Spin-Spin Interaction and Spin Squeezing in an Optical Lattice

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We show that, by displacing two optical lattices with respect to each other, we may produce interactions similar to the ones describing ferromagnetism and antiferromagnetism in condensed matter physics. We also show that particularly simple choices of the interaction lead to spin squeezing, which may be used to improve the sensitivity of atomic clocks. Spin squeezing is generated even with partially, and randomly, filled lattices, and our proposal may be implemented with current technology.

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Simulation of quantum many-body problems on a classical computer is difficult because the size of the Hilbert space grows exponentially with the number of particles. As suggested by Feynman [1], the growth in computational requirements is only linear on a quantum computer [2], which is itself a quantum many-body system, and such a computer containing only a few tens of quantum bits may outperform a classical computer. A quantum computer aimed at the solution of a quantum problem is expected to be easier to realize in practice than a general purpose quantum computer, because the desired solution is governed by physical interactions which are constrained, e.g., by locality [1,3]. In essence, such a quantum computer is a *quantum simulator* with the attractive feature that the experimentalist can control and observe the dynamics more precisely than in the physical system of interest. In this Letter we describe how atoms in an optical lattice may be manipulated to simulate spin-spin interactions which are used to describe ferromagnetism and antiferromagnetism in condensed matter physics. We also show that with a specific choice of interaction we may generate spin-squeezed states [4] which may be used to enhance spectroscopic resolution [5], e.g., in atomic clocks. Our tasks are achieved with only a few manipulations of the system and may thus be performed within a time much shorter than the coherence time of ground state atoms in optical lattices [6].

In Refs. [6,7] two different methods to perform a coherent evolution of the joint state of pairs of atoms in an optical lattice were proposed. Both methods involve displacement of two identical optical lattices with respect to each other. Each lattice traps one of the two internal states $|0\rangle$ and $|1\rangle$ of the atoms. Initially, the two lattices are on top of each other and the atoms are assumed to be cooled to the vibrational ground state in the lattices. The lattice containing the $|1\rangle$ component of the wave function is now displaced so that, if an atom (at the lattice site k) is in $|1\rangle$, it is transferred to the vicinity of the neighboring atom (at the lattice site $k+1$) if this is in $|0\rangle$, causing an interaction between the two atoms (see Fig. 1). The procedures described in this Letter follow the proposal in Ref. [6] where the atoms interact through controlled collisions. Also the optically induced dipole-dipole interactions proposed in [7] may be adjusted to fit into this framework. After the interaction, the lattices are returned to their initial position and the internal states of each atom may be subject to single particle unitary evolution. The displacement and the interaction with the neighbor yields a certain phase shift ϕ on the $|1\rangle_k|0\rangle_{k+1}$ component of the wave function; i.e.,

$$\begin{aligned} |0\rangle_k|0\rangle_{k+1} &\rightarrow |0\rangle_k|0\rangle_{k+1}\,, \\ |0\rangle_k|1\rangle_{k+1} &\rightarrow |0\rangle_k|1\rangle_{k+1}\,, \\ |1\rangle_k|0\rangle_{k+1} &\rightarrow e^{i\phi}|1\rangle_k|0\rangle_{k+1}\,, \\ |1\rangle_k|1\rangle_{k+1} &\rightarrow |1\rangle_k|1\rangle_{k+1}\,, \end{aligned} \tag{1}$$

where $|a\rangle_k$ ($a = 0$ or 1) refers to the state of the atom at the kth lattice site. In [6] it is suggested to build a general purpose quantum computer in an optical lattice based on the two-atom gates in Eq. (1) and single atom control, which is possible by directing a laser beam on each atom. We shall show that, even without allowing access to the individual atoms, the lattice may be used to perform a highly nontrivial computational task: simulation of magnetism.

