Convection versus Dispersion in Optical Bistability

S. Coen, M. Tlidi, Ph. Emplit, and M. Haelterman
Service d'Optique et Acoustique, Université Libre de Bruxelles, 50 Avenue F. D. Roosevelt, CP 194/5, B-1050 Bruxelles, Belgium
(Received 24 March 1999)

We analyze experimentally the combined action of convection and a diffusionlike process in a bistable nonlinear optical resonator. We show that the very existence of bistability in this device strongly depends on a competition mechanism between these two processes. This mechanism is described analytically in terms of the switching wave dynamics of a Fisher-Kolmogorov equation.

PACS numbers: 42.65.Sf, 42.65.Pc, 42.60.Da, 47.27.Te

Convection is well known to dramatically affect the dynamics of nonlinear distributed systems in such diverse fields as fluid mechanics [1–3], plasma physics [4], population dynamics, biochemistry [5], and nonlinear optics [6–9]. On the one hand, convection was shown to give rise to new dynamical behaviors such as self-pulsing instabilities [7] or pattern selection [9] in nonlinear optical resonators. On the other hand, convective transport mechanisms were shown to be responsible for the inhibition of instabilities such as the plasma beam instability [4], bistability [3,6], or intracavity modulational instability [8]. These cases put into play the so-called convectively unstable states where dynamically amplified local perturbations of a finite system are advected away more rapidly than their rate of spreading in such a way that the system, although linearly unstable, may be considered as stable [3,10]. As a general rule, the actual behavior of these systems is the result of a competition between convection and other physical mechanisms.

In the present Letter we investigate theoretically and experimentally a system whose basic dynamical behavior is drastically ruled by a competition mechanism between dispersion and convection. We consider an optical bistable system consisting of a simple passive fiber cavity synchronously pumped by a pulsed laser (see Fig. 1). Bistability in the system stems from the combined action of the cavity feedback and the Kerr nonlinearity of the fiber. Synchronous pumping means that the time-of-flight of the light pulses in the cavity is adjusted to the laser repetition time. Convection is introduced in this system through pumping synchronization mismatch due to inaccuracy in the cavity length. The remarkable feature of this device is that it allows, through the control of the cavity length, for a fine tuning of the amount of convection with respect to dispersion and, hence, to study the competition between these two fundamental processes. This contrasts with other devices such as practical reaction-diffusion systems in which diffusion is generally completely smeared out as soon as convection is introduced. In this respect, nonlinear resonators constitute ideal test beds for the experimental study of the competition between convection and diffusionlike processes that is liable to affect a broad range of physical systems. In this Letter, we illustrate the importance of this competition by studying the inhibition of the bistable response of an optical Kerr cavity under the influence of convection. Moreover, we show that the very existence of bistability drastically depends on this competition mechanism. These observations are confirmed by an analytical description of the phenomenon. The present work constitutes a practical illustration of the concept of nonlinearly convective unstable states proposed in Ref. [3].

The synchronously pumped nonlinear passive fiber ring cavity that we consider has been previously investigated for the study of modulational instabilities [11]. It is represented schematically in Fig. 1. We refer the reader to Ref. [11] for a detailed description of the experimental arrangement. It was shown in Ref. [12] that the intracavity field is ruled by the following dimensionless equation:

$$\partial_t E = S(\tau) - [1 + i(\Delta - |E|^2)]E - i\eta\partial^2_{\tau\tau}E - d\partial_\tau E\,, \quad (1)$$

where t is the slow time that describes the evolution of the field envelope E at the scale of the cavity photon lifetime, while τ is a fast time defined in a reference frame that travels at the group velocity of light in the fiber. The source term $S(\tau)$ represents the pump pulses profile. The damping term $-E$ represents the effect of the cavity losses. The parameter $\Delta = \delta_0/\alpha$ is the cavity phase detuning, α being the overall cavity loss (i.e., including intput/output coupler transmissions and fiber loss); and $\delta_0 = 2m\pi - \phi_0$, where ϕ_0 is the cavity round-trip phase shift, and m is the order of the closest cavity resonance. The dispersion parameter η will be taken either equal to zero to represent the absence of dispersion or equal to unity to represent the normal dispersion regime considered in our experiment. Finally, the convection parameter d represents the effect

