# Crossing Lattices, Vortex Chains, and Angular Dependence of Melting Line in Layered Superconductors

A. E. Koshelev
*Materials Science Division, Argonne National Laboratory, Argonne, Illinois 60439*
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We investigate vortex structure in very anisotropic layered superconductors at fields tilted with respect to the $c$ axis. We show that even a small in-plane field does not tilt the vortex lattice but, instead, penetrates inside the superconductor in the form of Josephson vortices (JVs). At high $c$-axis magnetic field the phase field of the JV is built up from the phase perturbations created by displacements of pancake vortices. The crossing-lattices ground state leads to linear dependence of the $c$-axis melting field on the in-plane field, in agreement with recent experimental observations. At small fields stacks of JVs accumulate additional pancake stacks creating vortex rows with enhanced density. This mechanism explains the mixed chains-lattice state observed by decorations.

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Significant progress has been achieved recently in understanding vortex states and melting transition in layered superconductors with very weak Josephson coupling such as $Bi_2Sr_2CaCu_2O_x$ (BSCCO). A magnetic field applied perpendicular to the layers penetrates such superconductors in the form of pancake vortices (PVs) [1]. PVs in different layers are coupled weakly via the Josephson and magnetic interactions and form aligned stacks at low fields and temperatures (PV stacks). The magnetic nature of coupling leads to an unusual melting scenario. In contrast to superconductors with strong Josephson coupling, both crystalline order and alignment of PVs are destroyed at the melting point leading to almost independent two-dimensional vortex liquids in the layers [2].

A rich variety of vortex structures were proposed for the magnetic field tilted with respect to the $c$ axis, such as the kinked lattice [3,4], crossing lattices of Abrikosov and Josephson vortices (JVs) (combined lattice) [3], tilted vortex chains [5], etc. Obviously, the thermodynamics of the melting transition at tilted fields is determined by the nature of the ground state. In moderately anisotropic superconductors an in-plane field simply tilts the vortex lattice inside the superconductor. This scenario does not work in the case of very weak Josephson coupling when the interlayer coupling is predominantly magnetic. The in-plane magnetic field interacts with the PVs only via the Josephson coupling. On the other hand, the PVs are mainly aligned by magnetic coupling, therefore a homogeneous tilt of the lattice costs magnetic energy. To save the magnetic coupling energy it is more favorable for the in-plane field to penetrate inside the superconductor in the form of JVs as in the Meissner state. The ground state of the system is then given by the dilute lattice of the JVs coexisting with the dense lattice of the PVs (crossing lattices or combined lattice). This ground state was first proposed for the opposite case of a small $c$-axis field [3] when the dilute pancake lattice coexists with the dense Josephson lattice. The JV in weakly coupled superconductors has a very wide nonlinear core (the region where the interlayer phase difference is large). This means that even at moderate $c$-axis fields the JV core contains many PVs (see Fig. 1a). We show that in this situation the phase field of the JV is built up from the phase perturbations created by pancake displacements. Such a JV has a smaller core size and smaller energy as compared to the ordinary JV. In the crossing-lattices state the free energy depends almost linearly on the in-plane field. As a consequence, the melting temperature and melting field of the pancake lattice also depend on the in-plane field in a linear way in agreement with recent experiments [6,7]. At high in-plane field the JV cores start to overlap. In the regime of strongly overlapping JVs the in-plane field produces only weak zigzag deformations in the PV lattice [8]. These deformations have a very little influence on the phase distribution and can be treated perturbatively, in contrast to isolated JV.

