Suppression of T_c in Superconducting Amorphous Wires

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The suppression of the mean field temperature of the superconducting transition, T_c, in homogeneous amorphous wires is studied. We develop a theory that gives T_c in situations when the dynamically enhanced Coulomb repulsion competes with the contact attraction. The theory accurately describes recent experiments on T_c suppression in superconducting wires, after a procedure that minimizes the role of nonuniversal mechanisms influencing T_c is applied.

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Disorder suppresses the superconductivity transition in morphologically homogeneous superconductors [1–4] because the diffusive character of the electron motion in dirty systems makes the Coulomb interaction more effective [5]. As a result, the attraction between the electrons in Cooper pairs becomes weaker, and the transition temperature, T_c, is lowered. In two dimensions (2D) the influence of disorder on T_c can be studied systematically by varying the film thickness d [6–8]. In uniform films T_c, being well defined, is suppressed as the sheet resistance, $R_\square$, increases with decreasing d. (For a review see Ref. [9].) When the geometry of the sample is such that its dimension is lowered towards the one-dimensional (1D) limit, the suppression of superconductivity should become more pronounced [10].

Recently, efforts have been made [11–13] to extend the experiments in films to narrow wires by fabricating a series of amorphous Pb wires of different thicknesses and widths. It has been found that the T_c suppression becomes stronger as the wires' width reduces below 1000 Å. The experiment of Refs. [12,13] is in the crossover region from 2D to 1D. Actually, the wires are in the 1D limit as far as superconducting fluctuations are concerned [14], but they are in the crossover region from 2D to 1D with respect to the diffusive motion of the electrons.

From the theoretical point of view, the problem of T_c suppression in 1D wires is rather intriguing. As is well known, the superconductivity transition is determined by a series of logarithmically divergent terms describing the electron scattering in the two-particle Cooper channel. In 2D systems the corrections due to the electron-electron (e-e) interactions combined with disorder are logarithmically divergent as well [5]. As the whole problem is controlled by logarithmic singularities, it can be studied by renormalization group (RG) methods [15]. In 1D, due to the reduced dimensionality, the effect of e-e interactions is more singular. It produces corrections that diverge as the square root of the frequency. The presence of two types of singularities demands a special analysis in the calculation of T_c. In this paper we develop a theory that describes adequately the effect of the dynamically enhanced e-e interaction on T_c in the crossover region from 2D to 1D and perform a detailed comparison with the experiment.

The mean field temperature, T_c, is defined as the temperature at which the electron scattering amplitude in the Cooper channel, Γ_c, becomes infinite. Fluctuations of the superconductivity order parameter lead to a broadening of the phase transition. However, its mean field temperature can be found experimentally by fitting the upper part of the resistive transition to the Aslamazov-Larkin theory [14]. The diagrammatic representation of the amplitude Γ_c is shown in Fig. 1. In addition to the contact BCS-interaction amplitude γ, the terms arising as a result of the interplay of the Coulomb interaction and disorder are also included in the Cooper ladder-diagram series. (The impurity scattering does not influence the e-e interaction mediated by phonons because in the long wavelength limit the lattice defects oscillate together with the ions [16].) The resulting equation for Γ_c is

$$\Gamma_c(\epsilon_n, \epsilon_l) = -|\gamma| + t\Lambda(\epsilon_n + \epsilon_l) - 2\pi T \sum_{m=0}^{M} [-|\gamma| + t\Lambda(\epsilon_n + \epsilon_m)] \times \frac{1}{\epsilon_m} \Gamma_c(\epsilon_m, \epsilon_l), \quad (1)$$

where $\epsilon_m = 2\pi T(m + 1/2)$ is the Matsubara frequency, and the summation over m is limited by $M = (2\pi T\tau)^{-1}$. In this equation γ, the bare value of the amplitude Γ_c, is rescaled in such a way that the Debye frequency as a cutoff energy is substituted by τ^{-1}, the inverse of the scattering time. Then, $\gamma = 1/\ln(T_{c0}\tau/1.14)$, where T_{c0} is the temperature of the superconducting transition in the bulk limit. The parameter $t = (e^2/2\pi^2\hbar)R_\square$ characterizes the level of disorder in a sample, where $R_\square$ is the sheet resistance. The amplitude Λ describing the combined action of the e-e interaction and disorder is given by

$$\Lambda(\omega_n) = u\,\frac{4\pi D}{La} \sum_{q_L, q_a} \frac{1}{Dq_L^2 + Dq_a^2 + \omega_n}, \quad (2)$$

where a and L are the width and the length of the wire, respectively. The parameter u describes the amplitude of

