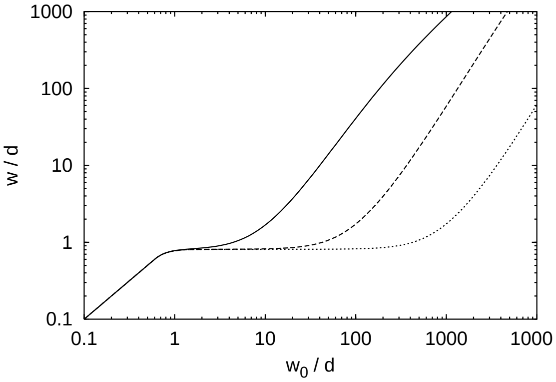

FIG. 3. Wall width (normalized to d) of a geometrically constrained magnetic wall calculated for model I, as a function of w_0/d; solid line: $S_1/S_0 = 10$; long-dashed line: $S_1/S_0 = 10^2$; short-dashed line: $S_1/S_0 = 10^3$.

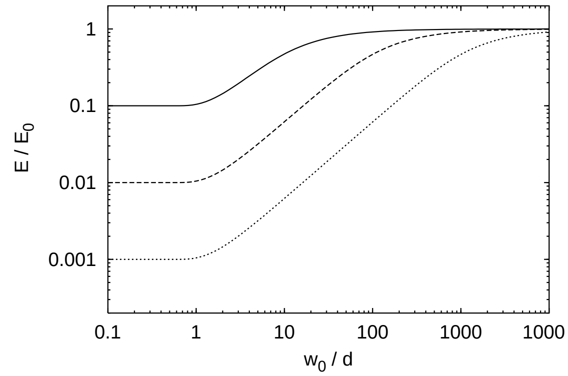

FIG. 4. Wall energy (normalized to E_0) of a geometrically constrained magnetic wall calculated for model I, as a function of w_0/d; solid line: $S_1/S_0 = 10$; long-dashed line: $S_1/S_0 = 10^2$; short-dashed line: $S_1/S_0 = 10^3$.

i.e., the wall width and energy are the same as the ones obtained for $S_1 = \infty$, [(7b) and (7c)], on the basis of approximations (5) and (6). Here the wall structure and wall width depend only on the geometry of the constriction and not at all on the material parameters, while the wall energy is of pure exchange character. If the ratio S_1/S_0 is large, this regime is achieved in a wide range of values of w_0/d, as appears clearly from Figs. 3 and 4.

Finally, for $w_0/d \geq S_1/S_0$, one gets

$$w \approx w_0 \left\{ 1 + \frac{18}{\pi^2} \frac{d}{w_0} \left[\left(\frac{S_1}{S_0} \right)^2 - \frac{S_1}{S_0} \right] \right\}^{-1}, \tag{14a}$$

$$E \approx \gamma_0 S_1 \left[1 - \frac{9}{\pi^2} \frac{d}{w_0} \frac{S_1}{S_0} + \frac{54}{\pi^4} \frac{d^2}{w_0^2} \left(\frac{S_1}{S_0} \right)^2 \right]. \tag{14b}$$

This is the case in which the wall structure is again determined primarily by the competition between the exchange and anisotropy energy terms, i.e., the first and third terms in the Euler equation (2), the additional term $\dot{\theta}\dot{S}/S$ due to the constriction being of secondary importance; therefore, w and E tend, respectively, towards w_0 and E_0 as w_0/d increases.

Let us now discuss the role of dipolar interactions, which we have neglected so far. A rough estimate of the dipolar contribution to the wall energy is given by $2\pi M_s^2$ multiplied by the wall volume, i.e., $E_d \approx 2\pi M_s^2 S_0 w$. If this energy is small as compared to the wall energy E calculated by neglecting the dipolar contribution, then we can expect that dipolar interactions will have only a small influence on the wall structure and on its width. For the most interesting case where $1 \leq w_0/d \leq S_1/S_0$, one finds that dipolar interactions can be neglected if $d \ll (\pi^2/4)\lambda$, where $\lambda \equiv A/(2\pi M_s^2)$ is the exchange length. For the typical value $\lambda \approx 3$ nm, this means $d \ll 7.5$ nm. Thus, for an atomic point contact, our approximation is well justified.

In conclusion, I have investigated the properties of a geometrically constrained magnetic wall. I have shown that the structure and the properties of such a wall differ considerably from those of an unconstrained wall, so that the geometrically constrained magnetic wall truly constitutes a new kind of magnetic wall, besides the well known Bloch and Néel walls. In particular, the wall width of a geometrically constrained magnetic wall can become very small if the characteristic length of the constriction is small, as is actually the case in an atomic point contact. This provides a simple, natural explanation for the large magnetoresistance which has been recently observed in atomic point contacts [4]. In addition, I have introduced a new definition of the wall thickness, which is the appropriate one for discussing the electrical resistance of magnetic wall. I have also proposed a simple approximation for solving the Euler equation, which allows one to obtain simplified, yet accurate, analytical results.

I express my gratitude to Gen Tatara, Hans-Peter Oepen, Yonko Millev, as well as to the late Alex Hubert, for helpful discussions.

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[1] F. Bloch, Z. Phys. **74**, 295 (1932).
[2] L. D. Landau and E. M. Lifshitz, Phys. Z. Sowjetunion **8**, 153 (1935).
[3] L. Néel, C.R. Acad. Sci. (Paris) **241**, 533 (1955).
[4] N. García, M. Muñoz, and Y.-W. Zhao, Phys. Rev. Lett. **82**, 2923 (1999).
[5] A. Hubert and R. Schäfer, *Magnetic Domains* (Springer-Verlag, Berlin, 1998).
[6] G. Tatara and H. Fukuyama, Phys. Rev. Lett. **78**, 3773 (1997).

[7] Explicitly, one obtains $w/w_0 = (\omega^3 + 3\delta\omega^2\sigma^2 + 3\omega\delta^2\sigma + \delta^3)^{-1}$ and $E/E_0 = \omega^3 + 3\omega^2\delta\sigma/2 + 3\delta/(2\sigma) - \delta^3/(2\sigma)$ for $\delta \leq 1$, and $w/w_0 = 1$ and $E/E_0 = 1/\sigma$ for $\delta \geq 1$. Here, $\sigma \equiv S_1/S_0$, $\delta \equiv 6d/(\pi^2 w_0)$, and $\omega \equiv \sigma[-\delta + \sqrt{\delta^2 + (1 - \delta^2)/\sigma^2}]$.