Instability of Dust Particles in a Coulomb Crystal due to Delayed Charging

S. Nunomura,* T. Misawa, N. Ohno, and S. Takamura
Department of Energy Engineering and Science, Graduate School of Engineering, Nagoya University, Nagoya 464-8603, Japan
(Received 23 April 1999)

A self-excited vertical oscillation of dust particles in a Coulomb crystal has been observed. It occurs near the plasma-sheath boundary of a dc plasma operated at a low plasma density and gas pressure. The excitation of this spontaneous oscillation is attributed to the finite charging time of particles. As particles move, their charge varies with the local plasma conditions, but with a delay due to the charging time. As particles traverse the vertical electric potential gradient in the sheath, they gain energy. A model of this mechanism is developed to predict the instability threshold and the energy gain.

PACS numbers: 52.25.Zb, 52.25.Ub, 52.35.–g

Coulomb crystals [1] formed in dusty plasmas have attracted interest in the field of plasma physics in recent years. After they were discovered in 1994 [2–5], their phase transition phenomena [6,7] and structural formation process [8,9] have been experimentally demonstrated, and many theoretical works have been carried out [10–12]. More recent investigations have included studies of lattice dynamics, including wave propagation [13–15].

One of the most important characteristics of dusty plasmas is the fact that the charge on a dust particle is not constant, but can change in time [16]. This is basically a quite natural feature of dusty plasmas. The charge can fluctuate in two ways. First, it varies randomly due to the collection of individual electrons and ions at random times. Second, it varies when a particle moves in an inhomogeneous plasma or a plasma that varies with time. Here we will present results related to the latter mechanism.

In this paper, a self-excited vertical oscillation of dust particles trapped in a dc plasma-sheath boundary is reported. We propose that this instability arises from the charge deviation from its equilibrium value, due to the delay in charging. The charge on dust has a memory of its history of motion, and this leads to non-Markovian kinetics [17].

Experiments were performed in a dc Ar plasma at a very low gas pressure less than 10 mtorr [18], generated by dc discharge between hot filaments and the grounded vacuum vessel [see Fig. 1(a)]. The plasma is confined by multiple cusps of magnetic field. Typical plasma parameters obtained with single probe measurements are $T_e \sim 1$ eV and $n_e \sim 10^8$ cm^{-3}. In order to levitate dust particles against gravity, a negatively biased mesh electrode with a diameter of 80 mm is inserted horizontally into the plasma. In addition, a negatively biased ring electrode with 40 mm diameter is located 5 mm above the mesh electrode to provide radial confinement of the particles. After dispersing dust particles into the plasma, some of them are trapped in a single horizontal layer in the plasma-sheath boundary area above the mesh electrode. These particles form a two-dimensional ordered structure, namely, a Coulomb crystal inside the ring electrode due to strong repulsive Coulomb interactions. A He-Ne laser illuminates the Coulomb crystal. The particle behavior is observed with an image-intensified CCD camera equipped with a macro lens.

The structure of the two-dimensional Coulomb crystal is shown in Fig. 1(b). Spherical particles of radius 2.5 ± 0.5 μm were arranged in a simple hexagonal structure on a horizontal plane. The mean interparticle distance is about 430 μm, so that the Coulomb coupling parameter is approximately several hundreds. This image was made with a gas pressure of 4.4 mtorr.

When the gas pressure is decreased below a critical value, an instability arises and dust particles in the crystal oscillate spontaneously in the vertical direction. Figure 1(c) shows a typical image of such a vertical oscillation at a gas pressure of 2.6 mtorr, in which five images are superimposed for a period of 0.17 s. The amplitude

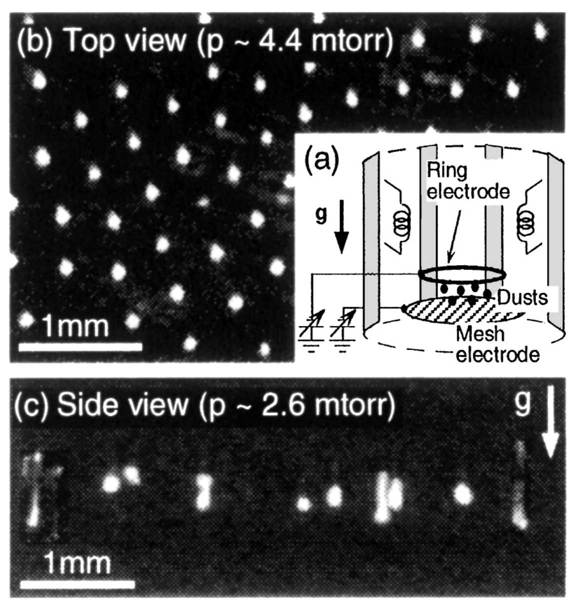

FIG. 1. (a) Schematic view of experimental apparatus. Two-dimensional Coulomb crystal composed of dust particles with the radius of 2.5 ± 0.5 μm was formed. (b) Top view in a higher gas pressure. (c) Side view of a self-excited vertical oscillation of dust particles in a low gas pressure.