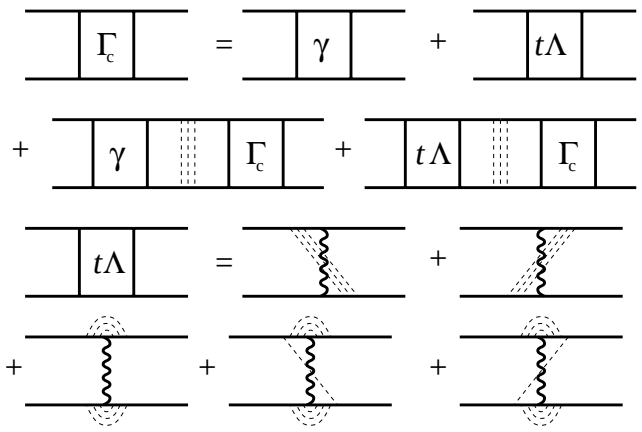

FIG. 1. The diagrammatic equation for the scattering amplitude Γ_c in the Cooper channel. The block γ denotes the BCS-interaction amplitude. The block $t\Lambda$ describes the interplay of the Coulomb interaction with disorder that leads to the suppression of T_c. The wavy line is the screened Coulomb interaction, dashed lines describe impurity scattering.

the e-e interaction when the momentum q transferred by this interaction is not too small compared with the transferred frequency ω, namely, when $q \gtrsim q_\omega = \sqrt{\omega/D}$. (As was explained in Refs. [3,9], the most divergent contributions from the region $q < q_\omega$ cancel each other out. This happens because in this region of small momenta the e-e interaction effectively depends only on the frequency, and therefore it can be gauged out.) Next, for amorphous Pb films the spin-orbit scattering time is expected to be only a few times longer than the elastic scattering time and therefore the part of the e-e interaction related to spin density fluctuations can be neglected. In that case, we may take u to be the value of the screened Coulomb interaction amplitude in the region of momenta $q \gtrsim q_\omega$, which gives $u \approx 1/2$.

In 2D the summation in Eq. (2) yields $\Lambda(\omega_n) \cong u \ln(1/\omega_n\tau)$. Therefore, Eq. (1) combines the usual BCS logarithms together with the ones arising due to disorder. Unlike the ladder diagrams in the BCS theory, the integrations in the different blocks of the diagrams in Fig. 1 cannot be factorized, because $\Lambda(\epsilon_n + \epsilon_m)$ matches the frequency arguments of two neighboring blocks. In order to solve this parquetlike equation with a logarithmic accuracy one uses the approximation $\ln(z + z') \cong \ln(\max\{z, z'\})$; see, e.g., Ref. [17]. Then, it is possible to apply the "maximum section" method. This procedure leads to the RG equation for the amplitude $\Gamma_c(\varepsilon, \varepsilon)$ [3,15]: $d\Gamma_c/dl_\varepsilon = ut - \Gamma_c^2$, where $l_\varepsilon = \ln(1/\varepsilon\tau)$. The integration of the RG equation gives the suppression of T_c by the Coulomb interaction in 2D disordered systems:

$$\ln\left(\frac{T_c}{T_{c0}}\right) = \frac{1}{|\gamma|} - \frac{1}{2\sqrt{ut}} \ln\frac{1 + \sqrt{ut}/|\gamma|}{1 - \sqrt{ut}/|\gamma|}. \qquad (3)$$

This formula accurately describes the experimental results in MoGe films [6], with $u = 1/2$ and using only one fitting parameter, γ [3].