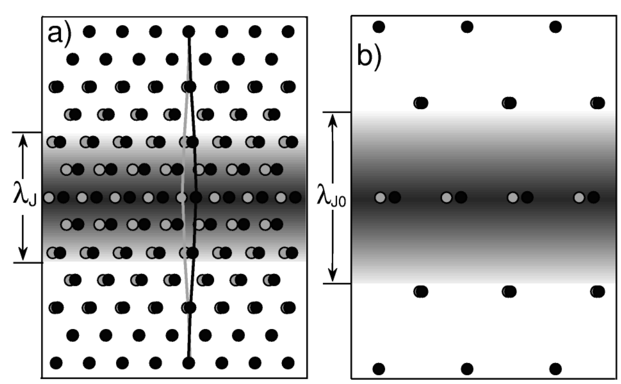


FIG. 1. JV coexistent with the lattice of PVs. Shaded area shows the JV core. Pancake positions in the two central layers are shown. (a) At large fields the JV core contains a large number of pancake rows. The phase field of the JV is determined by displacements of the PVs. (b) At small fields the JV core captures only one pancake row. Because of the interaction energy between the PV stacks and JVs, pancake density in this row is higher than in neighboring rows (mixed chains-lattice state).

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At very small $c$-axis fields the JV core contains only one row of PVs. In this case displacements of the PVs have a very little influence on the JV structure. These displacements, however, produce a finite interaction energy between the PV stack and the JV. Because of a very high anisotropy the JV lattice consists from densely packed stacks along $c$ axis (JV stacks) separated by relatively large distances. PV stacks crossing JV stacks have smaller energy as compared to other stacks, and it would be favorable to add extra PV stacks to the rows located on the JV stacks. However, this makes the PV lattice defective and increases its energy. On the other hand, when the distance between PVs, $a$, is much larger than the London penetration depth $\lambda$, the energy of defect in the lattice is exponentially small. This means that at a certain field there is a phase transition from the weakly deformed triangular lattice to the defective lattice. In the latter state PV rows located on the JV stacks will have a larger density of PVs than other rows (see Fig. 1b). This picture provides a natural explanation for the mixed chain-lattice state observed by Bitter decorations in BSCCO [9,10]. Initially these chains were attributed to the chains of tilted vortices, which were predicted on the basis of the anisotropic London theory [5]. Further detailed investigations [10] indicated that this interpretation does not agree with experiment. On the other hand, the interpretation based on the crossing lattices [11] qualitatively agrees with the experiment. We will calculate the crossing energy of the JV and PV stack which allow one to obtain a quantitative criterion for the transition to the "chain state."

First, we consider a structure of an isolated JV. In the Meissner state the JV is represented by a region of a singular phase distribution [12,13]. The phase difference between two central layers changes from 0 to $2\pi$ within the nonlinear core region. The core size $\lambda_{J0}$ and the JV energy $\varepsilon_{J0}$ are determined by the balance condition between the in-plane phase stiffness energy, $(J/2)(\nabla\phi_n)^2$, and the Josephson energy, $-E_J\cos(\phi_{n,n+1})$ [13],

$$\lambda_{J0} \approx \sqrt{J/E_J} = \gamma s, \qquad \varepsilon_{J0} = \pi\sqrt{JE_J}\,[\ln(\lambda/s) + 1.55],$$

where $\phi_n(\mathbf{r})$ is the phase of the order parameter in the $n$th layer, $\phi_{n,n+1} = \phi_{n+1} - \phi_n - (2\pi s/\Phi_0)A_z$ is the gauge invariant phase difference, $\lambda$ is the London penetration depth, $\gamma$ is the anisotropy parameter, $s$ is the interlayer spacing, $J = s\Phi_0^2/\pi(4\pi\lambda^2)$, and $E_J = J/(\gamma s)^2$.

We consider now a JV in the presence of a pancake lattice with lattice parameter $a \ll \lambda_{J0}$ (see Fig. 1a) for the case of very weak coupling, $\lambda_{J0} > \lambda$. If we put the JV into the PV lattice, its in-plane supercurrents will displace the PVs from their equilibrium positions. Lattice displacements induce extra phase variations, which add to the usual regular phase and renormalize the JV structure. We will show that at large $B_z$ the vortex phase has a smaller stiffness as compared to the regular phase and gives a dominating contribution to the phase perturbations. The coarse-grained vortex phase $\phi_{nv}(y)$, created by the unidirectional shear lattice displacement in the $n$th layer, $u_{nx}(y)$, is determined by $\nabla_y\phi_{nv} = 2\pi n_v u_{nx}$ with $n_v = B_z/\Phi_0$. The lattice deformations produce the phase sweeps between the opposite sides of the deformed region $\delta_y\phi_{nv} \equiv \phi_{nv}(\infty) - \phi_{nv}(-\infty)$ related to $u_{nx}(y)$ as $\delta_y\phi_{nv} = 2\pi n_v\int_{-\infty}^{\infty}u_{nx}(y)\,dy$. If we neglect the regular phase, these phase sweeps are given by $\delta_y\phi_{nv} = \pi\,\mathrm{sgn}(n)$ in the isolated JV. The vortex phase deformations in the JV are large and cannot be treated perturbatively.