and frequency are typically a few mm and 10–15 Hz, respectively. Initially the amplitude grows with time, as shown in Fig. 2(a), with a time scale of about 10 s. Later, the amplitude either saturates, or, if it is larger than about 2 mm, the particle falls down to the mesh electrode, as it did in Fig. 2(a). In saturation, the oscillation amplitude decreases with both gas density and n_e, as shown in Fig. 2(b). The crystal is unstable in the lower-left region of this diagram, but slightly beyond the contour labeled 0.1 mm the crystal is stable and the spontaneous excitation no longer occurs. One can cause a stable crystal to become unstable either by reducing the gas density or reducing the plasma density.

The equilibrium charge $Q_{d\text{-eq}}$ is determined by the balance of electron I_e and ion I_i currents flowing to the dust surface. For a dust particle immersed in a dc sheath, the orbital-motion limited currents are given by

$$I_e = -\pi r_d^2 e n_e \left(\frac{8k_B T_e}{\pi m_e}\right)^{1/2} \exp\left(\frac{e(V_s + V_f)}{k_B T_e}\right), \quad (1)$$

$$I_i = \pi r_d^2 e n_i \left(\frac{k_B T_e}{m_i}\right)^{1/2} \left(1 - \frac{eV_f}{m_i v_i^2/2 + |eV_s|}\right), \quad (2)$$

where V_f is the floating potential of the dust particle, compared to V_s, the space potential in the sheath, and $Q_{d\text{-eq}} = 4\pi\varepsilon_0 r_d V_f$. The equilibrium charge varies with height inside the sheath, because of the variation of n_e, n_i, and V_s inside the sheath.

Let us review the principle of dust levitation [9] before examining the mechanism of the vertical dust oscillation. It is well known that dust particles located in the plasma-sheath boundary are sustained against gravity by the electrostatic field in the sheath. Figure 3 shows spatial profiles of the dust particle charge $Q_{d\text{-eq}}$ and potential energy W_{tot}, which is composed of gravitational and electrostatic potential in the dc plasma sheath. These profiles are obtained numerically, assuming the space potential profile in the sheath is given by the Child-Langmuir law and using parameters measured in the experiment. Defining the plasma-sheath boundary at $z = 0$, we find that the charge has a minimum inside the sheath at $z \sim -0.7$ mm. This minimum arises from the decrease of I_i in the sheath, despite the exponential decrease in n_e. This variation in $Q_{d\text{-eq}}$ is one of the key factors in the self-excited oscillation.

A dust particle can be trapped in the potential well near the plasma-sheath boundary, as shown in curve W_{tot} in Fig. 3. The trapped particle may oscillate vertically inside this well if it is perturbed. Assuming a charge of a few thousand electrons, we use Fig. 3 to calculate the resonance frequency for small amplitude oscillations to be about 14 Hz, and the maximum amplitude to be approximately 2.5 mm, in agreement with experiment. The barrier potential corresponding to the maximum amplitude is a few V.

Here we propose a mechanism for particles to gain energy while oscillating vertically. The energy input must be sufficient to overcome the gradual damping by friction on the gas. We can eliminate plasma fluctuations and mechanical noise as candidate sources of energy because they are not coherent with the oscillation. So, we propose that the instability source is a delayed charge effect. In general, a dust particle in a plasma takes a finite time to arrive at its equilibrium charge, determined by surrounding plasma parameters and dust size. Accordingly, a dust particle may have a different charge from an equilibrium value if it moves quickly in an inhomogeneous plasma or sheath. Consider a particle bouncing vertically, as shown in Fig. 4(a). The particle gains energy if the charge $Q_{d\text{-down}}$ while falling down in a electrostatic potential is more negative than $Q_{d\text{-up}}$, the charge while the particle climbs up. This is the key concept for the instability.