In 1D the result of the summation in Eq. (2) yields a square root singularity in the amplitude $\Lambda(\omega_n)$. When one deals with singularities stronger than logarithmic ones, the approximations of the maximum section method cease to be valid, and a different method should be invented. In this Letter we treat the problem of finding T_c from Eq. (1) as a sort of an eigenvalue problem, which leads to an implicit equation for T_c. To see this, we will consider $\Gamma_c(\epsilon_n, \epsilon_m)$ as the matrix elements of a matrix $\hat{\Gamma}_c$, and will write the solution of Eq. (1) for Γ_c in matrix notations:

$$\hat{\Gamma}_c = \hat{\epsilon}^{1/2}(\hat{I} - |\gamma|\hat{\Pi})^{-1}\hat{\epsilon}^{-1/2}(-|\gamma|\hat{1} + t\hat{\Lambda}). \qquad (4)$$

Here $\hat{\Pi}(T) = \hat{\epsilon}^{-1/2}[\hat{1} - |\gamma|^{-1}t\hat{\Lambda}]\hat{\epsilon}^{-1/2}$, $\hat{\epsilon}_{nm} = \delta_{nm}(n + 1/2)$, $\hat{\Lambda}_{nm} = \Lambda(\epsilon_n + \epsilon_m)$, $\hat{1}_{nm} = 1$, and $\hat{I}$ is a unit matrix. Equation (4) is written in such a form that $\hat{\Pi}$ is a symmetric matrix. Notice, that the dependence of $\hat{\Pi}$ on the temperature T is not only through the dependence of $\hat{\Lambda}$ on the Matsubara frequencies, but also through the matrix rank $M = (2\pi T\tau)^{-1}$. The amplitude Γ_c diverges when the temperature is such that one of the eigenvalues of the matrix $\hat{\Pi}(T)$ is equal to $|\gamma|^{-1}$; i.e., at $T = T_c$ the equation

$$[|\gamma|^{-1}\hat{I} - \hat{\Pi}(T_c)]|\Psi\rangle = 0 \qquad (5)$$

holds. Thus, the equation determining T_c can be obtained from an eigenvalue problem. (One can also obtain an equation for T_c by considering a BSC-like gap equation with frequency dependent interaction vertex, $-|\gamma| + t\Lambda$.) The matrix elements of $\hat{\Pi} = \hat{\Pi}^0 + \hat{\Pi}^1$ are

$$\hat{\Pi}^0_{nm} = [(n + 1/2)(m + 1/2)]^{-1/2},$$
$$\hat{\Pi}^1_{nm} = -t[(n + 1/2)(m + 1/2)]^{-1/2}|\gamma|^{-1}\Lambda(\epsilon_n + \epsilon_m). \qquad (6)$$

As the matrix elements $\hat{\Pi}^0_{nm}$ are factorized with respect to n and m, all the eigenvalues of the matrix $\hat{\Pi}^0$, except one, are degenerate and equal to zero. The eigenvector corresponding to the nonzero eigenvalue is $\Psi^0_n = c/\sqrt{n + 1/2}$, and the equation $|\gamma|^{-1}\Psi^0_n = \sum_m \hat{\Pi}^0_{nm}\Psi^0_m$ leads to the BCS relation for T_{c0}:

$$|\gamma|^{-1} = l_0(T_{c0}), \qquad l_0(T) \equiv \sum_{m=0}^{M} \frac{1}{m + 1/2} = \ln\frac{1.14}{T\tau}. \qquad (7)$$

Our strategy now will be to calculate the corrections to this eigenvalue perturbatively in $\hat{\Pi}^1$ (notice that $\hat{\Pi}^1 \propto t$), and in this way to get an implicit equation for T_c. Since $\hat{\Pi}$ is symmetric we can perform this program using a standard perturbation theory,

$$|\gamma|^{-1} = l_0(T) + l_1(T) + l_2(T) + \ldots. \qquad (8)$$

The first order term can be obtained straightforwardly,

$$l_1 = \langle\Psi^0|\hat{\Pi}^1|\Psi^0\rangle \equiv -\frac{t}{l_0|\gamma|}\Sigma_2(T),$$
$$\Sigma_2(T) = \sum_{n,m=0}^{M} \frac{\Lambda(\epsilon_n + \epsilon_m)}{(n + 1/2)(m + 1/2)}. \qquad (9)$$