Our two level quantum systems conveniently describe spin 1/2 particles with the two states $|0\rangle_k$ and $|1\rangle_k$

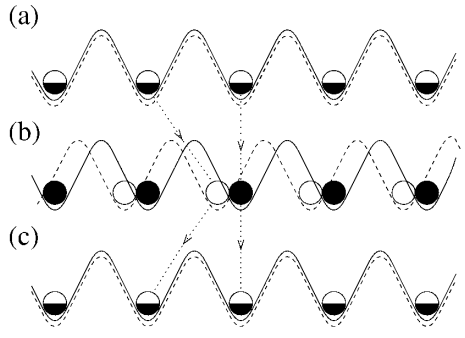

FIG. 1. (a) Two overlapping lattices trapping the two internal states $|0\rangle$ (black circle) and $|1\rangle$ (white circle). (b) The lattices are displaced so that if an atom is in the $|1\rangle$ state, it is moved close to the neighboring atom if this is in $|0\rangle$ causing an interaction between the two atoms. (c) The lattices are returned to their initial position, where the noninteracting atoms may be driven by external fields.

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representing $|m\rangle_k = |-1/2\rangle_k$ and $|1/2\rangle_k$, where states $|m\rangle_k$ are eigenstates of the $j_{z,k}$ operator $j_{z,k}|m\rangle_k = m|m\rangle_k$ ($\hbar = 1$). The phase-shifted component of the wave function in Eq. (1) is identified by the operator $(j_{z,k} + 1/2)(j_{z,k+1} - 1/2)$, and the total evolution composed of the lattice translations and the interaction induced phase shift may be described by the unitary operator e^{-iHt} with the Hamiltonian $H = \chi(j_{z,k} + 1/2)(j_{z,k+1} - 1/2)$ and time $t = \phi/\chi$. In a filled lattice the evolution is described by the Hamiltonian $H = \chi\sum_k(j_{z,k} + 1/2)(j_{z,k+1} - 1/2)$, and if we neglect boundary terms the Hamiltonian reduces to

$$H_{zz} = \chi \sum_{\langle k,l\rangle} j_{z,k} j_{z,l}\,, \tag{2}$$

where the sum is over nearest neighbors. By appropriately displacing the lattice, we may extend the sum to nearest neighbors in two and three dimensions. H_{zz} coincides with the Ising-model Hamiltonian [8,9] introduced to describe ferromagnetism. Hence, by elementary lattice displacements, we perform a quantum simulation of a ferromagnet (or antiferromagnet depending on the sign of χ).

A more general Hamiltonian of the type

$$H_f = \sum_{\langle k,l\rangle} \chi j_{z,k} j_{z,l} + \eta j_{x,k} j_{x,l} + \lambda j_{y,k} j_{y,l} \tag{3}$$

may be engineered using multiple resonant pulses and displacements of the lattices: A resonant $\pi/2$ pulse acting simultaneously on all atoms rotates the j_z operators into j_x operators, $e^{ij_{y,k}\pi/2} j_{z,k} e^{-ij_{y,k}\pi/2} = j_{x,k}$. Hence, by applying $\pi/2$ pulses, in conjunction with the displacement sequence, we turn H_{zz} into H_{xx}, the second term in Eq. (3). Similarly, we may produce H_{yy}, the third term in Eq. (3), and by adjusting the duration of the interaction with the neighbors we may adjust the coefficients χ, η, and λ to any values. We cannot, however, produce H_f by simply applying H_{zz} for the desired time t, followed by H_{xx} and H_{yy}, because the different Hamiltonians do not commute. Instead we apply a physical implementation of a well-known numerical scheme: the split operator technique. If we choose short time steps, i.e., small phase shifts ϕ in Eq. (1), the error will be only of order ϕ^2, and, by repeated application of H_{zz}, H_{xx}, and H_{yy}, we may stroboscopically approximate H_f.

For a few atoms the system may be simulated numerically on a classical computer. In Fig. 2 we show the propagation of a spin wave in a one-dimensional string of 15 atoms which are initially in the $|-1/2\rangle$ state. For illustrational purposes we assume that the central spin is flipped at $t = 0$. The Hamiltonian (3) which can be implemented without access to the individual atoms then causes a spin wave to propagate to the left and right. The figure shows the evolution of $\langle j_{z,k}\rangle$ for all atoms, obtained by repeatedly applying the Hamiltonians H_{zz}, H_{xx}, and H_{yy} with $\chi = \eta = \lambda$ and periodic boundary conditions. Small time steps $dt = 0.1\chi^{-1}$ result in a stroboscopic