Figure 5 shows numerical results for the temporal evolution of the vertical oscillation amplitude. These are obtained by solving the equation of motion, including the

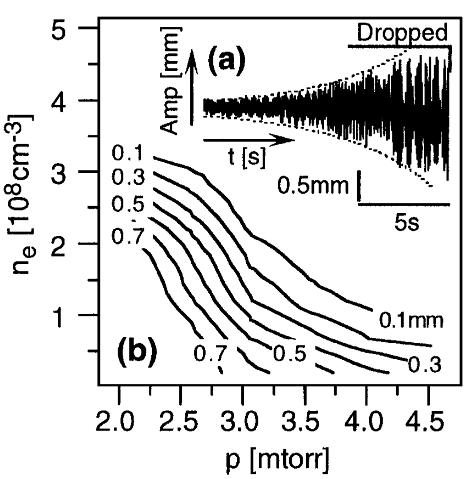

FIG. 2. (a) Temporal evolution for the growing oscillation. (b) Contour plots for the amplitude of self-excited dust oscillations in parameter spaces of the plasma density n_e and the gas pressure p.

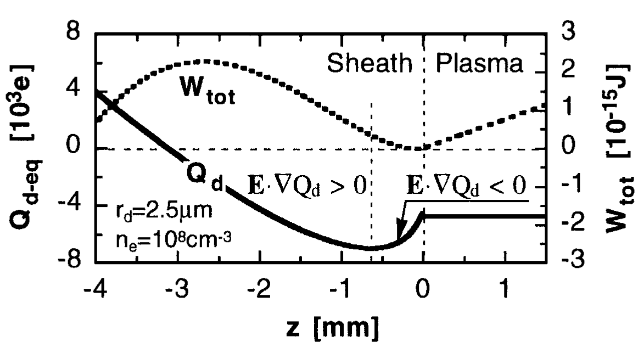

FIG. 3. Spatial profiles for the equilibrium charge (solid curve) on a dust particle and its potential energy (dotted curve) in the plasma-sheath boundary area. The plasma-sheath boundary is located at $z = 0$.

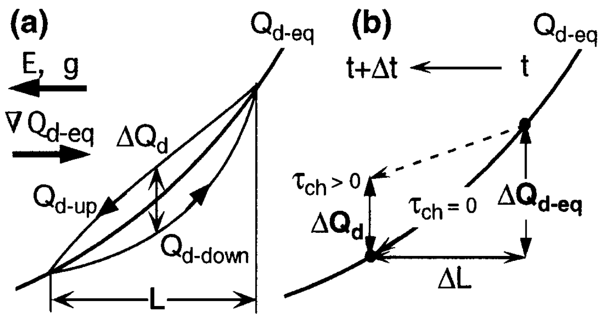

FIG. 4. Schematic for the energy gain model associated with delayed charging effect. (a) Orbits of the charge deviation in periodic bouncing motion. (b) Enlargement of the equilibrium charge curve in (a), showing the relation between deviation ΔQ_d and characteristic charging time τ_{ch}.

electrostatic, gravitational, and gas drag forces. Ion drag was also included, although it was not significant. To include the effect of charging time, the charge is obtained by integrating the electron and the ion flux to the particle, Eqs. (1) and (2), over time. The calculations were made for $r_d = 2.5\ \mu\text{m}$, $T_e = 1$ eV, and $n_e = 10^8\ \text{cm}^{-3}$. Initially, a small perturbation is applied to the dust particle located at the equilibrium position. The amplitude grew with time at low gas pressures, but diminished at large pressures, due to the friction with gas molecules. In the absence of gas drag, the growth rate was several seconds and the charge deviated from the equilibrium charge was a few percent.

Figure 6 shows numerically calculated contour plots of the amplitude when the oscillation growth has saturated. In saturation, the energy gain from the electrostatic acceleration is balanced by the energy loss. There are two sources of energy loss, first the friction with gas molecules and plasma particles, and second an electrostatic deceleration in the deep sheath that we describe later.

As in the experiment, Fig. 2(b), the numerical results show that the saturation amplitude diminishes with gas pressure. The numerical and experimental results both show an instability boundary dividing a stable region in the upper right from an unstable region in the lower left. Gas pressures above a critical value of a few mtorr always prevent the instability. This critical pressure decreases with plasma density. The numerical results reveal a third region, at values of n_e that were smaller than could be attained in the experiment. In this region, for n_e below a lower limit n_{ec}, dust levitation is not possible due to a weak electrostatic field in the sheath. The value of n_{ec}, determined by the balance between the gravitational and electrostatic forces, is estimated to be $\sim 2 \times 10^7\ \text{cm}^{-3}$.

