Surface Vibrations and the Pairing Interaction in Nuclei

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The induced pairing interaction arising from the exchange of low-lying collective surface vibrations among nucleons moving in time reversal states close to a Fermi energy is found to lead to values of the pairing gap which constitute a large fraction of those experimentally observed.

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It is well established that nucleons moving close to the Fermi energy in time reversal states have the tendency to form Cooper pairs which eventually condense [1]. This phenomenon, which parallels that which is at the basis of low-temperature superconductivity [2], modifies in an important way the nuclear properties.

While in the case of low-temperature superconductivity the attraction among electrons is generated by the exchange of lattice phonons, in the nuclear case the origin of the pairing interaction is related to the $^1S_0$ phase shift of the free nucleons, which is attractive at low relative momenta. Within the spirit of the BCS approximation [2] the matrix elements of the pairing interaction in nuclei are, in general, approximated by a constant inversely proportional to the mass number of the nucleus ($G \approx 25/A$ MeV). The consequences of this model have been extensively studied (cf., e.g., Ref. [1] and references therein).

In keeping with the fact that the free nucleon-nucleon interaction is strongly renormalized in nuclei, there exists a long tradition in the study of core polarization of the effective two-nucleon interaction (cf. Refs. [3–7] and references therein), in particular in the case of $J = 0$, $T = 1$ pairs (pairing correlations) where the role of exchange of high-lying quadrupole and hexadecapole modes (giant resonances) has been systematically studied (cf., e.g., Ref. [8] and references therein).

In the present paper we shall show that Cooper pair formation in nuclei can particularly benefit from the exchange of low-lying collective surface vibrations (cf. Refs. [1,9,10]), a mechanism which gives rise to pairing gaps which account in most cases for 50%–70% of the experimental values. Calculations have been carried out for a number of isotopic chains: $^{A}_{20}$Ca, $^{A}_{22}$Ti, and $^{A}_{50}$Sn. The results provide insight into the role the induced interaction plays in neutron and proton pairing correlations in nuclei. Calculations have also been carried out for the case of $^{42}_{21}$Sc and found to lead to strong proton-neutron pairing correlations.

The basic ingredients needed in the calculation of the induced pairing interaction (cf. inset of Fig. 1) are the single-particle energies ϵ_ν and the corresponding wave functions $\phi_\nu(\vec{r})$, as well as the energies $\hbar\omega_\lambda(n)$ and transition probabilities $B(E\lambda; 0 \to \lambda(n))$ of the vibrational modes. The quantities ϵ_ν and $\phi_\nu(\vec{r})$ were calculated assuming nucleons to move in an average field containing a spin-orbit term and parametrized in terms of a Saxon-Woods potential, with values of the depth, radius, diffusivity, and spin-orbit strength taken from Ref. [11]. The vibrations were calculated by diagonalizing multipole-multipole interactions within the framework of the random phase approximation (RPA) in the particle-hole basis provided by the solution of the single-particle Schrödinger equation. Vibrational states with multipolarity and parity $\lambda^\pi = 2^+, 3^-, 4^+, 5^-$ and collecting more than 90% of the energy weighted sum rule were calculated, and the corresponding energies $\hbar\omega_\lambda(n)$ and transition probabilities $B(E\lambda; 0 \to \lambda(n)) \sim \beta_\lambda^2(n)$ were determined. Typical values of these quantities are $\hbar\omega_\lambda \approx 1-2$ MeV (low-lying surface vibrations) and $\hbar\omega_\lambda \approx 15-20$ MeV (giant resonances), and $\beta_\lambda \approx 0.1$, for both types of collective modes.

With the knowledge of the quantities discussed above, the particle-vibration coupling matrix element (cf., e.g.,

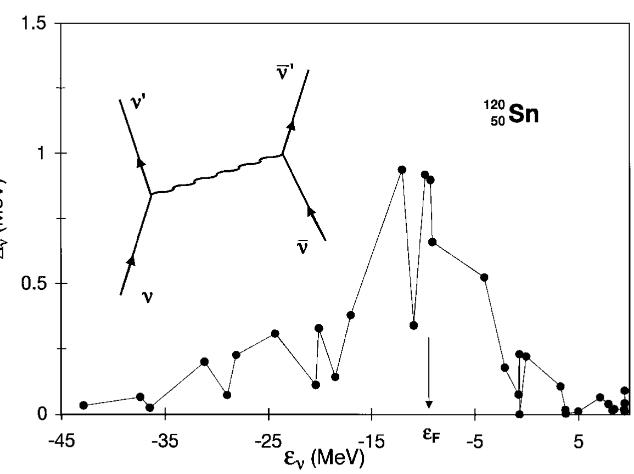

FIG. 1. State dependent pairing gap Δ_ν [cf. Eq. (3)] for the nucleus ^{120}Sn, calculated making use of the induced interaction defined in Eq. (2) (cf. inset, where particles are represented by arrowed lines and phonons by a wavy line).

