New Quantum Phase between the Fermi Glass and the Wigner Crystal in Two Dimensions

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(Received 21 December 1998; revised manuscript received 7 April 1999)

For intermediate Coulomb energy to Fermi energy ratios r_s, spinless fermions in a random potential form a new quantum phase which is neither a Fermi glass, nor a Wigner crystal. Studying small clusters, we show that this phase gives rise to an ordered flow of enhanced persistent currents for disorder strength and ratios r_s, where a metallic phase has been recently observed in two dimensions.

PACS numbers: 71.30.+h, 72.15.Rn

An important parameter for a system of charged particles is the Coulomb energy to Fermi energy ratio r_s. In a disordered two-dimensional system, the ground state is obvious in two limits. For large r_s, the charges form a kind of pinned Wigner crystal, the Coulomb repulsion being dominant over the kinetic energy and the disorder. For small r_s, the interaction becomes negligible and the ground state is a Fermi glass with localized one electron states. There is no theory for intermediate r_s, while many transport measurements following the pioneering works of Kravchenko *et al.* [1] and made with electron and hole gases give evidence of an intermediate metallic phase in two dimensions, observed [2], for instance, when $6 < r_s < 9$ for a hole gas in GaAs heterostructures. A simple model of spinless fermions with Coulomb repulsion in small disordered 2D clusters exhibits a new ground state characterized by an ordered flow of enhanced persistent currents for those values of r_s. In a given cluster, as we turn on the interaction, the Fermi ground state can be followed from $r_s = 0$ up to a first level crossing. A second crossing occurs at a larger threshold after which the ground state can be followed to the limit $r_s \rightarrow \infty$. There is then an intermediate state between the two crossings. In small clusters, the location of the crossings depends on the considered potentials, but a study over the statistical ensemble of the currents supported by the ground state gives us two well-defined values r_s^F and r_s^W: Mapping the system on a torus threaded by an Aharonov-Bohm flux, we denote, respectively, I_l and I_t the total longitudinal (direction enclosing the flux) and transverse parts of the driven current. One finds for their typical amplitudes $|I_t| \approx \exp-(r_s/r_s^F)$ and $I_l \approx \exp-(r_s/r_s^W)$ with $r_s^F < r_s^W$. Below r_s^F, the flux gives rise to a glass of local currents and the sign of I_l can be diamagnetic or paramagnetic, depending on the random potentials. Above r_s^F, the transverse current is suppressed while an ordered flow of longitudinal currents persists up to r_s^W, where charge crystallization occurs. The sign of I_l can be paramagnetic or diamagnetic, depending on the filling factor (as for the Wigner crystal), but does not depend on the random potentials (in contrast to the Fermi glass). One finds r_s^F and r_s^W in agreement with the values delimiting the new metallic phase when $0.3 < k_F l < 3$, k_F and l denoting the Fermi wave vector and the elastic mean free path, respectively. For $k_F l \geq 1$, I_l is strongly increased between r_s^F and r_s^W. This suggests that the intermediate phase of our model is related to the new metal observed in two dimensions by transport measurement which we shortly review.

In exceptionally clean GaAs/AlGaAs heterostructures, an insulator-metal transition (IMT) of a hole gas results [3] from an increase of the hole density induced by a gate. This occurs at $r_s \approx 35$, in close agreement with $r_s^W \approx 37$, where charge crystallization takes place according to Monte Carlo calculations [4], and makes highly plausible that the observed IMT comes from the quantum melting of a pinned Wigner crystal. The values of r_s where an IMT has been previously seen in various systems (Si-MOSFET, Si-Ge, GaAS) are given in Ref. [3], corresponding to different degrees of disorder (measured by the elastic scattering time τ). Those r_s drop quickly from 35 to a constant value $r_s \approx 8-10$ when τ becomes smaller. This is again compatible with $r_s^W \approx 7.5$ given by Monte Carlo calculations [5] for a solid-fluid transition in the presence of disorder. If the observed IMT is due to interactions, it might be expected that this metallic phase will cease to exist as the carrier density is further increased. This is indeed the case [2] for a hole gas in GaAs heterostructures at $r_s \approx 6$, where an insulating state appears, characteristic of a Fermi glass with electron-electron interactions.