To consider a structure of the JV quantitatively we calculate the phase stiffness due to the lattice deformations. This phase stiffness is mainly determined by the shear part of the elastic energy, which for dominating magnetic coupling can be written as [14,15]

$$\mathcal{E}_v = \frac{1}{2}\int\frac{d\mathbf{k}}{(2\pi)^3}\,[C_{66}k_\perp^2 + U_{44}(k_z)]\,|\mathbf{u}|^2, \qquad (1)$$

where $C_{66} = \frac{B_z\Phi_0}{(8\pi\lambda)^2}$ and

$$U_{44}(k_z) \equiv C_{44}(k_z)k_z^2 \approx \frac{B_z\Phi_0}{2(4\pi)^2\lambda^4}\ln\left(1 + \frac{r_{\mathrm{cut}}^2}{k_z^{-2} + r_w^2}\right),$$

with $r_{\mathrm{cut}} = \min(a,\lambda)$ and $r_w \approx u_{1x}(0) > s$. Using the relation between the phase perturbation and the lattice displacements, $\phi_v(\mathbf{k}) = 2\pi i n_v u/k_\perp$, we rewrite the elastic energy in terms of phase as

$$\mathcal{E}_v = \int\frac{d\mathbf{k}}{(2\pi)^3}\,\frac{J(B_z,\mathbf{k})}{2s}\,k_\perp^2|\phi_v(\mathbf{k})|^2, \qquad (2)$$

with the effective phase stiffness $J(B_z,\mathbf{k})$, $J(B_z,\mathbf{k}) = \frac{s(C_{66}k_\perp^2 + U_{44})}{(2\pi n_v)^2}$. In the range $k_\perp < 2/\lambda$ and $k_z > 1/r_w$ the phase stiffness is $\mathbf{k}$ independent,

$$J(B_z) \approx \frac{sU_{44}}{(2\pi n_v)^2} = J\,\frac{B_\lambda}{B_z}, \qquad B_\lambda \equiv \frac{\Phi_0}{4\pi\lambda^2}\ln\frac{r_{\mathrm{cut}}}{r_w};$$

i.e., magnetic field reduces the phase stiffness approximately by the factor $B_\lambda/B_z$. This justifies neglect of the regular phase in the region $B_z \gg B_\lambda$. The phase stiffness energy (2) has to be supplemented by the Josephson energy. Thermal motion of the PVs induces the fluctuating phase $\tilde{\phi}_{n,n+1}$ and suppresses the effective Josephson energy, $E_J \to CE_J$ where $C \equiv \langle\cos\tilde{\phi}_{n,n+1}\rangle$. Therefore the total energy in terms of $\phi_v$ is given by

$$\mathcal{E} = \sum_n\int d\mathbf{r}\left[\frac{J(B_z)}{2}(\nabla\phi_{vn})^2 - CE_J\cos(\phi_{vn,n+1})\right].$$

It has essentially the same form as in the Meissner state except for values of the phase stiffness and Josephson energy. Using known structure of an ordinary JV [12,13] we obtain for the core size and energy of the JV

$$\lambda_J(B_z) = \lambda_{J0}\sqrt{\frac{B_\lambda}{CB_z}}, \qquad \varepsilon_J(B_z) \approx \varepsilon_{J0}\sqrt{\frac{CB_\lambda}{B_z}}. \qquad (3)$$

The JV energy is reduced by the $c$-axis field, due to renormalizations of the phase stiffness and Josephson energy. The maximum lattice displacement $r_w$ and lattice deformation $u_{xy}$ in the vortex core can be estimated as $r_w \approx a\lambda/\lambda_{J0}$ and $u_{xy} \approx a\lambda/\lambda_{J0}^2$.