Refs. [1,12])

$$M^n_{\nu,\lambda\nu'} = \frac{\langle \nu' \| R_o \frac{\partial U}{\partial r} Y_\lambda \| \nu \rangle}{\sqrt{2j_\nu + 1}} \frac{\beta_\lambda(n)}{\sqrt{(2\lambda + 1)}} \quad (1)$$

can be calculated. The quantities entering in the reduced matrix element appearing in Eq. (1) are the nuclear radius R_o, the derivative of the Saxon-Woods potential, and a spherical harmonic of multipolarity λ. Once these matrix elements are known, one can calculate the induced pairing interaction matrix elements (cf. inset of Fig. 1)

$$\begin{aligned} v_{\nu\nu'} &= \langle (j_{\nu'} m_{\nu'})(j_{\nu'} \tilde{m}_{\nu'}) | v | (j_\nu m_\nu)(j_\nu \tilde{m}_\nu) \rangle_{\text{a.s.}} \\ &= \sum_{\lambda n} \frac{2}{(2j_{\nu'} + 1)} \frac{2(M^n_{\nu,\lambda\nu'})^2}{E_o - [e_\nu + e_{\nu'} + \hbar\omega_\lambda(n)]}, \end{aligned} \quad (2)$$

and thus determine the state dependent BCS pairing gap [2]

$$\Delta_\nu = -\sum_{\nu'} \frac{(2j_{\nu'} + 1)}{2} \frac{\Delta_{\nu'}}{2E_{\nu'}} v_{\nu\nu'}. \quad (3)$$

In the above equation the state $|j_\nu \tilde{m}_\nu\rangle$ is obtained by the operation of time reversal on the state $|j_\nu m_\nu\rangle$, while the subscript a.s. indicates the normalized, antisymmetric state of two particles. One of the factors of 2 in the numerator of Eq. (2) arises from the antisymmetry while the other factor of 2 is connected with the two time orderings of the process depicted in the inset of Fig. 1. In keeping with the fact that $\beta^2_\lambda(n) \ll \beta_\lambda(n)$ we have used lowest (second) order perturbation theory to calculate the induced interaction. The quantity $E_o = E_{\text{BCS}} - E_{\text{unp}}$ is the pairing energy, the difference between the BCS-ground state energy $E_{\text{BCS}} = \sum_\nu (2j_\nu + 1)(\epsilon_\nu - \epsilon_F)V^2_\nu + \sum_{\nu,\nu'} [(2j_\nu + 1)(2j_{\nu'} + 1)/4] (\Delta_\nu \Delta_{\nu'}/4E_\nu E_{\nu'}) v_{\nu\nu'}$ and the unperturbed ground state energy $E_{\text{unp}} = \sum_{\epsilon_\nu < \epsilon_F} (2j_\nu + 1)(\epsilon_\nu - \epsilon_F)$. This is in keeping with the fact that we use Bloch-Horowitz perturbation theory [13], where the energy denominator is the difference between the final energy of the system E_o and the energy of the intermediate state $e_\nu + e_{\nu'} + \hbar\omega_\lambda(n)$ ($e_\nu = |\epsilon_\nu - \epsilon_F|$). Equations (2) and (3) are thus coupled and have to be solved self-consistently. Consequently, the process in which two nucleons interact through the exchange of a vibrational mode is iterated to infinite order.

In Fig. 1 we show the calculated state dependent pairing gap for the nucleus ^{120}Sn. The average value of the matrix elements $v_{\nu\nu'}$ appearing in Eq. (3) associated with states lying around the Fermi energy is $\overline{v} \approx -0.14$ MeV, while the pairing energy E_o is equal to about 4 MeV. The absolute value $|\overline{v}|$ is to be compared with the value of the standard parametrization of the pairing coupling constant G ($\approx$0.2 MeV for $A = 120$). The resulting pairing gap around the Fermi energy is of the order of 1 MeV. This result reproduces within 30% the empirical value of 1.5 MeV, obtained from the mass table [14] making use of the relation

$$\Delta = \tfrac{1}{2}[B(N-2,Z) + B(N,Z) - 2B(N-1,Z)], \quad (4)$$

where $B(N,Z)$ is the binding energy of the nucleus with N neutrons and Z protons.