The prefactor $1/l_0$ appears in l_1 because the normalization factor c of the eigenvector Ψ^0_n is equal to $1/\sqrt{l_0}$. Since all

the eigenvalues of the operator $\hat{\Pi}^0$ are degenerate except the one under studying, it is also possible to find the higher order corrections using only the eigenvector $|\Psi^0\rangle$, without involving other eigenvectors. We demonstrate it here for the second order term, but a generalization to higher orders is straightforward. In the second order

$$l_2 = \sum_{i\neq 0} \frac{\langle\Psi^0|\hat{\Pi}^1|\Psi^i\rangle\langle\Psi^i|\hat{\Pi}^1|\Psi^0\rangle}{l_0} = \frac{1}{l_0}[\langle\Psi^0|\hat{\Pi}^1\hat{\Pi}^1|\Psi^0\rangle - (l_1)^2], \quad (10)$$

where

$$\langle\Psi^0|\hat{\Pi}^1\hat{\Pi}^1|\Psi^0\rangle \equiv \frac{t^2}{l_0|\gamma|^2}\,\Sigma_3(T),$$

$$\Sigma_3(T) = \sum_{nmk=0}^{M} \frac{\Lambda(\epsilon_n + \epsilon_k)\Lambda(\epsilon_k + \epsilon_m)}{(n + 1/2)(m + 1/2)(k + 1/2)}. \quad (11)$$

Inverting Eq. (8) perturbatively in t and having in mind that $|\gamma|l_0(T_{c0}) = 1$, we find

$$\ln\frac{T_c}{T_{c0}} = -t\Sigma_2(T_{c0}) + t^2\left(\Sigma_3(T_{c0}) + T_{c0}\frac{\partial\Sigma_2(T)}{\partial T}\bigg|_{T=T_{c0}}\Sigma_2(T_{c0})\right) + \ldots. \quad (12)$$

Since Eq. (12) gives an approximation for $\ln(T_c/T_{c0})$, while the measured quantity in experiments is T_c/T_{c0}, the first two terms of the perturbative series are sufficient for the description of the T_c suppression, if the parameter t is not too close to a critical value where T_c vanishes. [The parameter t should be inside the radius of convergence of the series (12). Outside this radius the superconductivity is completely suppressed.] In the 2D case Eq. (12) reproduces the first two terms of the expansion of the right-hand side of Eq. (3) in powers of ut/γ^2:

$$\ln\left(\frac{T_c}{T_{c0}}\right) = \sum_{n=1}^{\infty}\frac{1}{(2n+1)\gamma}\left(\frac{ut}{\gamma^2}\right)^n. \quad (13)$$

We note that expansion (13) does not contain a term $\propto t^2/\gamma^4$. There are several diagrams that give contributions to that order; however, finally they cancel each other [18]. The main advantage of Eq. (12) is that it is not restricted to a logarithmic accuracy, and can be applied to the description of the crossover from 2D to 1D systems.

In the experiment of Xiong *et al.* [13] the mean field temperature of the superconducting transition, T_c, has been measured systematically for uniform Pb wires of various widths. The effective strength of the disorder characterized by $R_\square$ has been controlled by the wire thickness d. Before going to a detailed comparison of the theory with the experiment a few remarks are in order. The theory described above deals with the universal mechanism related to large scale distances that are of the order of the thermal length $L_T \propto \sqrt{D/T}$. However, a number of other effects may also influence T_c when the thickness d is decreased. For example, the electron state quantization and the interaction of the electrons with the film's substrate can alter the parameters of the electron liquid. These nonuniversal effects of a short range origin are not addressed by the present theory. In some systems, e.g., MoGe (see Ref. [3,6]), the discussed effect, originated from the interplay of the Coulomb interaction and disorder, is dominant, and the theoretical curve matches the experimental data at 2D. Unfortunately, as it is shown in Fig. 2 the theoretical curve for Pb films does not follow the experiment. This fact indicates that the effects of a short range physics are not negligible here.

To minimize the role of the nonuniversal effects, and make the comparison between the experiment and theory possible, we proceed in the following way. For each width the theoretical curve has been multiplied by the function $x(R_\square) = T_c^{2D}(R_\square)_{ex}/T_c^{2D}(R_\square)_{th}$. This function is the ratio between the two curves presented in Fig. 2. Here, the basic idea is that, because the widths of the wires are considerably larger than any microscopical scale, the influence of the short range effects on T_c in wires remains the same as in 2D films. In this way, we believe, the effect of the long range physics determining the crossover from 2D to 1D systems can be captured by the present theory. To continue further we have to discuss another complication. Unlike the case of 2D films, the limit of $R_\square \to 0$ for a series of wires with a fixed width is somewhat ambiguous. For the discussed data, the extrapolation of T_c to the limit $R_\square \to 0$ at