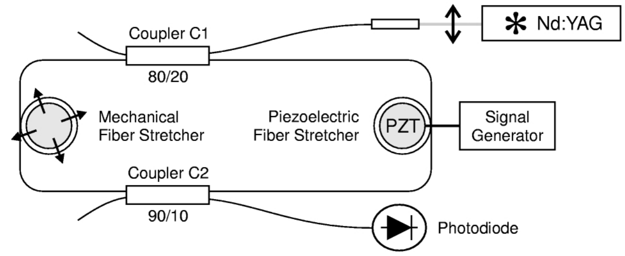

FIG. 1. Experimental setup.

 0031-9007/99/83(12)/2328(4)$15.00

of synchronization mismatch. With the normalization we used [12], $d = [2/(|\beta_2|L\alpha)]^{1/2}\Delta T$, where β_2 is the fiber dispersion coefficient, L is the cavity length, and ΔT is the synchronization mismatch, i.e., the difference between the laser repetition time and the cavity round-trip time.

Equation (1) is identical to the mean-field model derived by Haelterman *et al.* for the description of spatial transverse effects in the nonlinear Fabry-Perot resonator [6]. In that case, the convective term was introduced to account for oblique incidence of the pump beam. We briefly recall here the influence of convection on the features of this equation. In the case of continuous wave (cw) pumping (i.e., when convection plays no role since S is constant and all τ derivatives can be set to zero), the steady-state solution of Eq. (1) exhibits bistability, provided that $\Delta > \Delta_c = \sqrt{3}$. In the case of synchronous pumping with short pulses, bistability still exists. The only qualitative difference with respect to cw pumping is the presence, in the intracavity pulse profile, of switching waves that separate the lower and upper states of the bistable cycles [13]. However, in the limit of zero dispersion ($\eta = 0$) the pulsed pumping regime is fundamentally different as regards the effect of convection. Indeed, in the presence of convection ($\partial_\tau E \neq 0$) the steady-state pulse profile of Eq. (1) with $\eta = 0$ is given by the solution of the first order differential equation obtained by setting $\partial_t E = 0$. Such a solution being unique, one reaches the conclusion that bistability of the finite pulse is forbidden in the presence of convection. As was done in Ref. [6], this feature can be physically interpreted in terms of feedback. In a cavity, the feedback required for bistability stems from the point-to-point superposition of the pump field with the intracavity field at each pass in the input coupler of the cavity. When there is a synchronization mismatch, the pump field S at a given time τ superimposes with the intracavity field E at a time τ that is different at each round-trip so that, for a given point in the pulse envelope, there is effectively no feedback in the system. Bistability is therefore forbidden even if the envelopes of the pump and intracavity pulses overlap over a large part of their width in the cavity input coupler.

Dispersion is able to restore the feedback through the introduction of a nonlocality in the pulse envelopes, in a way akin to what happens with diffusion in chemical or biological systems. Indeed, owing to dispersion, the amplitude at a given time τ of the intracavity pulse at the beginning of the cavity round-trip influences, after propagation in the cavity, the amplitude distribution of the pulse in a whole region surrounding time τ. This region of influence may be large enough to compensate for the synchronization mismatch ΔT so as to restore the feedback and, hence, bistability.

We have studied experimentally this fundamental competition mechanism between dispersion and convection. In our experiment, we use a 7.4 m long all-fiber ring cavity in which light is launched through a standard 80/20 fiber coupler. The cavity is synchronously pumped by a mode-locked Nd:YAG laser emitting 180 ps (FWHM) pulses at 1064 nm with a repetition rate of 82 MHz. The cavity length is precisely controlled by means of a mechanical fiber stretcher that allows for synchronization tuning with a resolution of $\sim$50 fs. During our measurement of the bistable cycles, the input power was kept constant while the cavity detuning Δ was varied by applying a triangular signal to a piezoelectric fiber stretcher. Our results are illustrated in Fig. 2, which shows the bistable cycles obtained with several values of the synchronization mismatch for a mean input power of 2.8 W. These results reveal quite remarkably that a synchronization mismatch as small as $\Delta T = 600$ fs (i.e., only 0.3% of the pump pulse duration) is sufficient to make bistability disappear.

