Measurement of Parametric Correlations in Spectra of Resonating Quartz Blocks

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Random matrix theory predicts parametric correlations for chaotic or disordered systems which are as universal as their spectral counterparts. However, an experimental verification of parametric correlations is, in the vast majority of systems, a difficult task, because the motion of individual levels has to be followed as an external parameter is varied. We present the first statistically highly significant result for a parametric correlator that uses solely experimental information. To this end, we measure the motion of acoustic resonances in quartz blocks as a function of its uniform temperature. We find some deviation between theoretical prediction and experimental results whose possible origin is discussed.

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There is overwhelming evidence for the Bohigas-Gianonni-Schmit conjecture [1] stating that the spectral fluctuations of a quantum system whose classical counterpart is fully chaotic are described by random matrix theory (RMT) [2]. This is true for the energy correlators on the scale of the local mean level spacing up to a scale which is set by semiclassical dynamics. In particular, if the system is invariant under time reversal and free of Kramers degeneracies, the fluctuations are modeled by the Gaussian orthogonal ensemble (GOE). Furthermore, it has been shown that the statistical concepts of quantum chaos do also apply with high accuracy to classical waves, such as elastomechanical or electromagnetic waves in three dimensions; for a review, see Ref. [3]. This is remarkable because these systems show, unlike the scalar Schrödinger equation, mode conversion, i.e., ray splitting, and have in general different boundary conditions.

Do chaotic systems possess observables other than energy correlators which also show a high degree of universality? Moreover, if the answer is in the affirmative, do these features carry over to classical wave phenomena? In recent years, it has been shown that the parametric motion of energy levels in quantum systems [4] and the parametric correlations resulting thereof do show universal features; see the recent review in Ref. [3]. We consider a system as a function of some parameter X, such as geometric shape, pressure, temperature, etc. It is assumed that the system is fully chaotic and its spectral fluctuations are described by the GOE for every fixed value of X. Since the Wigner–von Neumann level repulsion governs the spectrum of chaotic quantum systems, a plot of the eigenenergies $E_n(X), n = 1, 2, \dots$ versus X shows avoided level crossings. The statistics of this "level motion" exhibits universal characteristics.

Unfortunately, a statistically significant measurement of those parametric correlations is a highly nontrivial task: to experimentally determine the functions $E_n(X)$, high resolution spectra have to be measured for various, preferably many, different values of the parameter X. We are not aware of any such measurement. The work of Simons *et al.* [5] on the hydrogen atom in a strong magnetic field is a combined experimental and theoretical study. Nevertheless, it is the strongest available test of parametric correlations in a real system. Most of the other evidence is numerical, such as the study of irregularly shaped quantum dots by Bruus *et al.* [6]. In elastomechanical systems, however, both requirements can be met: high resolution and a fine scan over the external parameter X. In this Letter, we present the first full-fledged experimental study of a parametric correlation function. Our system is a resonating, monocrystalline quartz block. As the external parameter X we use the uniform temperature of the block, T. The six independent elastic constants vary at considerably different rates with temperature [7]. As these constants determine the density of the material and thereby the velocity of elastic waves, the spectrum of the block also changes with temperature T. Thus, by measuring spectra at different temperatures, we experimentally obtain the functions $E_n(X)$, where, in our case, E_n stands for a frequency and $X = T$. In studying this system, we also address the second of the questions posed at the beginning of the previous paragraph.

Universal features can be expected only on the scale of the local mean level spacing $D(X)$ at any given X, implying that the energies have to be unfolded by setting $\varepsilon_n(X) = E_n(X)/D(X)$. The parametric level motion itself has to be unfolded, too. The appropriate scale is given by the level velocity $d\varepsilon_n(X)/dX$. The new, dimensionless parameter in the interval $[X_i, X_f]$ is defined by

$$x = \int_{X_i}^{X_f} \sqrt{\langle [d\varepsilon_n(X)/dX]^2 \rangle}\, dX\,, \qquad (1)$$

where the average is performed over all levels. In Ref. [8] the universality of this parameter unfolding is discussed in detail. Because of the importance of the avoided level crossings, it is useful to study the unfolded curvatures

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$k_n = \pi^{-1} d^2\varepsilon_n/dx^2$. Zakrzewski and Delande [9] conjectured the universal form

$$P(k) = \frac{1}{2(1 + k^2)^{3/2}} \tag{2}$$

for the distribution of these curvatures which was analytically confirmed by von Oppen [10]. We suppress the level index n. Since a much longer range must be covered in parameter space, it is harder to measure the velocity-velocity correlator introduced by Szafer and Altshuler [11],

$$c(x) = \left\langle \frac{d\varepsilon_n(\bar{x} - x/2)}{d\bar{x}} \frac{d\varepsilon_n(\bar{x} + x/2)}{d\bar{x}} \right\rangle. \tag{3}$$

After averaging over all levels, it is translationally invariant, i.e., depends only on the separation x between two points in the parameter space, but not on the midpoint $\bar{x}$.

