Libbrecht and Tanusheva Reply: Brener and Müller-Krumbhaar [1] raise an interesting point, and we agree with their main objection that our choice of the equilibrium condition at the interface is incorrect. However, using the modified boundary condition [their Eq. (3)] yields an electrically enhanced growth behavior that does not include runaway growth above a threshold potential, in contrast to our observations. Furthermore, it seems unlikely to us that extending the model beyond the static approximation is the answer to this dilemma. We estimate that $v\eta_1/D \lesssim (5\ \mu\text{m/sec})\,(0.05\ \text{m})/(2\times 10^{-5}\ \text{m}^2/\text{sec}) = 0.012$, which is much less than unity, thus suggesting the validity of the static approximation over the range of our experiments.

Including the electrical change in the solid chemical potential appears to be the most likely explanation of the phenomenon. This is easily calculated for the spherical case, where it arises simply from the electrostatic self-energy of a charged droplet. Including this effect yields the equilibrium vapor pressure

$$c(R) \approx c_{\text{sat}}\left(1 + \frac{d_0}{R} - \frac{R_{\text{es}}^2}{R^2} + \frac{R_{\text{pol}}^2}{R^2}\right)$$

to lowest order in the applied potential φ_0, where $R_{\text{es}} = (\varepsilon_0\varphi_0^2/2c_{\text{solid}}kT)^{1/2}$ and $R_{\text{pol}} = (\alpha\varphi_0^2/kT)^{1/2}$, with c_{solid} equal to the solid number density and α equal to the molecular polarizability. Solving the Smoluchowski equation for the spherical case then yields the droplet growth velocity

$$v(R) \approx \frac{Dc_{\text{sat}}}{Rc_{\text{solid}}}\left(\Delta_1 - \frac{d_0}{R} + \frac{R_{\text{es}}^2}{R^2}\right).$$

This result is identical to what we found previously [2], except that we have replaced R_{pol} with R_{es}. It then follows that the growth is electrically enhanced, and exhibits the observed runaway behavior above a threshold potential. For the experimental situation in question we have $R_{\text{es}} \approx R_{\text{pol}}/5$, which increases the value of the threshold potential by roughly a factor of 5. Since we could not measure the dendrite tip radius in our experiments, nor can we accurately calculate it given the uncertainties in the crystal parameters of ice, we can only roughly estimate the expected value of the threshold potential. Thus this new model remains consistent with our observations.

As pointed out by Brener and Müller-Krumbhaar, adopting this new picture leads one to the conclusion that the observed electrically enhanced dendrite growth behavior may appear even in the absence of a molecular polarizability. Thus this may be present in a much wider class of materials. Additional quantitative experiments would clearly be very useful for a better understanding of this phenomenon.

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[1] E. A. Brener and H. Müller-Krumbhaar, preceding Comment, Phys. Rev. Lett. **83**, 1698 (1999).

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