New Method for the Control of Fast Chaotic Oscillations

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We introduce a new method of controlling chaos that retains the essential features of occasional proportional feedback, but is much simpler to implement. We demonstrate control on a simple piecewise-linear Rossler circuit operating near 1 kHz and a Colpitts oscillator with a fundamental frequency of 19 MHz. As a result of the simplicity of our technique, control of chaos has been accomplished for the fastest chaotic system reported to date.

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In this Letter, we present a new chaos control algorithm that enables practical control in fast chaotic systems. Our approach uses a pulsewidth-modulated control signal derived from the transit time of the system state through a prescribed window. Although pulsewidth modulation has been considered previously [1,2], the present implementation is significant since its simplicity enables applications demanding very high frequency response and minimal latency.

The development of chaos control algorithms originated with Ott, Grebogi, and Yorke [3] in 1990. Since then, variants of the Ott-Grebogi-Yorke (OGY) control scheme have been used to control mechanical systems [4], electronic systems [5], solid-state lasers [6], chemical systems [7], and even heart tissue [8]. A common feature of all closed-loop chaos control algorithms is the use of very small perturbations to stabilize unstable steady states or unstable periodic orbits (UPO), which are abundant in chaotic attractors. Beyond just expanding the stable parameter space, one can take advantage of the natural complexity of the chaos by controlling the system to produce desired communication signals [9]. However, the feasibility of using chaos controllers in practical communication devices depends on the complexity and efficiency of the control algorithm, which will impact device size, power consumption, channel bandwidth, and overall system performance. In particular, controllers for communications devices must be fast, operating at radio frequencies (1 MHz to tens of GHz) and, in the application to semiconductor lasers, with latencies below 1 ns.

The chaos control process requires measurement of the system state, generation of a control signal, and the application of the control signal to an accessible system parameter. The total time it takes to accomplish these tasks is the *latency* of the controller. While the *frequency response* of the controller sets an upper limit on the natural frequency of devices that may potentially be controlled, the latency limits which of the UPOs, if any, may actually be stabilized. Latency was considered a key issue in the failure of an experiment to control a semiconductor laser [10].

The original OGY control scheme requires knowledge of the return map near a fixed point, x^*, corresponding to the targeted UPO. This technique exploits local linearity of the map about the fixed point. The map may be determined from an analytical model or from observations of time series. For a return near the fixed point, a system parameter is modified such that the system state is moved onto the stable manifold of the fixed point. This scheme requires performing several involved vector calculations to generate the control signal. These calculations, which usually are implemented with digital processors, can lead to large latencies. Hence, OGY is customarily used for only slow systems.

A simpler, scalar version of OGY, occasional proportional feedback (OPF), was introduced by Hunt [5] to enable control of very high period orbits. In OPF a system parameter, p, is perturbed by $\delta p = \gamma(x_s - x^*)\Theta(\tau)$, where x_s is a system state variable sampled when the system crosses a specified plane of section in phase space, x^* is an intersection point of the targeted UPO with the same plane, $\Theta(\tau)$ is a pulse of fixed duration τ, and γ is a gain factor. The control pulse is initiated only when the system state crosses the chosen plane of section within a predefined window W containing x^*. Different UPOs can often be found and stabilized simply by sweeping x^* through the attractor. Owing to its simplicity the OPF technique does not require a digital processor and can be implemented at higher speeds in an analog circuit. Chaos control using OPF has been demonstrated on systems with natural frequencies as high as 10^5 Hz, with controller latencies on the order of 10 μs [6].

Pyragas [11] suggested an alternate analog method of controlling chaos via synchronization with a delayed signal. The control signal is proportional to the difference of the current state and the state at some earlier time, with the delay being set to the period of the desired UPO. The method was later developed into extended time-delay autosynchronization (ETDAS) with high-speed applications in mind. Using ETDAS, the chaotic dynamics of a diode resonator driven at 10.1 MHz were stabilized [12], with a controller latency of 10 ns [10]. Until now, this is the fastest system reported to be stabilized using chaos control. Importantly, ETDAS also has an optical implementation which may be useful for controlling laser chaos (see Ref. [13] for a review).

We introduce a new chaos control scheme that eliminates several of the steps used by OPF and provides particularly simple signal generation. The result is an *occasional* feedback algorithm that can be implemented at unprecedented speeds. Our technique removes the sample and hold (x_s) and difference ($x_s - x^*$) operations performed by OPF. Instead, the technique uses a fixed amplitude control pulse but makes the pulse duration dependent on the transit time of the system through a specified volume in phase space. Explicitly, the method delivers perturbations such that

$$\delta p = \begin{cases} \gamma & \text{if } \mathbf{x}(t) \in W \\ 0 & \text{otherwise,} \end{cases} \tag{1}$$