In the measurements, we used a monocrystalline rectangular quartz block with dimensions $14 \times 25 \times 40\ \mathrm{mm}^3$. An octant of radius 8.0 mm was removed from one corner such that the block acquired the shape of a three-dimensional Sinai billiard. This geometry ensured, first, a full breaking of all D_3 point-group symmetries present in the crystal and, second, *spectral* fluctuation properties of GOE type. The experimental setup described in Refs. [12,13] was employed, which guarantees a very high resolution of the data. The quality factor $Q = f/\Delta f$ where f is the frequency and Δf the width of a given resonance is, on average, roughly 10^5. This setup is supplemented with a high quality thermostat which controls the temperature inside the pressure chamber with a precision of better than 0.01 °C during the measurements. A description will be given elsewhere [14]. To trace the motion of the levels as a function of the temperature, we measured a set of transmission spectra, all in the same frequency interval, at different, but fixed and controlled temperatures. For the accumulation of data, the optimal window in the two dimensional space defined by frequency and temperature had to be chosen. Importantly, the strongly increasing level density sets an upper limit for the frequency. It turned out to be most efficient to measure at temperatures ranging from 45 to 145 °C with increments of 1.0 °C in the frequency interval from 980 to 1071 kHz. Thus, 101 spectra were taken, each containing 709 resonances to be traced. The first of these resonances was approximately the 2400th, counted from the ground state.

To remove the dependence on the level density, each spectrum is unfolded individually as in Ref. [12]. The experimental cumulative frequency density $N(f)$, the "staircase" is fitted with a third order polynomial $N_{\mathrm{av}}(f)$, which then defines the dimensionless frequency scale. To check that the spectra exhibit random matrix fluctuations on this scale, we work out the spacing distribution and the spectral rigidity. Both agree with the GOE prediction, the latter up to interval lengths of about $L_{\max}^{(P)} \simeq 50$ in units of the mean level spacing. However, there is still another structure in our data. In Fig. 1, the difference $N(f) - N_{\mathrm{av}}(f)$ is plotted. The large scale oscillations are not consistent with pure random matrix fluctuations—they are rather due to the acoustic equivalence of "bouncing ball" modes [15]. In quantum chaos, those modes can be understood in a semiclassical approximation as a family of orbits of a particle bouncing between parallel walls. The corresponding modes in our system are richer and more complicated due the existence of longitudinal and transverse waves and the anisotropy of the crystal. As there is no theory available, we have to remove this structure phenomenologically. We use a Fourier transform method similar to the one described in Ref. [16]. The Fourier transform shown in Fig. 1 reveals the influence of the bouncing-ball-like modes as big peaks near the origin. We choose a cutoff of $t_c = 100\ \mu\mathrm{s}$ and transform these peaks back into frequency space. This gives a smooth approximation of the oscillations $N_{\mathrm{osc}}(f)$ which is also subtracted from $N(f) - N_{\mathrm{av}}(f)$. The spectral rigidity worked out for this spectrum shows good agreement with GOE prediction up to interval lengths of about $L_{\max}^{(F)} \simeq 80$ in units of the mean level spacing. Since this number is slightly dependent on t_c, we use it only as a guideline in studying the influence of the bouncing-ball-like modes on the statistical observables. We mention in passing that this intermediate result actually improves the confirmation of GOE spectral fluctuations in elastomechanical systems performed in Ref. [12]. In particular, since the level motion is followed over a sizable parameter range, we are confident that no single level was missed in the experiment.

After the unfolding, a fitting procedure is used to determine the resonance positions. The thus obtained level motion versus temperature is illustrated in Fig. 2 by showing seven of the all together 709 functions $\varepsilon_n(T)$. To construct the dimensionless parameter x defined above, all functions $\varepsilon_n(T)$ are fitted and the average $\langle [d\varepsilon_n(T)/dT]^2 \rangle$ is evaluated. Since no theoretical prediction for an individual function $\varepsilon_n(T)$ exists, polynomial fits are