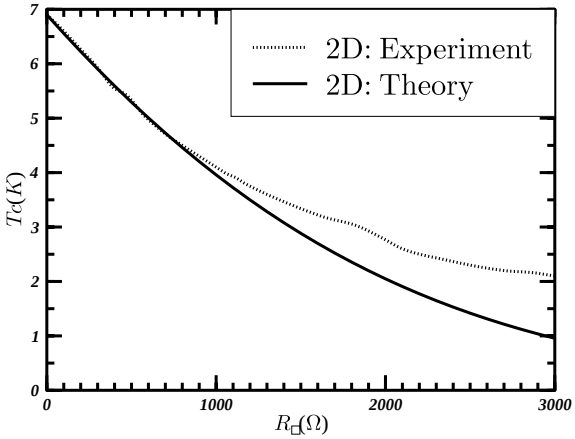

FIG. 2. Comparison between the theory (solid line) and the experimental results [13] (dashed line) in 2D Pb films. The fitting parameter $\gamma = -0.16$ is determined from the initial slope of $T_c(R_\square)$. The deviation between the theory and the experiment at large $R_\square$ shows that the interplay of the Coulomb interaction and disorder is not the only mechanism influencing T_c.

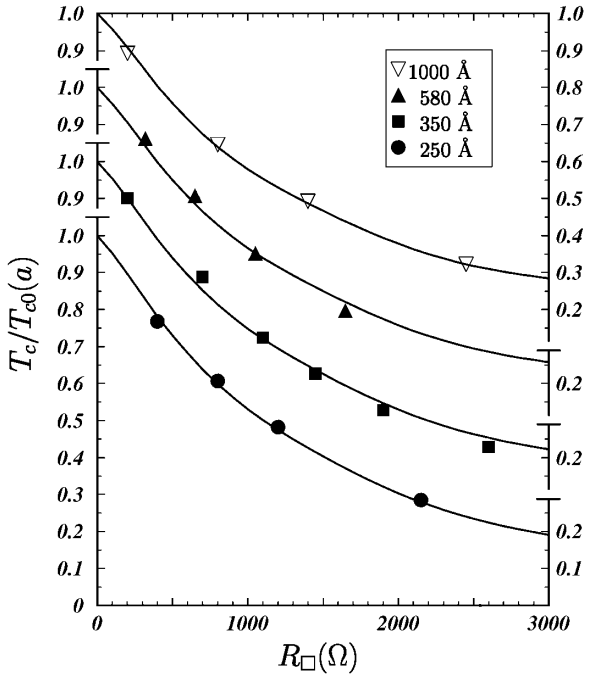

FIG. 3. Comparison between the theory (solid line) and the experimental data [13] for wires of different width. The short range effects are excluded assuming that they are the same as in 2D. For each wire width, a, the theoretical curves and the experimental data points are normalized by the extrapolated value $T_{c0}(a) = T_c(R_\square \to 0)$. For all widths we use $\gamma = -0.16$ as in 2D.

a fixed width yields values that are not equal to the transition temperature in the bulk limit. (Moreover, the extrapolated values behave in an irregular way as a function of the wire width.) Under this circumstance, we have normalized the theoretical curves in such a way that in the limit $R_\square \to 0$ the fitting curves for each width, a, start from the extrapolated $T_{c0}(a) = T_c(R_\square \to 0)$. After this normalization procedure and rescaling the theoretical curves by $x(R_\square)$, the data for wires of different widths have been plotted together with the theoretical curves in Fig. 3. The fitting parameter $\gamma = -0.16$, determined from the initial slope of $T_c(R_\square)$ in 2D films, was the same for all wire widths. Notice that at $R_\square \gtrsim 2000\ \Omega$ the suppression of T_c for the wire of the smallest width is about 1.5 times stronger than for the widest one. The agreement between theory, i.e., Eq. (12), and experimental data for all wires of different widths turns out to be very good.

To summarize, we have developed a theory that describes the suppression of the mean field temperature of the superconducting transition in amorphous systems. The theory is based on the consideration of the suppression of the contact attraction due to phonons, by the dynamically enhanced Coulomb repulsion. It is suitable for the description of the crossover region between 2D and 1D. By treating the problem as an eigenvalue problem, we overcame the difficulties occurring because of the coexistence of different singularities in the equation determining T_c. In order to compare the available experimental results with the theory, we analyzed the data in a way that minimizes the role of nonuniversal effects of a short range origin. We believe that the theory could be tested further with superconducting wires fabricated from other materials, where the initial slope of $T_c(R_\square)$ is larger than in Pb films.

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