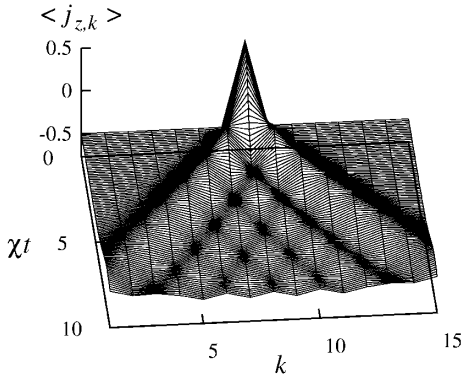

FIG. 2. Propagation of a spin wave in a one-dimensional string. The central atom is flipped at $t = 0$, and repeated application of H_{zz}, H_{xx}, and H_{yy} results in a wave propagating to the left and right. The figure shows the evolution of $\langle j_{z,k}\rangle$ for all atoms (k).

approximation almost indistinguishable from the results of a direct numerical integration of H_f.

A host of magnetic phenomena may be simulated on our optical lattice: spin waves, solitons, topological excitations, two magnon bound states, etc. Models for magnetic phenomena have interesting thermodynamic behavior, and we propose to carry out calculations for nonvanishing temperature by optically pumping a fraction of the atoms to the $|1/2\rangle$ state. The randomness of the pumping introduces entropy into the system and produces a microcanonical [9] realization of a finite temperature. Other procedures for introducing a nonvanishing temperature are described in Ref. [3]. The results of the simulation may be read out by optical diffraction of light, sensitive to the internal atomic states. Although individual atoms may not be resolved, optical detection may also be used directly to resolve magnetic structures on a spatial scale of a few lattice periods.

We now show how to generate spin-squeezed states using the same techniques as discussed above. Signals obtained in spectroscopic investigations of a sample of two level atoms are expressed by the collective spin operators $J_i = \sum_k j_{i,k}$, and their quantum mechanical uncertainty limits the measurement accuracy, and, e.g., the performance of atomic clocks. In standard spectroscopy with N uncorrelated atoms starting in the $|-1/2\rangle$ state, the uncertainties $\Delta J_x = \sqrt{\langle J_x^2\rangle - \langle J_x\rangle^2}$ and ΔJ_y are identical, and the standard quantum limit resulting from the uncertainty relation of angular momentum operators

$$(\Delta J_x)^2(\Delta J_y)^2 \geq |\langle J_z/2\rangle|^2 \tag{4}$$

predicts a spectroscopic sensitivity proportional to $1/\sqrt{N}$. Polarization rotation spectroscopy and high precision atomic fountain clocks are now limited by this sensitivity [10,11]. In [4] it is suggested to produce spin-squeezed states which redistribute the uncertainty unevenly between components such as J_x and J_y in (4), so that measurements, sensitive to the component with reduced uncertainty,

become more precise. Spin squeezing resulting from absorption of nonclassical light has been suggested [12] and demonstrated experimentally [13]. Reference [4] presents an analysis of squeezing obtained from the nonlinear couplings $H = \chi J_x^2$ and $H = \chi(J_x^2 - J_y^2)$. For neutral atoms, such a coupling has been suggested in the spatial overlap of two components of a Bose-Einstein condensate [14]. Spin squeezing in an optical lattice has two main advantages compared to the condensates: The interaction can be turned on and off easily, and the localization at lattice sites increases the density and thus the interaction strength. The product J_x^2 involves terms $j_{x,k}j_{x,l}$ for all atoms k and l, and this coupling may be produced by displacing the lattices several times so that the $|1/2\rangle$ component of each atom visits every lattice site and interacts with all other atoms. In a large lattice such multiple displacements are not desirable. We shall show, however, that substantial spin squeezing occurs through interaction with only a few nearby atoms, i.e., for Hamiltonians

$$H = \sum_{k,l} \chi_{k,l} j_{x,k} j_{x,l} \tag{5}$$

and

$$H = \sum_{k,l} \chi_{k,l} (j_{x,k} j_{x,l} - j_{y,k} j_{y,l}), \tag{6}$$

where the coupling constants $\chi_{k,l}$ between atoms k and l vanish except for a small selection of displacements of the lattices [15].