Interactions between the JVs are given by the anisotropic London theory with the anisotropy ratio $\gamma_{\text{eff}} = \gamma\sqrt{B_\lambda/(B_z C)}$. At finite density the JVs form a triangular lattice, which is stretched along the layers with the ratio $\gamma_{\text{eff}}$, i.e., the JVs form stacks along the $c$ axis with the period $c_z = \sqrt{\beta\Phi_0/(\gamma_{\text{eff}} B_x)}$ and these stacks are separated by the distance $c_y = \sqrt{\gamma_{\text{eff}}\Phi_0/(\beta B_x)}$. Within the anisotropic London theory two states with $\beta = 2\sqrt{3}$ and $\beta = 2/\sqrt{3}$ have the same energy [16]. These states correspond to the stretching of the regular triangular lattice with the closed-packed direction oriented along the layer and along the $c$ axis. The lattice energy $F_{JL}$ is given by

$$F_{JL}(B_z, B_x) = \frac{B_x^2}{8\pi} + \frac{B_x}{\Phi_0}\frac{\varepsilon_0}{\gamma}\sqrt{\frac{B_\lambda C}{B_z}}\ln\frac{c_z}{s}. \qquad (4)$$

We now compare this energy with the energy change corresponding to the homogeneous tilt of the lattice $\delta F_{\text{tilt}}$, which in magnetically coupled superconductors is determined by the tilt modulus $C_{44}^{(0)}$ at $k_z = 0$ [14,15]

$$\delta F_{\text{tilt}} = \frac{C_{44}^{(0)}}{2}\left(\frac{B_x}{B_z}\right)^2, \qquad C_{44}^{(0)} = \frac{B_z^2}{4\pi} + 3.68\frac{\Phi_0^2}{(4\pi\lambda)^4}. \qquad (5)$$

Comparing Eqs. (4) and (5) we see that the crossing-lattices state is a favorable state when the tilt angle of the field $\theta \approx B_x/B_z$ exceeds the critical value

$$\theta_0 = \frac{6.8}{\gamma}\sqrt{\frac{4\pi\lambda^2 B_z C}{\Phi_0}\ln\frac{r_{\text{cut}}}{r_w}}\ln\frac{c_z}{s}. \qquad (6)$$

At $\theta = \theta_0$ the system experiences a first order phase transition from the homogeneously tilted lattice to the crossing lattices of PVs and JVs. Taking $\gamma = 500$ and $\lambda = 200\ \text{nm}/\sqrt{1 - (T/T_c)^2}$ at $B_z = 130$ G and $T = 70$ K we obtain $\theta_0 \approx 3°$.

To derive the shift of the melting point by the JV lattice we write the thermodynamic condition for melting

$$F_{\text{cr}}(B_{zm}) + \delta F_{\text{cr}}(B_{zm}, B_x) = F_l(B_{zm}) + \delta F_l(B_{zm}, B_x),$$

where $F_{\text{cr}}(B_z)$ and $F_l(B_z)$ are the free energies of the crystal and liquid states for field along the $c$ axis, and $\delta F_{\text{cr}}(B_z, B_x)$ and $\delta F_l(B_z, B_x)$ are corrections due to $B_x$. For small $B_x$ we can write $B_{zm}(B_x) = B_{m0} + \delta B_m(B_x)$ and expand $F_{\text{cr}}(B_{zm}) - F_l(B_{zm})$ with respect to $\delta B_m(B_x)$, $F_{\text{cr}}(B_{zm}) - F_l(B_{zm}) \approx \delta B_m(B_x)\Delta M$, where $\Delta M = M_l - M_{\text{cr}} > 0$ is the magnetization jump at the melting point. This gives

$$\delta B_m(B_x) \approx -\Delta F(B_{m0}, B_x)/\Delta M, \qquad (7)$$

where $\Delta F(B_{m0}, B_x) = \delta F_{\text{cr}}(B_{m0}, B_x) - \delta F_l(B_{m0}, B_x)$. Similarly, we can relate the shift of the melting temperature $\delta T_m(B_x)$ with the melting entropy jump $\Delta S$ per single PV, $\delta T_m(B_x) \approx -s\Phi_0\Delta F(B_{m0}, B_x)/(B_{m0}\Delta S)$.