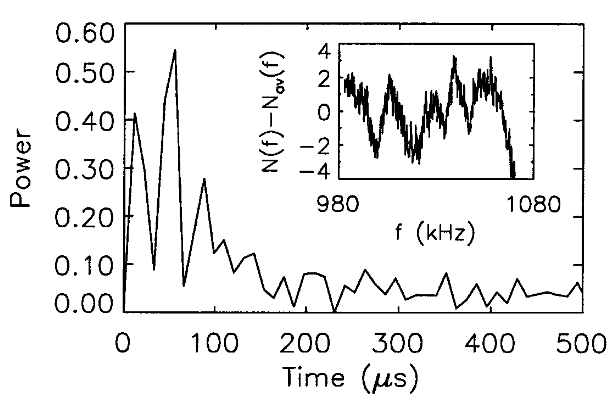

FIG. 1. Fourier transform of the difference between experimental cumulative frequency density $N(f)$ and its polynomial part $N_{\mathrm{av}}(f)$, this difference is shown as an inset.

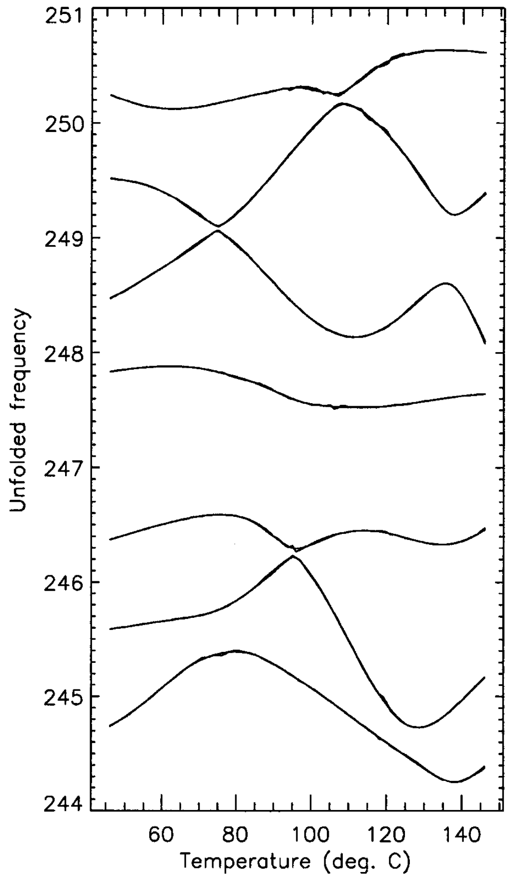

FIG. 2. Experimentally obtained motion of a set of seven consecutive levels, together with a fit.

used. The fitting algorithm is combined with a noise reduction scheme. By repeating this procedure many times in different ways, it was carefully checked that the noise reduction had no influence on the various statistical observables that we analyze. Finally, the level positions $\varepsilon_n(x)$ are obtained as functions of the unfolded temperature x.

Having performed the unfolding procedures for both the frequency and the temperature, we can compare statistical observables with the predictions of random matrix theory. We checked that the distribution of the level velocities $d\varepsilon_n(x)/dx$ is well approximated by a Gaussian, in agreement with the GOE prediction [3]. Figure 3 displays the distribution of the curvatures k_n defined above. For large $|k|$, up to values of $|k| \simeq 25$, the experimental result is well described by the $1/|k|^3$ tail of $P(k)$ in Eq. (2). This meets our expectation, because this tail is directly related to the linear behavior of the spacing distribution for small spacings [17]. Thus, this is consistent with the claim made above that the *spectral* fluctuations are of the GOE type. In the center of the distribution, however, a sizable deviation from the prediction (2) can be seen. Li and Robnik [18] show that such deviations in the center of the distribution can be due to a nonuniversal unfolding. However, we do not expect such effects to play an important role because we use the unfolding procedure (1).

As our most important result, Fig. 4 shows the velocity-velocity correlator $c(x)$ defined in Eq. (3). To obtain this statistically highly significant result up to $x \simeq 1.8$ required the high resolution of our experimental setup. The experimental result is compared with the numerical simulation of random matrices performed by Mucciolo [19]. It should be stressed that this numerical result is obtained after the twofold unfolding described above. Thus, it is independent of the distribution function for the matrix elements and of the global features of the level motion, i.e., universal. In the region between $x \simeq 0.4$ and $x \simeq 1.1$, a sizable deviation between theory and experiment can be seen. In Ref. [5], a qualitatively similar deviation was seen in the case of the hydrogen atom in a strong magnetic field. It was argued that quasiregular features could be a candidate for an explanation of this effect. In our case, such effects are less likely to play a role. We stress that we removed the bouncing-ball-like modes which can be viewed as such quasiregular features in our system. However, these modes do have a visible effect on $c(x)$. To demonstrate this, Fig. 4 also shows the experimental $c(x)$ for the spectra that still contain the bouncing-ball-like modes, i.e., the unfolding was done with a polynomial only. As can be seen, there is a deviation which is qualitatively different, most strikingly, the correlation dies out at smaller values of x. In any case, the result for $c(x)$ after removing the bouncing-ball-like modes is the one that has to be compared with the theoretical prediction because the GOE describes chaotic fluctuations only. We emphasize that the deviation between theory and experiment is robust. We also tried other unfolding techniques. In particular, the deviation is robust against variations of the Fourier cutoff t_c used to remove the bouncing-ball-like modes.

