Vacuum Backreaction on a Pair-Creating Source

G. A. Vilkovisky
Lebedev Physics Institute and Research Center in Physics, Leninsky Prospect 53, Moscow 117924, Russia
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A solution is presented to the simplest problem involving the vacuum backreaction on a pair-creating source. The backreaction effect is nonanalytic in the coupling constant and restores completely the energy conservation law. The vacuum changes the *kinematics* of motion, as relativity theory does, and imposes a new upper bound on the velocity of the source.

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The phenomenon of the vacuum instability caused by nonstationary external fields received much attention owing to its significance in the black hole physics (see the references in [1]), but the phenomenon itself is quite general. A charged source of a nonstationary field is capable of creating from the vacuum real particles having the same type of charge (electrical, gravitational, etc.). When the frequency of the source exceeds the threshold of pair creation, it emits a flux of energy and charge carried by the created particles. However, the problem with external field is physically incomplete since it does not answer the question where the energy of the created particles comes from. It is clear that the vacuum particle production is only a mechanism of the energy transfer. The energy comes ultimately from the source of the external field, and there emerges a question: how much energy can be extracted from a source through the vacuum mechanism?

An attempt to answer this question without taking into account the backreaction of the vacuum on the motion of the source leads only to a contradiction with the energy conservation law. The radiation of black holes is only one (although the most glaring) example. Typically, the radiation rate grows unboundedly with the energy of the source, and, at a sufficiently high energy, the source appears to give more than it has. This is the case even in QED [1,2].

Of course, one expects that the corrections stemming from the self-consistent equations for the expectation values of the field will remove the contradiction, but one should realize that the vacuum radiation is a purely quantum effect, and, therefore, a quantum correction to the external field will result only in a higher-order correction to the radiation energy. The backreaction effect capable of restoring the energy conservation law can only be nonanalytic in the coupling constant.

Below I present the solution of the self-consistent problem for the simplest model of a pair-creating source. The question receives an answer but the significance of this answer seems to surpass the significance of the question.

The model is an electrically charged spherical shell expanding in the self field. Below, $r = \rho(t)$ is the law of expansion of the shell, e and M are, respectively, the full charge and mass of the shell, and $\mathcal{E}$ is its energy in excess of the rest energy ($c = 1$). It is assumed that before some time instant $t = t_{\text{start}}$ the shell was kept at the state of maximum contraction $r = r_{\min}$ and next was let go. The world line of the shell is shown in Fig. 1.

Since the shell moves with acceleration, it creates particles from the vacuum. It radiates at a short stage of its evolution near $t = t_{\text{start}}$ where the acceleration is maximum. The bigger the energy $\mathcal{E}$, the bigger is this acceleration, and the more violent is the creation of particles. Therefore, it is interesting to consider the ultrarelativistic shell $(\mathcal{E}/M) \gg 1$.

Without predetermining the law of motion $\rho(t)$, one may assume that the shell expands monotonically with an increasing velocity which at $t = \infty$ reaches some finite value $\dot{\rho}(\infty)$. Then $\dot{\rho}(\infty)$ may serve as a measure *at late time* of the acceleration at t_{start}. As $(\mathcal{E}/M) \to \infty$, the velocity $\dot{\rho}(t)$ approaches 1 at all t except in a small sector near $t = t_{\text{start}}$. The world line of the shell approaches then the broken line in Fig. 1. These assumptions are valid for the classical motion of the shell, and they cannot be invalidated by the quantum corrections since these corrections are small.

Let $\Delta\mathcal{E}$ and Δe be the energy and charge emitted by the shell for the whole of its history. Using the methods

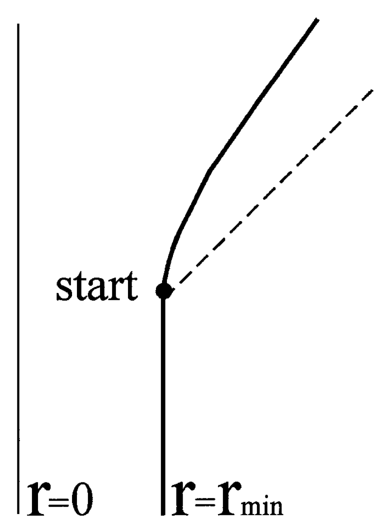

FIG. 1. The world line of the shell on the r, t plane. The broken line is the outgoing light ray.