In Fig. 2, we show the value of the state dependent pairing gap averaged over levels lying within an energy interval of the order of $\pm 2\Delta$ around the Fermi energy, for a number of Sn isotopes in comparison with the corresponding values obtained from Eq. (4). In all cases, theory accounts for a consistent fraction of the empirical values of the pairing gap. If one were to reproduce this empirical value of Δ, one would need to add to $v_{\nu\nu'}$ an approximately constant quantity, which changes only slightly from isotope to isotope, and whose average value is $G_o \approx 0.06$ MeV (i.e., parametrized as $G_o \approx x/A$ MeV would have $x \approx 7$).

The question of the convergence of perturbation theory in the case of finite nuclei has been previously discussed in the literature, in particular within the framework of the nuclear field theory [12,15]. It was found that the useful expansion parameter is $1/\Omega$, Ω being the effective degeneracy of the single-particle levels participating in the collective vibration. As a rule (cf., e.g., [16]), the lowest order contributions in $1/\Omega$ were found to provide an accurate approximation to the exact solution, provided $\Omega \geq 20$. From detailed calculations carried out for

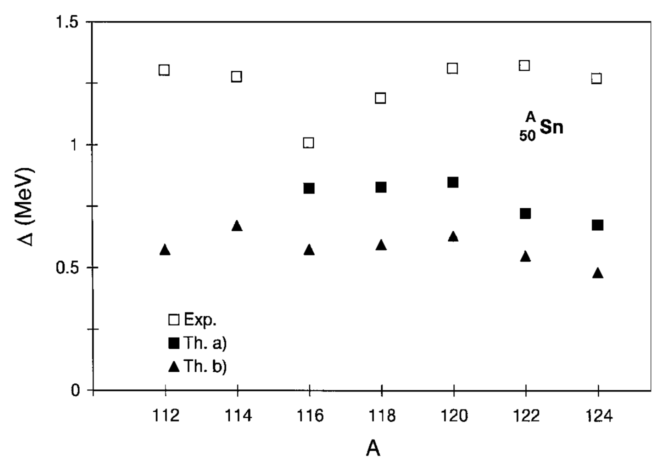

FIG. 2. Average value of the state dependent pairing gap associated with levels lying close to the Fermi energy of $^{A}_{50}$Sn isotopes, calculated as discussed in the text, making use of the pairing gap defined in Eq. (3), in comparison with the empirical pairing gap [cf. Eq. (4)]. The results of two calculations are shown, associated with RPA solutions which fit two different sets of transition probabilities connecting the lowest-lying quadrupole and octupole vibrations with the ground state. The first set (also used in the calculation shown in Fig. 1 for ^{120}Sn) was taken from Ref. [26], and the corresponding result for Δ_ν denoted by Th. a (solid squares). The second set is from Ref. [27], and the associated values of Δ_ν are denoted by Th. b (solid triangles).

systems with two neutrons moving outside closed shells, where a full diagonalization of the particle-vibration Hamiltonian giving rise to the induced interaction is also simple to carry out, we have found that the results of perturbation theory agree within 10%–15% with the "exact" results.

In Fig. 3 we display the results of calculations carried out for the isotopes ^{A}Ca and ^{A}Ti, in comparison with the corresponding results of Eq. (4). The average value of $v_{\nu\nu'}$ associated with levels lying close to the Fermi energy is, in this case, of the order of -0.2 MeV, while E_o is of the order of -3 MeV. As in the previous case, the induced interaction leads to pairing gaps which account for a consistent fraction of the empirical value, and which furthermore display a similar dependence with A, a behavior which reflects the shell dependence of the collective surface modes. In particular, the low predicted value of Δ in ^{50}Ca as compared to ^{42}Ca is due to the fact that the "core" ^{48}Ca is more rigid than the core ^{40}Ca. We have also determined the induced proton-neutron pairing interaction in ^{42}Sc, arising from the exchange of the low-lying collective surface vibrations of the core ^{40}Ca. The calculated value of 1.5 MeV (cf. also the result obtained for ^{42}Ca, Fig. 2) is close to the empirical value (1.6 MeV) obtained making use of Eq. (4). Again, in all these cases (Ca, Ti, and Sc isotopes), as in the case of Sn isotopes, the empirical values of Δ calculated making use of Eq. (4) are well reproduced making use of the pairing matrix elements $v_{\nu\nu'} + G_o$.