In this paper, we take advantage of exact diagonalization techniques for large sparse matrices (Lanczos method), where tiny changes of energy can be precisely studied. This restricts us to small clusters and low filling factors. Fortunately, the dependence on particle number has proved to be remarkably weak in many cases. In the clean limit, calculations [6] with 6–8 particles give the condensation of the electron gas into an incompressible quantum fluid when a magnetic field is applied. Pikus and Efros [4] have obtained $r_s^W \approx 35$ from 6×6 clusters with six particles, close to $r_s^W \approx 37$ obtained by Tanatar and Ceperley for the thermodynamic limit [4]. In the disordered limit which we consider, there is another reason for expecting weak finite size effects. When the energy levels do not depend very much on the boundary conditions, the periodic repetition of the same cluster cannot drastically differ from the thermodynamic limit obtained from an ensemble of

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different clusters. This usual localization criterion applies for insulators such as the Fermi glass or the pinned Wigner crystal. Small cluster approximations should then be sufficient for small and large r_s. This explains why the critical factors r_s which we will discuss are close to the thermodynamic limit given by the experiments. Finite size effects can be important only if one has a metal for intermediate r_s.

We consider a simple model of $N = 4$ Coulomb interacting spinless fermions in a random potential defined on a square lattice with $L^2 = 36$ sites. The Hamiltonian reads

$$H = -t \sum_{\langle i,j \rangle} c_i^\dagger c_j + \sum_i v_i n_i + U \sum_{i \neq j} \frac{n_i n_j}{2 r_{ij}}. \qquad (1)$$

$c_i^\dagger$ (c_i) creates (destroys) an electron in the site i, t is the strength of the hopping terms between nearest neighbors (kinetic energy), and r_{ij} is the interparticle distance for a 2D torus. The random potential v_i of the site i with occupation number $n_i = c_i^\dagger c_i$ is taken from a box distribution of width W. The interaction strength U yields a Coulomb energy to Fermi energy ratio $r_s = U/(2t\sqrt{\pi n_e})$ for a filling factor $n_e = N/L^2$. The disorder to hopping energy ratio W/t is chosen such that $k_F l$ takes values where the IMT has been observed [1–3]. A Fermi golden rule approximation for τ gives [7] $k_F l \approx 192 \pi n_e (t/W)^2$. One has $n_e = 1/9$, $W/t = 5$, 10, and 15 corresponding to $k_F l = 2.7$, 0.67, and 0.3, respectively.

The boundary conditions are always taken periodic in the transverse y direction, such that the system becomes a torus enclosing an Aharonov-Bohm flux ϕ in the longitudinal x direction. Imposing $\phi = \pi/2$ ($\phi = \pi$ corresponds to antiperiodic condition), one drives a persistent current of total longitudinal and transverse components given by

$$I_l = -\frac{\partial E(\phi)}{\partial \phi} \bigg|_{\phi = \pi/2} = \frac{\sum_i I_i^l}{L} \qquad (2)$$

and $I_t = \sum_i I_i^t / L$, respectively. The local current I_i^l flowing at the site i in the longitudinal direction is defined by $I_i^l = 2\,\mathrm{Im}(\langle \Psi_0 | c_{i_x+1,i_y}^\dagger c_{i_x,i_y} | \Psi_0 \rangle)$ and by a corresponding expression for I_i^t. The response is paramagnetic if $I_l > 0$ and diamagnetic if $I_l < 0$. We begin by showing behaviors characteristic of a single cluster when r_s varies.

Figure 1 corresponds to $k_f l \leq 1$ ($W/t = 15$). Looking at the low energy part of the spectrum, one can see that, as we gradually turn on the interaction, classification of the levels remains invariant up to first avoided crossings, where a Landau theory of the Fermi glass is certainly no longer possible. Looking at the electronic density $\rho_i = \langle \Psi_0 | n_i | \Psi_0 \rangle$ of the ground state $|\Psi_0\rangle$, we have checked that it is mainly maximum in the minima of the site potentials for the Fermi glass. After the second avoided crossing, ρ_i is negligible except for four sites forming a lattice of charges as close as possible to the Wigner crystal triangular network in the imposed square lattice. The degeneracy of the crystal is removed by the disorder, the array being pinned in four sites of favorable energies.