Here we develop the basic principles of the self-excited oscillation into a theoretical model. This model predicts a threshold for the instability induced by the delay in charging. Figure 4 shows schematics for the energy supply mechanism due to delayed charging. The key concept is that the energy supply depends on the relationship between a direction of the electrostatic field $\mathbf{E}$ and a gradient of the equilibrium charge on the dust particle $\nabla Q_{d\text{-eq}}$. There are two possibilities. If $\mathbf{E} \cdot \nabla Q_{d\text{-eq}} < 0$ the particle gains energy. This occurs near the sheath boundary. On the other hand, if $\mathbf{E} \cdot \nabla Q_{d\text{-eq}} > 0$ the particle loses energy. This occurs deep in the sheath, as we mentioned earlier.

An instability requires that the energy gain exceeds the energy loss. First we estimate the energy gain. The difference in the charge ΔQ_d for a particle moving up, compared to moving down, can be estimated as illustrated in Fig. 4(a). This difference arises from the finite charging time τ_{ch} [16] of the dust particle. The energy gain W_{gain} is proportional to ΔQ_d during the bounce, $W_{\mathrm{gain}} \simeq \Delta Q_d E L$, where E and L are the electrostatic field at the equilibrium height and oscillation amplitude. It is clear that ΔQ_d is zero only in the case of $\tau_{\mathrm{ch}} = 0$, that is, instantaneous charging. The difference ΔQ_d is thought to vary

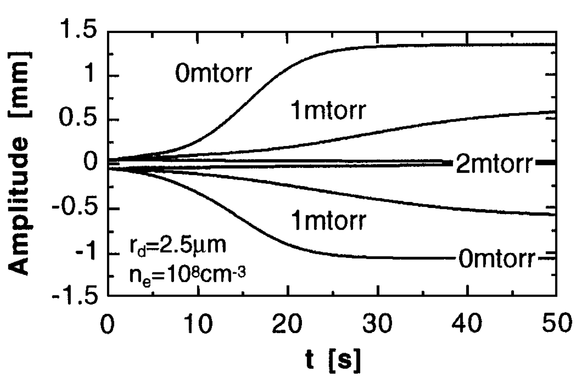

FIG. 5. Numerical results for the time evolution for amplitudes of the oscillating particle. These are obtained by integrating the equation of motion taking into account the history of the charge.

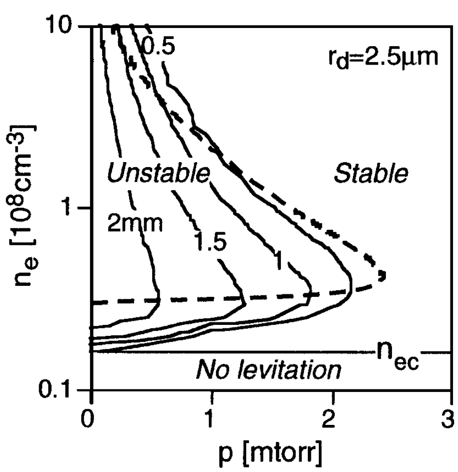

FIG. 6. Numerical results for the saturation amplitude of self-excited dust oscillations. For comparison, the broken curve shows the stability boundary obtained theoretically from Eq. (6). Here, $n_{ec} = 3 \times 10^7\ \text{cm}^{-3}$, $A = 5$ mtorr, and $B = 10^4\ \text{cm}^{3/2}$.

as $\exp(-T/\tau_{ch})$ so that $\Delta Q_d = \Delta Q_{d\text{-eq}} \exp(-T/\tau_{ch})$. Here, T is the period of oscillation and $\Delta Q_{d\text{-eq}}$ is the difference of the equilibrium charge at the top and bottom turning points in the bounce motion. We estimate $\Delta Q_{d\text{-eq}}$ as $\Delta Q_{d\text{-eq}} \simeq v_d T \nabla Q_{d\text{-eq}}$, where v_d is the typical particle velocity during the oscillation. We also need the following scalings on the key parameters against T_e, n_e, and r_d:

$$\nabla Q_{d\text{-eq}} \propto T_e^{1/2} n_e^{1/2} r_d \,, \tag{3}$$

$$\tau_{ch} \propto T_e^{1/2} n_e^{-1} r_d^{-1}, \tag{4}$$

$$T \propto [(T_e/r_d)^2 - (\alpha T_e/n_e)^{1/2}]^{-1/2} (T_e/n_e)^{1/2}. \tag{5}$$

Here, α is a coefficient that depends on n_{ec}.