Expectation values of relevant angular momentum operators and the variance of the spin operator $J_\theta = \cos(\theta)J_x + \sin(\theta)J_y$ may be calculated for an initially uncorrelated state with all atoms in $|-1/2\rangle$, propagated by the simple coupling (5). If each atom visits one neighbor $\chi_{k,l} = \chi\delta_{k+1,l}$, we get the time dependent variance of the spin component $J_{-\pi/4} = (1/\sqrt{2})(J_x - J_y)$ by a lengthy, but straightforward, calculation:

$$(\Delta J_{-\pi/4})^2 = \frac{N}{4}\left[1 + \frac{1}{4}\sin^2(\chi t) - \sin(\chi t)\right]. \tag{7}$$

The mean spin vector is in the negative z direction and has the expectation value

$$\langle J_z \rangle = -\frac{N}{2}\cos^2(\chi t). \tag{8}$$

For small values of χt, $\Delta J_{-\pi/4}$ decreases linearly with χt, whereas $|\langle J_z\rangle|$ decreases proportional to $(\chi t)^2$; hence, $\Delta J_{-\pi/4}$ falls below $|\langle J_z/2\rangle|$, and the spin is squeezed.

In Fig. 3 we show numerical results for 15 atoms in a one-dimensional lattice with periodic boundary conditions. Figure 3(a) shows the evolution of $(\Delta J_\theta)^2$ when we apply the coupling (5) and visit 1, 2, and 3 neighbors. We assume the same phase shift for all collisions; i.e., all nonvanishing $\chi_{k,l}$ are identical. The squeezing angle $\theta = -\pi/4$ is optimal for short times $\chi t \ll 1$. For longer times the optimal angle deviates from $-\pi/4$, and we plot the variance $(\Delta J_\theta)^2$ minimized with respect to the angle θ. If $(1/\sqrt{2})(e^{-i\theta/2}|1/2\rangle + e^{i\theta/2}|-1/2\rangle)$ is rotated into $|1/2\rangle$, sub-binomial counting statistics of the $|1/2\rangle$ population provides an easy accessible experimental signature of squeezing in J_θ.

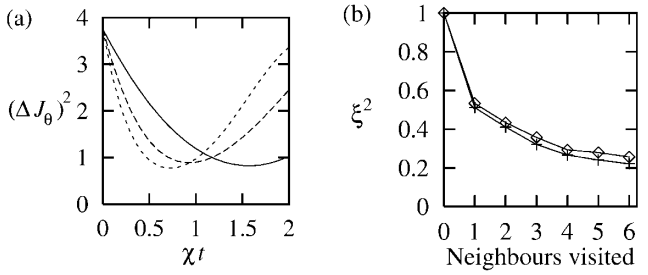

FIG. 3. Squeezing in a one-dimensional lattice with 15 atoms. (a) Evolution of $(\Delta J_\theta)^2$ with the coupling (5) and interaction with 1, 2, and 3 neighbors (full, dashed, and short-dashed line, respectively). (b) The optimal value of the squeezing parameter ξ^2 obtained with the coupling (5) ($\diamond$) and (6) (+).

In [5] it is shown that, if spectroscopy is performed with N particles, the reduction in the frequency variance due to squeezing is given by the quantity

$$\xi^2 = \frac{N\langle \Delta J_\theta\rangle^2}{\langle J_z\rangle^2}. \tag{9}$$

In Fig. 3(b) we show the minimum value of ξ^2 obtained with the couplings (5) and (6) as functions of the number of neighbors visited. Figure 3(b) shows that the coupling (6) produces better squeezing than (5). The coupling (5), however, is more attractive from an experimental viewpoint. Since all $j_{x,k}$ operators commute, we do not have to apply several displacements with infinitesimal durations to produce the desired Hamiltonian. We may simply displace the atoms so that they interact with one neighbor to produce the desired phase shift ϕ, and then go on to interact with another neighbor.

Similar to the analytic expression for ξ^2 obtained from (7) and (8), the results shown in Fig. 3(b) are independent of the total number of atoms as long as this is much larger than the number of neighbors visited. When all lattice sites are visited, we approach the results obtained in Ref. [4], i.e., a variance scaling as $N^{1/3}$ and a constant for the couplings (5) and (6).