$\delta F_{\text{cr}}(B_{m0}, B_x)$ is determined by the energy of the Josephson lattice (4) (we estimated that entropic correction to $F_{JL}$ is relatively weak). In magnetically coupled superconductors the melting transition is accompanied by strong misalignment of PVs in the different layers [2]. This means that the Josephson coupling in the liquid state is strongly suppressed and the influence of the in-plane magnetic field in the liquid is much weaker than in the crystal, $\Delta F(B_{m0}, B_x) \approx F_{JL}(B_{m0}, B_x) - B_x^2/(8\pi) \propto B_x$. From Eqs. (4) and (7) we obtain that the shift in $B_{mz}$ is approximately linear with $B_x$ and

$$\frac{\partial B_m}{\partial B_x} = -\frac{\Phi_0}{4\pi\lambda^2\gamma\Delta B}\sqrt{\frac{B_\lambda C}{B_z}}\ln\frac{c_z}{s}, \qquad \Delta B \equiv 4\pi\Delta M. \qquad (8)$$

The linear dependence holds until the JV cores start to overlap at $B_x \approx \gamma\sqrt{B_\lambda B_z/C}$. In clean superconductors the linear dependence also breaks down at large enough $B_z$ when the decoupling transition occurs within the crystalline phase [17]. The melting field of the tilted vortex lattice in moderately anisotropic superconductors depends quadratically on the in-plane field [18] in agreement with experiment [19]. Therefore the linear dependence is an indication of the crossing-lattices state. The angular dependence of the melting field was measured in Ref. [6]. Figure 2 shows data from Fig. 4 of this paper for $T = 70$ and 80 K replotted in the coordinates $H_{mz} = H_m \cos\theta$ vs $H_{m\parallel} = H_m \sin\theta$. In both cases the linear dependence $H_{mz}(H_{m\parallel})$ is clearly observed. The same linear behavior of the melting field has been reported recently by Ooi *et al.* [7]. For $T = 70$ K we have $B_m = 133$ G, $\Delta B = 0.35$ G, and $\partial B_m/\partial B_x = -0.014$. Using the same parameters as before we obtain from Eq. (8) an estimate $\partial B_m/\partial B_x \approx -0.03$, in reasonable agreement with experiment.

We now consider the case of small $c$-axis fields, $B_z \ll \Phi_0/(\gamma s)^2$. In this case the JV core contains only one row of PVs (see Fig. 1b). Displacements of PVs do

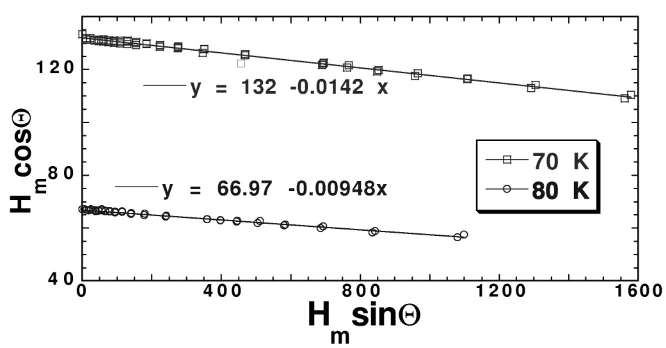


FIG. 2. Dependence of the $c$ component of the melting field on the in-plane field for BSCCO at $T = 70$ and 80 K. The data are taken from Ref. [6]. For both cases a linear dependence is clearly observed indicating the crossing-lattices ground state.