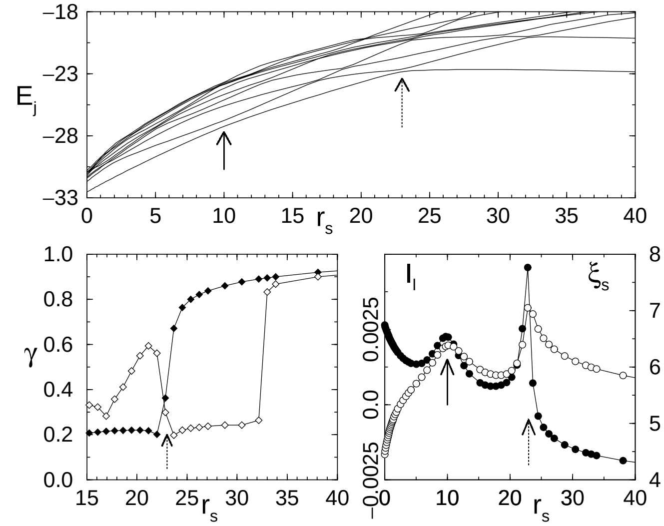

FIG. 1. Behavior of a single cluster for $k_F l \leq 1$ ($W/t = 15$) as a function of r_s. Top: low energy spectrum (a $1.9 r_s$ term has been subtracted); arrows show avoided crossings between the ground state and the first excited state. Bottom left: jumps of γ at the second crossing where the ground state (solid diamonds) and the first excited state (open diamonds) are interchanged. Bottom right: longitudinal current I_l (left scale, open circles) and number of occupied sites ξ_s (right scale, solid circles).

For the same cluster, we have calculated $C(r) = N^{-1} \times \sum_i \rho_i \rho_{i-r}$ and the parameter $\gamma = \max_r C(r) - \min_r C(r)$ used by Pikus and Efros [4] for characterizing the melting of the crystal. $\gamma = 1$ for a crystal and 0 for a liquid. Calculated for the ground state and the first excited state, $\gamma(r_s)$ allows us to identify the second crossing with the melting of the crystal. Moreover, one can see that the crystal becomes unstable in the intermediate phase, while the ground state is related to the first excitation of the crystal (Fig. 1, bottom left). Around the crossings, the longitudinal current I_l and the participation ratio $\xi_s = N^2 (\sum_i \rho_i^2)^{-1}$ of the ground state (i.e., of the number of sites that it occupies) are enhanced (Fig. 1, bottom right). The general picture is somewhat reminiscent of strongly disordered chains [8], where level crossings associated with charge reorganizations of the ground state are accompanied by enhancements of the persistent currents. Figure 1 is representative of the ensemble, with the restriction that the location of the crossings fluctuates from one sample to another as well as the sign (paramagnetic or diamagnetic) of I_l below the first crossing, in contrast to 1D.

Figure 2 corresponds to $k_f l \geq 1$ ($W/t = 5$). The previous level crossings are now almost suppressed by a stronger level repulsion and charge crystallization occurs more continuously. There is instead a broad enhancement of I_l which, in contrast to Fig. 1, is not accompanied by a corresponding increase of ξ_s, which smoothly decreases from 20 of the 36 possible sites down to 4 when charge

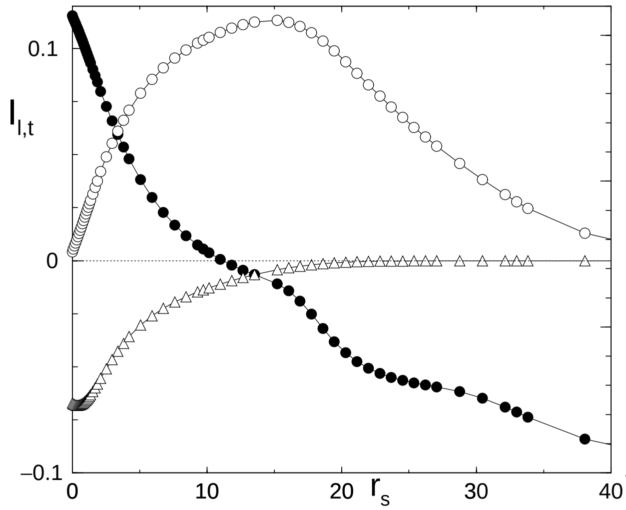

FIG. 2. Behavior of a single cluster for $k_F l \geq 1$ $(W/t = 5)$ as a function of r_s. Left coordinates: currents I_l (open circles) and I_t (triangles). Right coordinates: ξ_s (solid circles).