So far we have assumed that the lattice contains one atom at each lattice site and that all atoms are cooled to the vibrational ground state. It has been suggested that this may be achieved by filling the lattice from a Bose-Einstein condensate [16]. The present experimental status in optical lattices is that atoms can be cooled to the vibrational ground state in 2D [17]. A mean filling factor of unity in 3D is reported in [18], but when at most a single atom is permitted at each lattice site a mean occupation of 0.44 is achieved.

The interaction in a partially filled lattice may be described by the Hamiltonian $H = \sum_{k,l} \chi_{k,l} h_k (j_{z,k} + 1/2) h_l (j_{z,l} - 1/2)$, where the stochastic variable h_k is

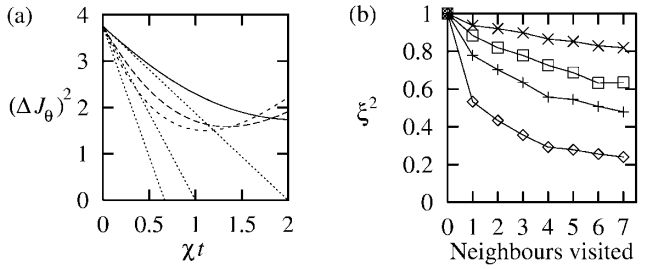

FIG. 4. Spin squeezing in a partially filled one-dimensional lattice containing 15 atoms. (a) Evolution of $(\Delta J_\theta)^2$ in a lattice with a filling factor $p = 50\%$ and displacements to 1, 2, and 3 neighboring sites (full, dashed, and short-dashed curve, respectively). Dotted lines represent the predictions from Eq. (10). (b) Minimum attainable squeezing parameter ξ^2 for filling factors $p = 100\%$ ($\diamond$), 50% (+), 25% ($\square$), and 10% ($\times$) as functions of the number of sites visited.

1 (0) if a lattice site is filled (empty). If we displace the atoms so that $\chi_{k,l}$ is symmetric in k and l, we produce the Hamiltonian $H = \sum_{k,l} \chi_{k,l} h_k j_{x,k} h_l j_{x,l}$. This Hamiltonian models magnetism in random structures, and it might shed light on morphology properties, and, e.g., percolation [19]. Here we shall restrict our analysis to spin-squeezing aspects, since these are both of practical interest, and they represent an ideal experimental signature of the microscopic interaction.

In Fig. 4 we show the result of a simulation of squeezing in a partially filled one-dimensional lattice. Each lattice site contains an atom with a probability p, and the size of the lattice is adjusted so that it contains 15 atoms. In Fig. 4(a) we show the decrease in the variance of J_θ, averaged over 20 realizations and minimized with respect to θ. Lines indicate the predictions from the time derivatives at $t = 0$:

$$\frac{d}{dt}(\Delta J_{-\pi/4})^2 = -\frac{1}{2}\sum_{k,l} \chi_{k,l}\langle h_k h_l\rangle , \qquad (10)$$

where $\langle h_k h_l\rangle$ denotes the ensemble average over the distribution of atoms in the lattice, i.e., the two atom correlation function. In Fig. 4(b) we show the minimum value of ξ^2 for different filling factors p as a function of the number of neighbors visited. The calculations confirm that, even in dilute lattices, considerable squeezing may be achieved by visiting a few neighbors.

In conclusion, we have suggested a method to simulate condensed matter physics in an optical lattice, and we have shown how the dynamics may be employed to produce spin squeezing. We emphasize the moderate experimental requirement for our scheme. With the two internal states represented as hyperfine structure states in alkaline atoms, all spin rotations may be performed by Raman or rf pulses acting on all atoms simultaneously, and lattice displacements may be performed by simply rotating the polarization of the lasers [7]. With the parameters in [6], the duration of the sequence in Fig. 1 can be as low as a few microseconds. Following our suggestion, spin squeezing may be produced in dilute optical lattices, and implementation is possible with current technology. The decrease in projection noise has several promising applications in technology and quantum physics, and it provides an experimental signature of the microscopic interaction between the atoms.

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