of Refs. [1,2], these quantities can be calculated with an arbitrary law of motion $\rho(t)$. For the ultrarelativistic motion under the assumptions above the result is [In the high-frequency approximation [2] which in the present case is provided by the condition $(\mathcal{E}/M) \gg 1$, the radiation flux does not depend on the mass of the vacuum particles.]

$$\frac{\Delta\mathcal{E}}{\mathcal{E}} = \frac{\Delta e}{e} = -\frac{\kappa^2}{24\pi}\log\sqrt{1-\dot\rho(\infty)} + \kappa^2 O(1)\,, \qquad \dot\rho(\infty)\to 1\,, \tag{1}$$

where $O(1)$ denotes the terms that remain finite as $\dot\rho(\infty)\to 1$, and $\kappa^2 > 0$ is the constant of coupling of the electromagnetic field of the shell to the vacuum charges (8 times the fine structure constant for the electron-positron vacuum). Inserting in (1) the $\dot\rho(\infty)$ calculated from the *classical* law of motion

$$\frac{M}{\sqrt{1-\dot\rho^2}} + \frac{1}{2}\frac{e^2}{\rho} = M + \mathcal{E}\,, \tag{2}$$

one obtains the result

$$\frac{\Delta\mathcal{E}}{\mathcal{E}} = \frac{\Delta e}{e} = \frac{\kappa^2}{24\pi}\log\frac{\mathcal{E}}{M}\,, \qquad \frac{\mathcal{E}}{M}\to\infty\,, \tag{3}$$

which manifestly contradicts the energy conservation law.

The self-consistent problem to be solved for obtaining the correct result is as follows. Any spherically symmetric electromagnetic field is determined by a single function $\mathbf{e}(t,r)$, which is the charge contained at the time instant t inside the sphere of radius r. In terms of this function, the electric field E and electromagnetic current j^α are, respectively, of the form

$$E = \frac{\mathbf{e}(t,r)}{r^2}\,, \qquad -4\pi r^2 j^\alpha = \left(\nabla^\alpha r\frac{\partial}{\partial t} + \nabla^\alpha t\frac{\partial}{\partial r}\right)\mathbf{e}(t,r)\,. \tag{4}$$

The $\mathbf{e}(t,r)$ must satisfy the condition of regularity of the electric field at $r=0$, $\mathbf{e}(t,0)=0$, and the normalization condition $\mathbf{e}(t,\infty)=e$. With these boundary conditions one is to solve the expectation-value equations

$$j^\alpha + \frac{\kappa^2}{2\pi}\gamma(-\Box)j^\alpha = j^\alpha_{\text{bare}}\,, \tag{5}$$

$$\gamma(-\Box) = \frac{1}{12}\left[\int_{4m^2}^\infty d\mu^2\left(1-\frac{4m^2}{\mu^2}\right)^{3/2}\frac{1}{\mu^2-\Box} - \int_0^\infty d\mu^2\frac{1}{\mu^2+m^2} + \frac{8}{3}\right] \tag{6}$$

with the retarded resolvent $1/(\mu^2-\Box)$ [1,2]. [The spectral-mass function in Eq. (6) is the one for the standard loop [1,2] but the detailed form of this function is unimportant. Its important properties are positivity, the presence of a threshold, and the behavior at large spectral mass.] In (5) and (6), m is the mass of the vacuum particles, and j^α_{bare} is expressed through

$$\mathbf{e}_{\text{bare}}(t,r) = e\theta\big(r-\rho(t)\big) \tag{7}$$

by the same formula as in (4). The set of equations is closed by adding the equation of motion of the shell

$$\frac{d}{dt}\left(\frac{M\dot\rho}{\sqrt{1-\dot\rho^2}}\right) = e\,\frac{\mathbf{e}_+(t)+\mathbf{e}_-(t)}{2\rho^2}\,, \tag{8}$$

where, as appropriate for the charged surface, the force exerted on the shell is determined by one-half of the sum of the electric fields on both sides of the shell:

$$\mathbf{e}_\pm(t) = \mathbf{e}\big(t,\rho(t)\pm 0\big)\,. \tag{9}$$

Reserving the procedure of solving for an extended publication, I present only the final result. The solution for the force in (8) is

$$\mathbf{e}_+(t)+\mathbf{e}_-(t) = e + e\frac{\kappa^2}{24\pi}\log\big(1-\dot\rho^2(t)\big) + \kappa^2 O(1)\,. \tag{10}$$