In order to develop an analytical description of this phenomenon, it is useful to give more details on the nonlinear intracavity wave dynamics in the presence of convection. In the pulsed regime, the multivalued response of the cavity is associated with the existence, in the profile of the intracavity pulse, of switching waves that link adjacent temporal domains in which the system is respectively on the upper and lower states of the bistable cycle. Such switching waves (SW's) have already been studied numerically in the spatial domain by Rozanov *et al.* in the framework of a study of transverse effects in diffractive nonlinear Fabry-Perot resonators [13]. Following that paper we can infer that in a cw pumped cavity the SW's consist of steady-state fronts that propagate at a speed determined by the input pump amplitude. For a given cavity detuning, only one value of the pump amplitude, say S^*, gives rise to a stationary SW. For higher (lower) pump amplitudes, the SW's exhibit a nonzero velocity resulting in the extension of the upper (lower) state domain. The maximum values of the SW velocity are obtained at the two limit points of the bistable cycle, say, $S_{\rm up}$ and $S_{\rm down}$. Note that, since we consider here the temporal domain, in contrast with Refs. [6,13], the term *velocity* means the translation speed within the traveling reference time frame τ.

In the pulsed regime in the absence of convection, the SW's in the steady-state intracavity pulse profile are located at the values of τ corresponding to the critical value of pump amplitude, S^*. Therefore when increasing

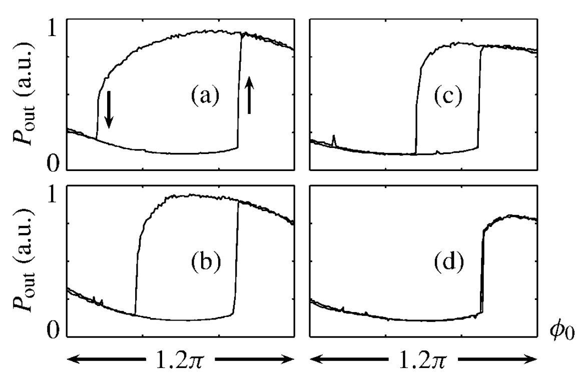

FIG. 2. Bistable cycles for (a) a perfect synchronization, and with a synchronization mismatch of (b) 150 fs, (c) 300 fs, and (d) 600 fs. The mean input power was set to 2.8 W.

(decreasing) the pump pulse amplitude the two SW's move apart (closer together). This means that if the system is close to up-switching (down-switching) the SW's are far apart (close together). This feature is illustrated in Fig. 3(a), which shows several experimental pulse profiles measured at the cavity output by a streak camera in the conditions of Fig. 2(a).

With a nonzero synchronization mismatch, the convective term in Eq. (1) is responsible for a drift motion of the SW's at the velocity d. The up-stream (down-stream) SW moves closer to (away from) the pulse peak where the pump amplitude is higher (lower) than S^*. In that region the natural velocity (i.e., the velocity in the absence of convection) of the SW is therefore such that the upper (lower) state domain gets favored and tends to grow. It thus appears that the natural SW velocity counteracts the effect of convection. Consequently, in the intracavity pulse profile, stationary SW's are formed that result from a balance between both velocities. The resulting steady-state pulse profile is then asymmetric as illustrated in Fig. 3(b), which shows the pulse profile near down-switching when the synchronization mismatch was shorter than 100 fs. Of course, stationary SW's are formed only if the convection velocity d is not larger than the maximum value of the natural velocity of the SW, that is, the velocity at up-switching $S = S_{\mathrm{up}}$. If this condition is not satisfied, there is no mean to compensate for the fast convective motion of the up-stream SW that thus always goes across the pulse maximum so that the upper state domain is washed away. In this situation bistability is inhibited by convection.