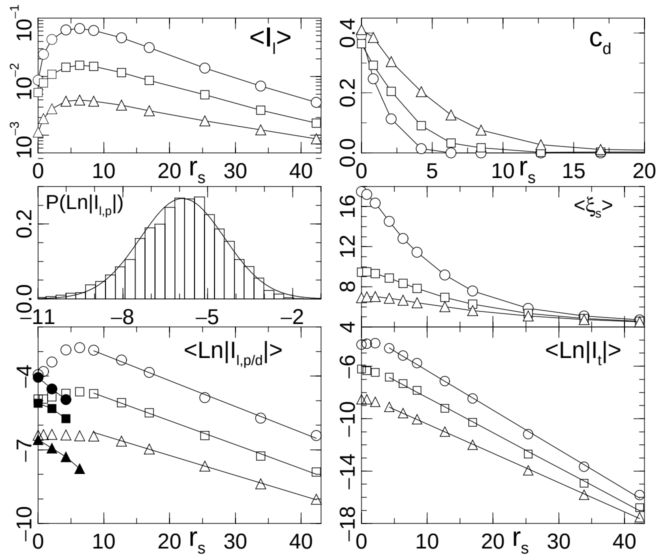

FIG. 3. Statistical study of an ensemble of clusters for $W/t = 5$ (circles), 10 (squares), and 15 (triangles) as a function of r_s. Top left: mean value $\langle I_l \rangle$ of the longitudinal current. Top right: fraction C_d of diamagnetic samples. Middle left: distribution of the logarithms of the paramagnetic current $I_{l,p}$ at $r_s = 1.7$ and $W/t = 15$. Middle right: mean number $\langle \xi_s \rangle$ of sites occupied by the ground state. Bottom left: longitudinal paramagnetic (open symbols) and diamagnetic (solid symbols) currents. Bottom right: transverse currents. The straight lines are exponential fits giving r_s^F and r_s^W values shown in Fig. 4.

crystallization becomes perfect. A transition of the persistent current, from a disordered array of loops towards an ordered flow as r_s increases, has been noticed [9] by Berkovits and Avishai. To illustrate this phenomenon, the total transverse current I_t is shown in Fig. 2. One can see that I_t is suppressed at $r_s \approx 5$ while I_l continues to increase up to $r_s \approx 15$. We have checked that a disordered array of loops persists up to $r_s \approx 5$, followed by an ordered flow of enhanced longitudinal currents persisting up to $r_s \approx 15$. The disordered array of loops gives rise to a diamagnetic or paramagnetic current I_l, depending on the microscopic disorder. The ordered flow gives rise to a paramagnetic I_l. However, Coulomb repulsions do not always yield a paramagnetic response. For instance, 4×6 clusters with $N = 6$ always become diamagnetic at large r_s. One can only conclude that the sign of the response in 2D does not depend on the random potential when r_s is sufficient for suppressing I_t. In 1D, Legett's theorem [10] states that the sign of I_l depends on the parity of N only, for all disorder and interaction strength. The proof is based on the nature of "nonsymmetry dictated nodal surfaces," which is trivial in 1D, but which has a quite complicated topology in higher dimensions. It is likely that such a theorem could be extended in 2D when the transverse flow is suppressed.