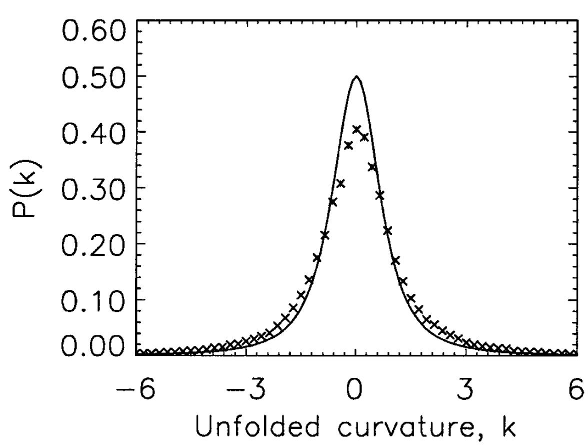

FIG. 3. Experimentally obtained curvature distribution $P(k)$, shown as crosses, and the analytical GOE result (2) as a solid line. There is a deviation in the center, whereas the tails agree well.

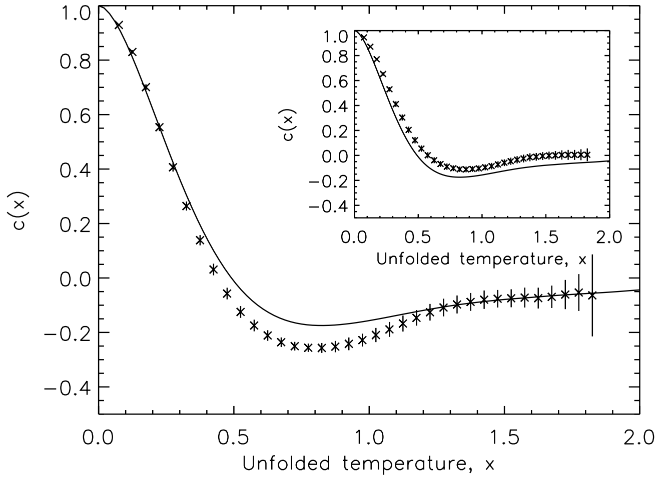

FIG. 4. Experimentally obtained velocity correlator $c(x)$ as crosses versus unfolded temperature x, compared with the numerical GOE result of Ref. [19]. There is a considerable deviation for medium values of x. As an inset, the experimental $c(x)$ is shown for the spectra from which the bouncing-ball-like modes have not been removed.

Because of the unfolding in the parameter space, the deviations cannot be due to specific effects associated with the temperature. Nevertheless, to definitely exclude this possibility, we also performed a second experiment, in which the mass of the block was used as the external parameter. Using 10 μm size powder, thin layers of quartz were ground off the block. However, we found exactly the same results as in the first experiment. Furthermore, we performed yet another measurement of $c(x)$, but in a different system [20]. We used aluminum plates and, once more, found the same results. We notice that there are no bouncing-ball-like modes in these plates. Thus, we are led to the conclusion that bouncing-ball-like modes, although important for the parametric correlations, cannot be responsible for the deviation under consideration. Its origin is still unclear. It is possible that our system is simply not perfectly well described by the models of Refs. [9,10]. We emphasize that this deviation cannot be obtained in the random matrix model by simply modifying the distribution function of the matrix elements. Because of universality, such changes will always be absorbed by the twofold unfolding, provided those additional terms do not generate scales comparable to the mean level spacing or the local scale of the parameter motion. More experimental information is now becoming available by a recent study of parametric correlations in microwave billiards [21].

In conclusion, we have presented a detailed experimental study of the statistics of parametric level motion. Thanks to our high resolution setup, the statistical significance of our results, in particular for the velocity-velocity correlator, far exceeds that of any other study that uses solely experimental information. For the curvature distribution and the velocity-velocity correlator, we found deviations between theory and experiment, although the *spectral* correlators are of GOE type on large scales. We show that the latter is strongly influenced by bouncing-ball-like modes. Thus, the parametric statistics seems to be somewhat less universal than the spectral correlators.

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