Since κ^2 is small, and $O(1)$ is uniformly bounded, the term $\kappa^2 O(1)$ in (10) can be discarded. The force of the vacuum backreaction depends on the velocity. Nevertheless, the equation of motion (8) with this force admits the energy integral:

$$M\int_1^{1/\sqrt{1-\dot\rho^2}}\frac{dx}{1-(\kappa^2/12\pi)\log x} + \frac{1}{2}\frac{e^2}{\rho} = \mathcal{E}\,, \tag{11}$$

which at $\kappa^2 = 0$ goes over into the classical law (2). There is no problem with the singularity of the integral in (11). It is never reached. As in (2), for a given energy, the velocity $\dot\rho$ reaches its maximum value at $\rho=\infty$ but the value is now different:

$$\int_1^{1/\sqrt{1-\dot\rho^2(\infty)}}\frac{dx}{1-(\kappa^2/12\pi)\log x} = \frac{\mathcal{E}}{M}\,. \tag{12}$$

As in (2), $\dot\rho(\infty)$ grows with $\mathcal{E}/M$ *but not up to* 1:

$$\dot\rho(\infty) = 1 - \frac{1}{2}\exp\left(-\frac{24\pi}{\kappa^2}\right), \qquad \frac{\mathcal{E}}{M}\to\infty \tag{13}$$

and this is the principal consequence of the vacuum backreaction.

The insertion of (13) in (1) restores the conservation laws:

$$\frac{\Delta\mathcal{E}}{\mathcal{E}} = \frac{\Delta e}{e} = \frac{1}{2} + \kappa^2 O(1), \qquad \frac{\mathcal{E}}{M}\to\infty\,. \tag{14}$$

Up to 50% of energy and charge can be extracted from a source by raising its initial energy. But the main result is, of course, (11). The vacuum does not change the electric potential as one could expect. It changes *the kinematics of motion* as relativity theory does. Furthermore, within a given type of coupling, this change is universal. It does not depend on the parameters of the source, only on the coupling constant κ^2. The vacuum appears as a medium in which the velocity of light is less than c, but there is one special thing about this medium: it cannot be escaped.

Like the vacuum radiation itself, its backreaction is a semiclassical effect, and the avoidance of the ultraviolet problem deserves a note. This problem manifests itself in the fact that the distribution $\mathbf{e}(t, r)$ that solves the expectation-value equations is singular on the shell's surface:

$$\mathbf{e}(t, r)\Big|_{r\to\rho(t)\pm 0} = \mp e\,\frac{\kappa^2}{24\pi}\log|r - \rho(t)| \tag{15}$$

(Fig. 2). Nevertheless, the force exerted on the shell is finite and is unambiguously obtained by making the sum (10) in the spectral integral. This is equivalent to giving the shell a Compton width. The ultraviolet problem concerns the spacelike vicinity of the shell but not its motion. It is important that the infinite jump of $\mathbf{e}(t, r)$ across the shell is constant. Owing to this fact, the fluxes across the shell are finite, and so is the force of their backreaction.

Another object of worry is the vicinity of $r = 0$ since $r_{\min} = (e^2/2\mathcal{E})$ while $\mathcal{E}$ increases. However, one can consider two procedures of raising the energy: keeping e fixed and keeping $r_{\min}$ fixed. Only in the first case does the shell probe small scales whereas the results above are the same in both cases. The details of radiation at early time may be sensitive to the small scales but the resultant distribution of charges at late time may not.

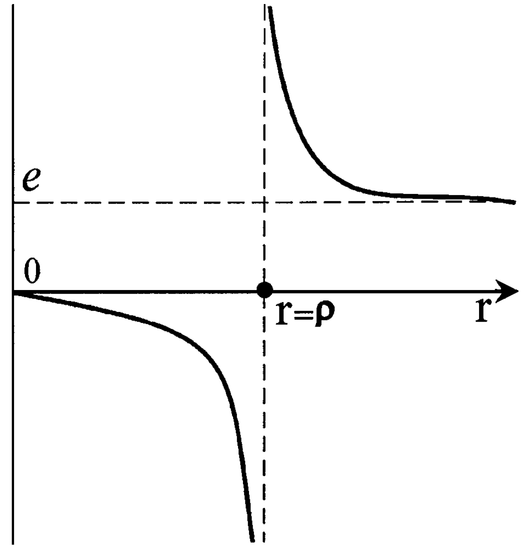

FIG. 2. The distribution $\mathbf{e}(t, r)$ for a given t.

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