We now present a statistical study of an ensemble of 10^3 clusters for $W/t = 5$, 10, and 15. At the top left of Fig. 3, one can see an increase of the mean I_l by about 1 order of magnitude when $r_s \approx 7$ for $W/t = 5$. We note that the persistent currents [11] measured in an ensemble of mesoscopic rings are typically higher by a similar amount than the theoretical prediction neglecting the interactions. At the right top of Fig. 3, the fraction of diamagnetic clusters is given as a function of r_s, showing that the enhancement of the mean is partially related to the suppression of the diamagnetic currents. This suppression is faster for weak disorders. The mean number ξ_s of sites occupied by the ground state is given at the middle right of Fig. 3, showing a negligible increase when $W/t = 15$ at low r_s and a regular decay otherwise. The paramagnetic $I_{l,p}$ and diamagnetic $I_{l,d}$ longitudinal currents and $|I_t|$ have log-normal distributions for all values of r_s when $W/t \geq 5$. The stronger the disorder, the better the log-normal shape of the distribution (see middle left of Fig. 3). The average of the logarithms gives the typical values shown in the bottom part of Fig. 3. On the left, the longitudinal currents I_l are given, the diamagnetic responses $I_{l,d}$ (solid symbols) being separated from the paramagnetic responses $I_{l,p}$ (open symbols), while the transverse currents I_t are given at the right side. The log averages exponentially decay as $I_{l,d} \propto |I_t| \propto \exp-(r_s/r_s^F)$ and $I_{l,p} \propto \exp-(r_s/r_s^W)$ when r_s is large enough. The variances of $\log|I_t|$ and $\log I_l$ increase as r_s/r_s^F and r_s/r_s^W above r_s^F and r_s^W, respectively. The values of r_s^F and r_s^W extracted from the exponential fits (straight lines in Fig. 3) are given in Fig. 4, where a sketch of the phase diagram is proposed.

Figures 3 and 4 show that a simple model of spinless fermions can account for the critical carrier densities and disorder strengths where the IMT occurs. The comparison between the curve $r_s(\tau)$ given in Ref. [3] (summarizing the factors r_s where the IMT has been observed) and the curve $r_s^W(k_F l)$ of Fig. 4 (characterizing the suppression of I_l) is striking. The value $r_s = 6$, where the reentry has been

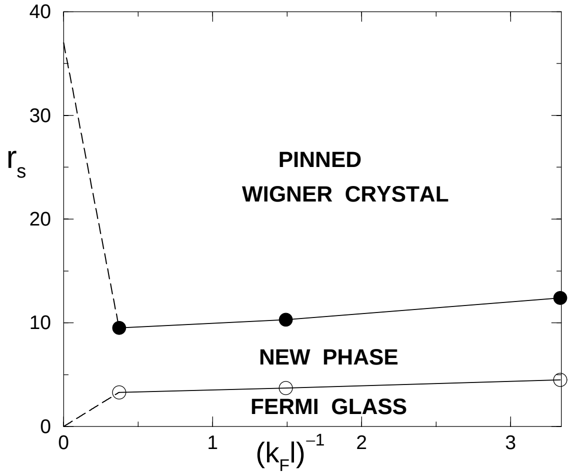

FIG. 4. Proposed phase diagram for 2D spinless fermions in a random potential. r_s^W (solid circles) and r_s^F (open circles) obtained from Fig. 3 bottom.

observed in Ref. [2], is compatible with the curve $r_s^F(k_F l)$ characterizing the suppression of I_t. The spin degrees of freedom are not included in our model, the orbital part of the wave function being totally antisymmetrized. This restriction is quite important for short range screened interactions, but is certainly less severe for long range interactions and low densities. However, there is much experimental evidence [12] that spin effects play a role. But even for 2D spinless fermions, we conclude that there is a new quantum phase between the Fermi glass and the Wigner crystal, identified by a plastic flow of currents without charge crystallization. Small cluster studies do not allow us to know with certainty if the identified r_s factors correspond to real transitions delimiting a metallic phase, or to some simpler crossover phenomena. However, a recent extension of finite size scaling [13] to the many body ground state (assuming certain unavoidable approximations) strongly suggests that a real transition towards a metallic phase occurs at the first threshold ($r_s \approx 4$). We have not indicated in the proposed phase diagram the difference between $k_F l \geq 1$ (where I_l has a strong enhancement) and $k_F l \leq 1$ (where I_l persists up to r_s^W without noticeable enhancement). A finite size scaling study will again be necessary for understanding if a transition can be driven by an increase of $k_F l$ at intermediary r_s. ξ_s and I_l convey similar information when $k_F l < 1$ while the increase of I_l is accompanied by a decrease of ξ_s when $k_F l > 1$. This suggests that transport for intermediary r_s results more from a collective motion of charges than from a delocalization of individual charges.

This work is partially supported by a TMR network of the EU.

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