Now we estimate the energy loss due to friction with gas molecules. The loss rate is $W_{loss} \doteqdot \pi r_d^2 n_n v_n m_n v_d L$, where v_n is the thermal velocity of gas molecules and $n_n = p/k_B T_n$ is the number density at pressure p.

We can now rewrite the criterion for an instability, $W_{gain} > W_{loss}$, by substituting the above equations. This yields the necessary condition on p and n_e to excite the spontaneous oscillations,

$$p \leq \frac{A}{\{1 - n_e^{*-1/2}\}^{1/2}} \exp\left\{\frac{B n_e^{*1/2}}{\{1 - n_e^{*-1/2}\}^{1/2}}\right\}, \tag{6}$$

where, n_e^* is the normalized plasma density by n_{ec}, and A and B are appropriate coefficients depending on T_e and r_d. The broken curve in Fig. 6 shows the result of Eq. (6) with an equality condition, thereby showing the boundary between stability and instability. In plotting the broken curve, we chose values for A and B to achieve the best agreement with the numerical results and observations. The fact that we were able to achieve agreement suggests that qualitatively our model includes the appropriate physical mechanisms, including the delay in charging.

In summary, we observed a self-excited dust oscillation in Coulomb crystal at low values of plasma density and gas pressure. An instability mechanism induced by the delay in charging is proposed. Numerical and theoretical analyses including the delayed charge show enough agreement with experiment to conclude that delayed charge effects play an important role for the spontaneous vertical oscillation in a dust crystal.

The authors would like to thank Professor J. Goree, Dr. Y. Uesugi, and K. Asano for discussions. This work was supported in part by a Grant-in-Aid for Scientific Research from the Japan Ministry of Education, Science, Sport and Culture (No. 3254).

*Present address: Department of Physics and Astronomy, University of Iowa, Iowa City, IA 52242.

[1] H. Ikezi, Phys. Fluids **29**, 1765 (1986).
[2] H. Thomas, G. E. Morfill, V. Demmel, J. Goree, B. Feuerbacher, and D. Mohlmann, Phys. Rev. Lett. **73**, 652 (1994).
[3] J. H. Chu and Lin I, Phys. Rev. Lett. **72**, 4009 (1994).
[4] Y. Hayashi and K. Tachibana, Jpn. J. Appl. Phys. **33**, 804 (1994).
[5] A. Melzer, T. Trottenberg, and A. Piel, Phys. Lett. A **191**, 301 (1994).
[6] V. A. Schweigert, I. V. Schweigert, A. Melzer, A. Homann, and A. Piel, Phys. Rev. Lett. **80**, 5345 (1998).
[7] H. Thomas and G. E. Morfill, Nature (London) **379**, 806 (1996).
[8] J. B. Pieper, J. Goree, and R. A. Quinn, Phys. Rev. E **54**, 5636 (1996).
[9] S. Nunomura, N. Ohno, and S. Takamura, Phys. Plasmas **5**, 3517 (1998).
[10] H. C. Lee and D. Y. Chen, Phys. Rev. E **56**, 4596 (1997).
[11] H. Totsuji, T. Kishimoto, and C. Totsuji, Phys. Rev. Lett. **78**, 3113 (1997).
[12] S. Hamaguchi, R. T. Farouki, and D. H. E. Dubin, Phys. Rev. E **56**, 4671 (1997).
[13] M. Zuzic, H. M. Thomas, and G. E. Morfill, J. Vac. Sci. Technol. A **14**, 496 (1996).
[14] A. Homann, A. Melzer, R. Madani, and A. Piel, Phys. Lett. A **242**, 173 (1998).
[15] F. Melandso, T. Aslaksen, and O. Havens, Planet. Space Sci. **41**, 321 (1993).
[16] C. Cui and J. Goree, IEEE Trans. Plasma Sci. **22**, 151 (1994).
[17] Y. Chutov and P. Schram, Contrib. Plasma Phys. **39**, 127 (1999).
[18] S. Nunomura *et al.*, in *Proceedings of the 1998 ICPP&25th EPS Conference on Controlled Fusion and Plasma Physics, Praha, 1998* [ECA **22C**, 2509 (1998)].