This analysis provides us with a way to calculate the maximum value of the synchronization mismatch compatible with bistable operation of the device, say, $\Delta T_{\max}$. This value is simply given by equalizing the drift velocity $d = [2/(|\beta_2|L\alpha)]^{1/2}\Delta T$ with the maximum SW velocity $v_{\max}$ obtained in $S = S_{\mathrm{up}}$, i.e., $\Delta T_{\max} = (|\beta_2|L\alpha/2)^{1/2} v_{\max}$. However, the inherent complex nature of the chromatic dispersion term in Eq. (1) prevents us from deriving a minimizing potential to describe analytically the SW dynamics of our nonlinear cavity. We had then to resort to an approximate calculation of $v_{\max}$.

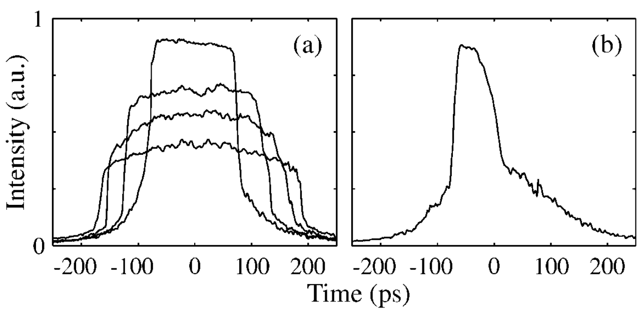

FIG. 3. (a) Pulse profiles at zero synchronization mismatch. When approaching down-switching, the SW's go closer together. (b) Asymmetric pulse shape observed with a small cavity mismatch near down-switching.

We start from Eq. (1) where we consider S constant, $d = 0$, and $\eta = 1$. To calculate analytically the velocity of the SW solutions of Eq. (1), we restrict our analysis to nascent bistability. This means that we consider Eq. (1) in the vicinity of the critical point of optical bistability, namely, the inflection point (with infinite slope) of the intensity response curve corresponding to the critical detuning for the onset of optical bistability $\Delta = \Delta_c = \sqrt{3}$. To this end, we introduce a small parameter $\epsilon \ll 1$ that measures the distance from the critical point and we express the cavity detuning in the form $\Delta = \Delta_c(1 + \epsilon^2 I)$, where I is a quantity of order one. Next, we define new time scales $t_\epsilon = \epsilon^2 t$, $\tau_\epsilon = 3^{1/4}\epsilon\tau$ to take into account the slowing down of the dynamics around the critical point. Finally, we decompose the electric field into its real and imaginary part $E = u + iw$, and we define a moving frame $\xi = \tau_\epsilon - Vt_\epsilon$ that travels at the velocity $V = 3^{1/4}v/\epsilon$ (to be determined) of the SW. In this moving frame, the SW solution is stationary so that Eq. (1) eventually reduces to a couple of ordinary differential equations for u and w with V as a parameter:

$$S - u - w(u^2 + w^2 - \Delta) + \epsilon^2(Vu' + \sqrt{3}\,w'') = 0\,, \tag{2}$$

$$-w + u(u^2 + w^2 - \Delta) + \epsilon^2(Vw' - \sqrt{3}\,u'') = 0\,, \tag{3}$$

where primes denote derivatives with respect to ξ. Our strategy is now to seek solutions of Eqs. (2) and (3) in the vicinity of the nascent bistability. To this end, we expand all variables in terms of ϵ as $Z = Z_c(1 + \epsilon Z_0 + \epsilon^2 Z_1 + \epsilon^3 Z_2 + \ldots)$, where Z is any of the three quantities $[u(\xi), w(\xi), S]$ and Z_c is the corresponding coordinate of the critical point: $u_c = (3/4)S_c$, $w_c = -(\sqrt{3}/4)S_c$, $S_c = 2\sqrt{2}/3^{3/4}$.