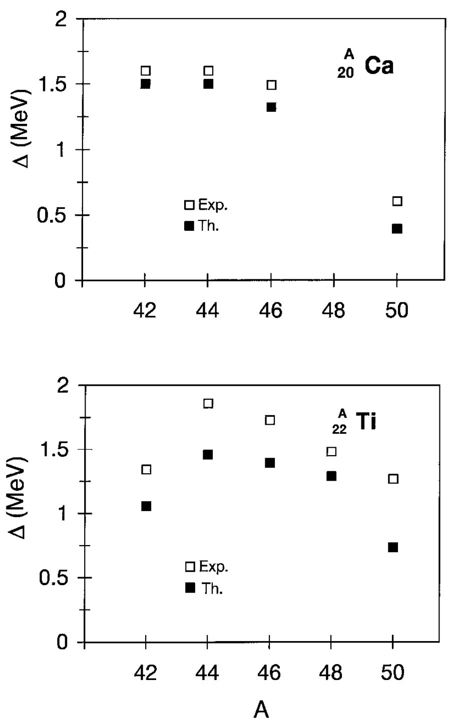

FIG. 3. Average value of the neutron pairing gap of the ${}^{A}_{20}$Ca isotopes and of the proton pairing gap of ${}^{A}_{22}$Ti isotopes, in comparison with the empirical pairing gap [cf. Eq. (4)]. The input experimental data used in the calculation of the vibrational states were taken from Ref. [28]. The gap of the Ca isotopes has been calculated making use of Eq. (3). In the case of the Ti isotopes, the matrix of the induced interaction was diagonalized. The empirical value of the pairing gap was calculated making use of the relation $\Delta = \frac{1}{2}[B(N,Z-2) + B(N,Z) - 2B(N,Z-1)]$, the proton analogous to the expression given in Eq. (4) and used to calculate the neutron pairing gap.

We have repeated the calculations described above but this time including only the coupling of the lowest-lying surface vibrations ($n = 1, \lambda^{\pi} = 2^{+}, 3^{-}, 4^{+}, 5^{-}$). The calculated values of Δ coincide, in most cases, within 20%, with those obtained from the full calculation. The main contributions arise from the exchange of low-lying quadrupole and octupole collective vibrations. These results may provide, at the microscopic level, insight into and eventually justification for the success found by surface and density dependent pairing interactions used in the literature to describe the low-energy nuclear structure (cf., e.g., Refs. [17–20]).

It is expected that, because the low-lying surface vibrations are built, to some extent, by the valence nucleons, the results discussed above will be somewhat modified by properly taking into account the Pauli principle (cf. also Refs. [12,15] and references therein) between the interacting nucleons and the correlated particle-hole excitations associated with the vibrational modes. To check this point we have recalculated the pairing gap of ^{42}Ca making use of the phonons of the core ^{40}Ca, obtaining a value of 0.9 MeV, a result which still accounts for about 60% of the experimental value.

The induced pairing interaction constitutes also a basic element in the study of the superfluidity of the inner crust of neutron stars (cf. Ref. [21] and references therein). Calculations of the associated pairing gap have been carried out for infinite [22,23] and for semi-infinite nuclear matter [24], thus neglecting finite size effects. A consistent treatment of this phenomenon should, instead, take into account the interplay between nuclei and the sea of free neutrons in which they are immersed, that is, make use of a calculational scheme as that developed in Ref. [25]. Within this context, the contribution of high-lying modes (giant resonances) to the induced interaction is expected to be qualitatively similar to the estimates carried out above for the case of isolated nuclei, while that associated with the exchange of low-lying (collective) surface vibrations should be very different. This is because the properties of giant resonances should not depend in any significant way on the fact that the system is finite or not. On the other hand, the low-lying surface modes are expected, in the case of nuclei immersed in a sea of free neutrons, to be strongly modified, becoming less collective than in the case of a free isolated nucleus. In any case, a proper calculation of the pairing matrix elements needed to describe superfluidity in the inner crust of a neutron star is still an open problem.

We conclude that the exchange of low-lying surface vibrations among nucleons moving in time reversal states close to the Fermi energy gives rise to an induced pairing interaction which leads to pairing gaps that account for a consistent fraction of those experimentally observed. This result is likely to have consequences in the analysis of phenomena like the quenching of the pairing gap taking place as a function of the angular momentum and of the energy (temperature) content of the nuclear system. In keeping with these results, and because collective vibrations couple democratically to all nucleons, regardless of the isospin quantum number, the induced pairing force mechanism is expected to lead to consistent proton-neutron pairing correlations.

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