where $\mathbf{x}(t)$ is the state vector, and W is the specified volume in phase space. The control pulse turns on with fixed amplitude γ when the system state enters the control window W and turns off when the system leaves the window. As with OPF, the polarity and amplitude of γ are chosen to overcome growth of the trajectory in the unstable direction; however, unlike OPF, the polarity and amplitude of γ are set initially and do not vary with each crossing through W. The strength of the perturbation is dependent on the fixed amplitude of γ *and* on the width of the control pulse, which is determined by the transit time of the system through the window on each crossing. Since the control signal power must approach the noise limit as the system settles into the UPO, the boundary of the window must be positioned such that the transit time through the window approaches zero when the system follows a UPO. As an example, for a simple rectangular window in two states, one vertex of the rectangle may be placed on the desired UPO as shown in Fig. 1(a). In this example, the trajectories further from the targeted UPO exhibit longer transit times through the window. The effect of the control γ is to push these trajectories toward the UPO. An experimental realization of this window is achieved with a circuit such as shown in Fig. 1(b).

For stabilization of UPOs in the ideal case of zero latency, infinite bandwidth, and sufficient γ for the controller output, the placement and shape of the window W will have the following properties: (i) The intersection of W and the targeted UPO has zero length. (ii) The transit time through W increases along the local unstable directions.

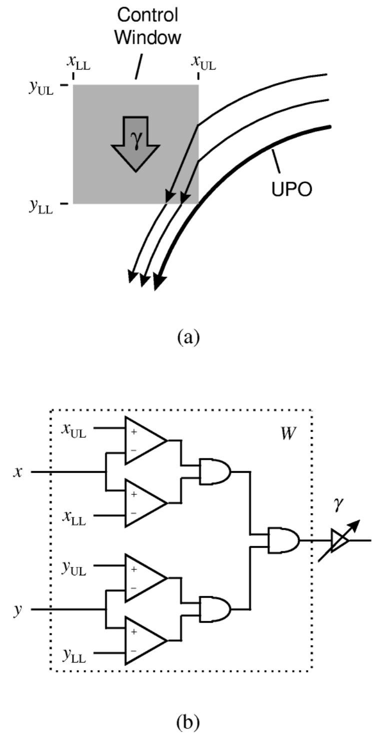

FIG. 1. (a) Control using a rectangular window in two system states. Trajectories that enter the window are perturbed toward the UPO with a total energy determined by the transit time through the window. (b) Circuit realization of controller.

If the controller latency is significant compared to the period of the UPO, or if γ is too small, deviations from these conditions may be expected. For many systems such as Rössler's simply folded band attractor [14], the trajectory diverges from the UPO by passing through alternate sides of the saddle on each orbit. Thus, a single window placed on either side of the saddle is sufficient to attain control. For more complex systems, multiple windows W_i may be required. Each W_i has associated with it a corresponding amplitude γ_i.

We confirmed experimental operation of this control technique using an electronic controller to stabilize UPOs in a low-frequency chaotic circuit. The controller circuit, shown in Fig. 1(b), uses standard op amps (TL084) for the buffers, comparators, and amplifier. The logic gate was realized using diodes (1N914). The controller circuit provides a rectangular window in two state voltages, and the window voltage limits are set using a versatile, computer-controlled interface.

We applied the controller to a piecewise-linear circuit described by Carroll [15]. Uncontrolled, this circuit exhibits the attractor shown in Fig. 2(a), which is a simply folded band structure similar to that seen in Rössler's oscillator [14]. The window was scanned through the attractor to find and stabilize different UPOs. Shown in Fig. 2(b) is a period-1 UPO for this circuit that was stabilized using the electronic controller configured with

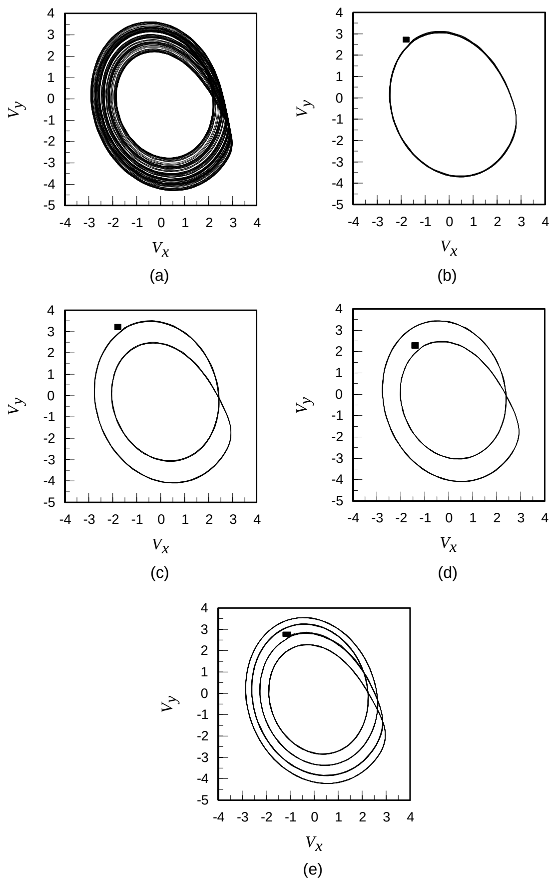

FIG. 2. Experimental phase plots for the piecewise linear Rössler circuit: (a) uncontrolled; (b) period 1; (c) period 2; (d) period 2, with a different window placement; and (e) period 4. The rectangle in (b)–(e) shows the window placement relative to the UPO.