At first order in ϵ, we find $S_0 = 0$ and $w_0(\xi) = -u_0(\xi)$. At second order, we find $S_1 = (3/4)I$. Finally, at third order we obtain a differential equation for u_0:

$$u_0'' + Vu_0' + f(u_0) = 0\,, \tag{4}$$

where $f(u_0) = -2u_0^3/3 + Iu_0 + 4S_2/3$ in which S_2, being defined through the relation $S = S_c[1 + (3/4)\epsilon^2 I + S_2\epsilon^3]$, represents the role of the pump amplitude. Equation (4) has the form of the well-known Fisher-Kolmogorov equation [14,15]. In the present context, the cubic polynomial $f(u_0)$ is directly related to the characteristic S shape of the bistable intensity response of the cavity. The roots of $f(u_0)$ correspond indeed to the homogeneous steady-state solutions of Eq. (4). Note that, as expected, it is only when $I > 0$, i.e., $\Delta > \Delta_c$, that $f(u_0)$ has three real roots, namely, u_0^{l}, u_0^{i}, and u_0^{u}. Assuming $u_0^{\mathrm{l}} < u_0^{\mathrm{i}} < u_0^{\mathrm{u}}$, these solutions correspond, respectively, to the lower, intermediate, and upper branch of the bistable cycle. With these notations, the switching wave solution of Eq. (4) that connects the lower and the upper states reads

$$u_0(\xi) = u_0^{\mathrm{l}} + (u_0^{\mathrm{u}} - u_0^{\mathrm{l}})[e^{\kappa\xi}/(1 + e^{\kappa\xi})]\,, \tag{5}$$

where $\kappa = (u_0^{\mathrm{u}} - u_0^{\mathrm{l}})/\sqrt{3}$. The corresponding velocity is

$V = \sqrt{3}\, u_0^{\mathrm{i}}$. From Eq. (4), we can easily obtain the velocity at up-switching, i.e., in $S_2 = (I/2)^{3/2}$, where $u_0^{\mathrm{i}} = u_0^{\mathrm{l}} = (I/2)^{1/2}$. With the units of Eq. (1) we find $|v_{\max}| = [(\Delta - \Delta_c)/2]^{1/2}$ that leads, in real units, to the following expression of the maximum synchronization mismatch:

$$\Delta T_{\max} = \sqrt{|\beta_2| L\alpha/2}\, \sqrt{(\Delta - \Delta_c)/2}\,. \qquad (6)$$

This analytical expression indicates how the various physical parameters influence the role of convection in the onset of bistability. In particular, it illustrates the competition mechanism between convection and dispersion discussed above. Equation (6) shows indeed that a small dispersion leads to a high sensitivity to convection. In the limit of zero dispersion, $\beta_2 = 0$, the slightest amount of convection inhibits bistability since one finds in this case $\Delta T_{\max} = 0$.

We have checked the validity of our analytical model [Eq. (5)] by comparing it with the SW solutions of Eq. (1) calculated numerically. The results are summarized in Fig. 4(a), which shows the maximum SW velocity (i.e., the velocity at up-switching) versus the cavity detuning. As expected, a good agreement is obtained only in the vicinity of the critical detuning Δ_c. However, even for larger values of the detuning as those considered in the experiment, the analytical description of the role of the detuning is still verified qualitatively. A qualitative agreement is also obtained in the profile of the SW as illustrated in Figs. 4(b) and 4(c), which show the SW intensity profile for $\Delta = 4$ as obtained [4(b)] from numerical solution of Eq. (1) and [4(c)] from Eq. (5). As can be seen the width of the SW is predicted correctly. The discrepancy observed for the SW velocities in Fig. 4(a) may be attributed to the fact that the overshoot oscillations that are observed numerically at large detunings are not described by the analytical model Eq. (5) as can be seen in Figs. 4(b) and 4(c).

It is interesting to evaluate the value of $\Delta T_{\max}$ from the parameters of the experiment. Through the measurement of the cavity resonance width, we have evaluated $\alpha \simeq 0.45$, and from the bistable cycle at zero synchronization mismatch [see Fig. 2(a)] we calculate a phase detuning at up-switching of $\delta_0 \simeq 2.15$ so that $\Delta = \delta_0/\alpha \simeq 4.8$. Numerical simulation of Eq. (1) then leads to a maximum SW velocity of $v_{\max} \simeq 3.5$ that gives $\Delta T_{\max} \simeq 640$ fs, which is in good agreement with the experimental observation.