not strongly influence the JV structure but create a small pinning energy for the PV stack by the JV. Because of this energy one can expect at some field a phase transition from the weakly deformed triangular lattice to the chain state represented by regions of good lattice intercepted by the vortex chains located on the JV stacks. In the chain state the degeneracy in the JV lattice [16] is broken and the state with smaller $c_z$ corresponding to $\beta = 2/\sqrt{3}$ has smaller energy. The distance between the chains is simply given by the distance between the JV stacks $c_y = \sqrt{\sqrt{3}\,\gamma\Phi_0/(2B_x)}$ [11]. For $B_x \approx 30$ G and $\gamma = 200$ this gives $c_y \approx 10\ \mu$m, which roughly agrees with the distance between the chains observed in the decoration experiments [9,10]. We now obtain the criterion for the transition to the chain state.

First, consider one PV stack crossing one JV located between the layers 0 and 1. The crossing energy $E_\times$ appears due to displacements of PVs $u_n$ under the action of the in-plane currents $j_n$ induced by the JV

$$E_\times \approx \sum_n \left( \frac{U_{44}}{2n_v} u_n^2 - \frac{s\Phi_0}{c} j_n u_n \right) \qquad (9)$$

with $j_n \approx \frac{c\Phi_0}{8\pi^2\lambda^2} \frac{C_n}{(n-1/2)\gamma s}$, where $C_n \to 1$ at $n \to \infty$. For estimates we neglected the weak logarithmic $k_z$ dependence of $U_{44}(k_z)$. Minimization of (9) with respect to $u_n$ gives

$$u_n \approx \frac{2C_n\lambda^2}{(n - \frac{1}{2})\gamma s \ln(\lambda/r_w)},$$

$$E_\times = -\frac{A\Phi_0^2}{4\pi^2\gamma^2 s \ln(A_1\gamma s/\lambda)}.$$

Numerical calculations give $A = \sum_{n=1}^{\infty} (\frac{C_n}{n-1/2})^2 \approx 2.1$ and $A_1 \approx 3.5$. At finite $B_x$ each PV stack intersects $1/c_z$ JVs per unit length and the interaction energy of the PV stack with the JV stack is given by

$$\varepsilon_{Jp} = \frac{E_\times}{c_z} = \sqrt{\frac{\sqrt{3}\,\gamma B_x}{2\Phi_0}} \frac{A\Phi_0^2}{4\pi^2\gamma^2 s \ln(A_1\gamma s/\lambda)}. \qquad (10)$$

The transition to the chain state is expected when the energy gain (10) to put an extra PV stack on the JV stack exceeds the energy loss due to formation of the edge interstitial in the Abrikosov lattice $\varepsilon_{\mathrm{EI}}$ at $a \gg \lambda$ [20], $\varepsilon_{\mathrm{EI}} \approx 1.14\varepsilon_0\sqrt{a/\lambda}\exp(-a/\lambda)$. Comparison of the two energies gives the following equation for the PV lattice spacing $a_t$ at which the transition takes place:

$$\sqrt{\frac{a_t}{\lambda}} \exp\left(-\frac{a_t}{\lambda}\right) \approx \sqrt{\frac{\gamma s^2 B_x}{\Phi_0}} \frac{6.86\lambda^2}{\gamma^2 s^2 \ln(3.5\gamma s/\lambda)}. \qquad (11)$$

The transition field $B_t = 2\Phi_0/\sqrt{3}\,a_t^2$ grows slowly with $B_x$. For $\gamma = 200$, $\lambda = 220$ nm, and $B_x = 30$ G we obtain $B_t \approx 36$ G. The transition can be driven by the field or by the temperature (due to the $T$ dependence of $\lambda$).

In conclusion, we have shown that in very anisotropic layered superconductors the vortex ground state in tilted fields is given by the crossing lattices of the JVs and PV stacks. This ground state explains (i) the linear dependence of the $c$ component of the melting field on the in-plane field and (ii) the mixed chains-lattice state observed by Bitter decorations at small fields.

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