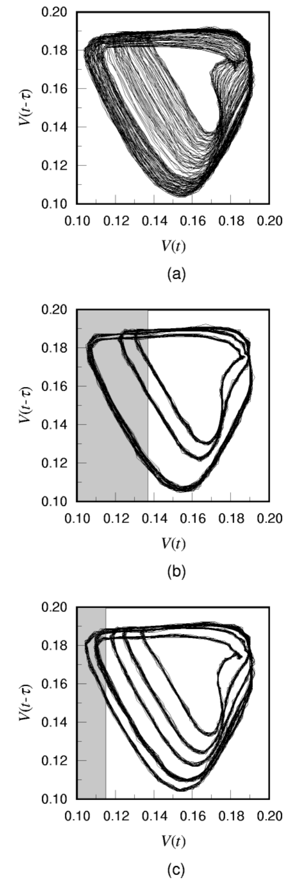

FIG. 3. Experimental phase plots for a 19.1 MHz Colpitts oscillator: (a) uncontrolled; (b) period 4; and (c) period 6. The plots are made from an embedding of the observed time series data, sampled at 1 ns intervals, using an embedding time $\tau = 13$ ns. The control windows are shown in gray.

the window shown as the black rectangle. This UPO has a frequency of 1.2 kHz. In Fig. 2(c) and 2(d), the same period-2 UPO is stabilized using two different window positions. In Fig. 2(e), a period-4 UPO is stabilized using a single window; however, stabilizing even higher period UPOs may require more windows. Strong evidence that the stabilized orbits shown in Fig. 2 are actual UPOs was provided by a local minimum in the control signal power as a function of window placement.

The simplicity of this control technique makes application to fast chaotic systems appealing. To this end, we also demonstrated control of a much faster, chaotic Colpitts oscillator. The oscillator was based on the circuit given by Kennedy [16]; however, we scaled the component parameters so that the fundamental frequency was near 19 MHz. A projection of the attractor generated by an embedding of time series data from this circuit is shown in Fig. 3(a). For both simplicity and low latency, a controller featuring a window on only one state variable was constructed. Windowing on a single state variable provides less selectivity of UPOs compared to a window on more state variables. Nevertheless, we found that a single variable window was sufficient to achieve control of at least two different UPOs for the Colpitts oscillator. The controller was built using fast, commercially available buffers (BUF600), comparators (Max 9685), logic gates (MC10H104), amplifiers (AD8012), and a custom printed circuit board designed for rf operation. The controller has a frequency response of 220 MHz measured with a sine wave input, and the latency was measured to be 4.4 ns.

Figures 3(b) and 3(c) show the observed phase portraits of the Colpitts oscillator for stabilized period-4 and period-6 UPOs, and the respective control windows are shown in gray. For both periodic states shown in Fig. 3, the fundamental, period-1 frequency is 19.1 MHz. These UPOs were obtained for the same circuit values but for different window placements within the attractor. The γ amplitude adjustment was not readily accessible for this experiment. Thus the same amplitude and polarity of γ were used to stabilize both states. Controller latency and low perturbation strength are thought to be the primary reasons for penetration of the UPOs into the control windows. It is seen from Fig. 3 that the period-4 UPO penetrates farther into its control window than the period-6 UPO. This may indicate greater instability of the period-4 UPO relative to the period-6 UPO. With a γ adjustment and a control window using more than one state variable, we believe it is possible to reduce the overlap of the UPO trajectories with their control windows, as well as to stabilize other UPOs for this system.

Local minima in the control power as a function of window position were not observed for the stabilized orbits shown in Fig. 3. Alternatively, evidence that the stabilized states are UPOs of the uncontrolled system was found by matching the periodic states to portions of the time series data for the uncontrolled state. The stabilized and free running oscillator time series are found to overlap, to within the noise uncertainty, for at least one full period before the uncontrolled state wanders off the periodic state. We also note that the stabilized states in Figs. 3(b) and 3(c) are completely bounded by the uncontrolled state in Fig. 3(a).

In summary, we have shown that a transit-time pulse-width modulation technique is sufficient to perform chaos control on two chaotic systems, operating at different frequencies. The simplicity of the technique allowed us to implement a controller with wide frequency response ($>$200 MHz) and very small latency ($<$5 ns). Control of distinct UPOs was demonstrated at over 19 MHz for a fast electronic system. At present we are seeking faster chaotic systems upon which to attempt stabilization of UPOs with our existing controller. We expect that an integrated circuit form of our controller can exceed a bandwidth of 1 GHz and latency below 1 ns, suggesting its use in control of semiconductor lasers.

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