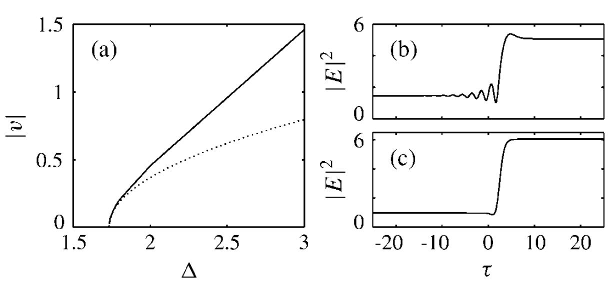

FIG. 4. (a) Solid line represents the velocity of the SW at up-switching. The dotted line shows the analytical prediction. (b), (c): Temporal profile of the SW obtained for $\Delta = 4$ at up-switching. (b) shows the exact numerical result while (c) shows the analytical prediction.

In conclusion, by means of a simple all-fiber optical resonator, we have experimentally demonstrated that dispersion can efficiently overcome the inhibiting effect of convection in optical bistability. We have shown that dispersion competes with convection through the introduction of a nonlocality in a way akin to what happens with diffusion in chemical or biological systems. This competition mechanism has been described in terms of switching wave dynamics. This approach allowed us to develop an analytical description of the phenomena based on the reduction of the complex mean-field model of nonlinear optical cavities to a Fisher-Kolmogorov equation. The structural simplicity of the studied device and the generality of the phenomenology involved as well as of the mathematical treatment give to our study a universal character. Our results are thus liable to improve the knowledge and the understanding of the behaviors of distributed bistable systems encountered in nonlinear optics as well as in other fields of nonlinear science.

We acknowledge fruitful discussions with R. Lefever. This research was supported by the Fonds National de la Recherche Scientifique (Belgium) and the Inter-University Attraction Pole program of the Belgian government under Grant No. P4-07.

[1] L. D. Landau and E. M. Lifshitz, *Fluid Mechanics* (Pergamon, New York, 1959).
[2] K. L. Babcock, G. Ahlers, and D. S. Cannell, Phys. Rev. Lett. **67**, 3388 (1991).
[3] J. M. Chomaz, Phys. Rev. Lett. **69**, 1931 (1992).
[4] P. A. Sturrock, Phys. Rev. **112**, 1488 (1958).
[5] J. D. Murray, in *Mathematical Biology*, Biomathematics Texts Vol. 19, edited by S. A. Levin (Springer-Verlag, Berlin, 1993), 2nd ed.
[6] M. Haelterman, G. Vitrant, and R. Reinisch, J. Opt. Soc. Am. B **7**, 1309 (1990).
[7] M. Haelterman and G. Vitrant, J. Opt. Soc. Am. B **9**, 1563 (1992).
[8] M. Santagiustina, P. Colet, M. San Miguel, and D. Walgraef, Phys. Rev. Lett. **79**, 3633 (1997).
[9] M. Santagiustina, P. Colet, M. San Miguel, and D. Walgraef, Opt. Lett. **23**, 1167 (1998).
[10] E. Infeld and G. Rowlands, *Nonlinear Waves, Solitons and Chaos* (Cambridge University Press, Cambridge, England, 1990).
[11] S. Coen and M. Haelterman, Phys. Rev. Lett. **79**, 4139 (1997).
[12] M. Haelterman, S. Trillo, and S. Wabnitz, Opt. Commun. **91**, 401 (1992).
[13] N. N. Rozanov, V. E. Semenov, and G. V. Khodova, Kvant. Elektron. Mosk. **9**, 354 (1982) [Sov. J. Quantum Electron. **12**, 193 (1982)].
[14] R. A. Fisher, Ann. Eugenics **7**, 353 (1937).
[15] A. N. Kolmogorov, I. G. Petrovskiĭ, and N. S. Piskunov, Bjul. Moskovskovo Gos. Univ. **17**, 1 (1937).