Electromodulation of the Bilayered $\nu = 2$ Quantum Hall Phase Diagram

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We make a number of precise experimental predictions for observing the various magnetic phases, and the quantum phase transitions between them, in the $\nu = 2$ bilayered quantum Hall system. In particular, we analyze the effect of an external bias voltage on the quantum phase diagram, finding that a finite bias should readily enable the experimental observation of the recently predicted novel canted antiferromagnetic phase in transport and spin polarization measurements.

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Recent theoretical work [1–3] predicts the existence of a novel canted antiferromagnetic (C) phase in the $\nu = 2$ bilayered quantum Hall system under quite general experimental conditions, and encouraging experimental evidence in its support has recently emerged through inelastic light scattering spectroscopy [4,5] and transport measurements [6]. Very recent theoretical works have shown that such a C phase may exist [7] in a multilayer superlattice system (with $\nu = 1$ per layer) and that in the presence of disorder-induced-interlayer tunneling fluctuations the C phase may break up into a rather exotic spin Bose glass phase [8] with the quantum phase transition between the C phase and the Bose glass phase being in the same universality class as the two-dimensional superconductor-insulator transition in the dirty boson system. In this Letter we consider the effect of an external electric field induced *electromodulation* (through an applied gate bias voltage) of the $\nu = 2$ bilayered quantum phase diagram. We assume that the gate bias voltage does not change the total electron density $\nu = 2$ of the system but provides an electrostatic potential between the layers. Our goal is to provide precise experimental predictions which will facilitate direct and unambiguous observations of the various magnetic phases and, more importantly, the quantum phase transitions among them. We find the effect of a gate bias to be quite dramatic on the $\nu = 2$ bilayered quantum phase diagram. In particular, a finite gate bias makes the C phase more stable which could now exist even in the absence of any interlayer tunneling in contrast to the situations considered in Refs. [1–3] where the interlayer tunneling induced finite symmetric-antisymmetric gap was crucial in the stability of the C phase. Thus, a finite gate bias, according to our theoretical calculations presented here, has a qualitative effect on the $\nu = 2$ bilayered quantum phase diagram—it produces a *spontaneously interlayer-coherent canted antiferromagnetic phase* which exists even in the absence of any interlayer tunneling. The prediction of this spontaneously coherent canted (CC) phase is one of the new theoretical results of this paper. The theoretical construction of the bilayered $\nu = 2$ quantum phase diagram and predicting its experimental consequences in the presence of the bias voltage are our main results. Details of the calculations reported here will be presented elsewhere [9].

The bilayered $\nu = 2$ system is characterized by five independent energy scales: the cyclotron energy, ω_c (we take $\hbar = 1$ throughout); the interlayer tunneling energy characterized by Δ_{SAS}, the symmetric-antisymmetric energy gap; the Zeeman energy or the spin-splitting Δ_z; the intralayer Coulomb interaction energy; and the interlayer Coulomb interaction energy. The application of the external electric field adds another independent energy scale, the bias voltage, to the problem. Neglecting the largest (ω_c, which we take to be very large) energy scale, one is still left with four independent dimensionless energy variables to consider in constructing the $\nu = 2$ bilayered quantum phase diagram in the presence of finite bias. In the absence of any bias the quantum phase diagram is surprisingly simple, allowing for only three qualitatively different quantum magnetic phases, as established by a microscopic Hartree-Fock theory [1–3,7], a long wavelength field theory based on the quantum $O(3)$ nonlinear sigma model [2,3] and a bosonic spin theory [8]. These three magnetic phases are the fully spin polarized ferromagnetic phase (F), which is stabilized for large values of Δ_z (or for strong intralayer Coulomb interaction), the paramagnetic symmetric or spin singlet (S) phase, which is stabilized for large values of Δ_{SAS} (or for strong interlayer Coulomb interaction), and the intermediate C phase, where the electron spins in each layer are tilted away from the external magnetic field direction due to the competition between ferromagnetic and singlet ordering. Note that the S phase is fully pseudospin polarized with Δ_{SAS}, the symmetric-antisymmetric gap, acting as the effective pseudospin splitting. The C phase is a true many-body symmetry-broken phase not existing in the single particle picture (and is stabilized by the interlayer antiferromagnetic exchange interaction). The single particle theory predicts a level crossing and a direct first order transition between the S phase and the F phase (nominally at $\Delta_z = \Delta_{\mathrm{SAS}}$) as the Zeeman splitting increases. Coulomb

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interaction creates the new symmetry broken C phase, which prevents any level crossing (and maintains an energy gap throughout so that there is always a quantized Hall effect) between F and S phases, and makes all phase transitions in the system continuous second order transitions. The canted phase is canted in both the spin and pseudospin spaces.

One key experimental difficulty in observing the predicted phase transitions (in the absence of any external bias voltage) is that a given sample (with a fixed value of Δ_{SAS}, determined by the system parameters such as well widths, separations, etc.) is always at a fixed point in the quantum phase diagram calculated in Refs. [1–3,7,8] because Δ_z, Δ_{SAS}, and the Coulomb energies are all fixed by the requirement $\nu = 2$ and the sample parameters. Therefore, a given experimental sample in this so-called *balanced* condition (i.e., no external bias, equal electron densities in the two layers on the average) is constrained to lie in the F or C or S phase, and the only way to see any phase transitions is to make a number of samples with different parameters lying in different parts of the phase diagram and to investigate and compare properties, as was done in the light scattering experiments of Refs. [4,5]. This is obviously an undesirable situation because what one really wants is to vary an experimental control parameter (e.g., an external electric field) to tune the system through the phase boundaries and study the quantum phase transition instead of studying different samples. (Theoretically this tuning is easily achieved by making Δ_z, Δ_{SAS}, and the Coulomb energies continuous variables in the phase diagram [1–3,7,8], but experimentally, of course, this cannot be done.) In this Letter we show that an externally applied electric field through a gate bias, which takes one away (*off balance*) from the balanced condition and introduces [10] unequal layer electron densities is potentially an extremely powerful experimental tool in studying the $\nu = 2$ bilayered quantum phase transitions. Our results indicate that using an external gate bias as a tuning parameter, a technique already extensively used [6,11,12] in experimental studies of bilayered structures, should lead to direct experimental observations of the predicted quantum phases in $\nu = 2$ bilayered systems and the continuous transitions between them in both transport measurements [6,11,12] and in spin polarization measurements through NMR Knight shift experiments [13].

We have used two complementary techniques, the direct Hartree-Fock theory [1–3,7] and the effective bosonic spin [8] theory, to evaluate the bilayered $\nu = 2$ quantum phase diagram including the effect of a finite bias voltage. The resulting bias dependent phase diagrams (in the $\Delta_z - \Delta_{\mathrm{SAS}}$ space) for the Hartree-Fock theory and the bosonic spin theory are shown in Figs. 1 and 2, respectively. Although there are some quantitative differences between the phase diagrams in the two models (to be discussed below), the main qualitative features are the same: increasing bias voltage enhances the *phase space* of the C phase mostly at the cost of the F phase, and for large enough bias the C phase becomes stable even for $\Delta_{\mathrm{SAS}} = 0$; this CC phase is spontaneously coherent. We note that the CC phase (i.e., the bias induced C phase along the $\Delta_{\mathrm{SAS}} = 0$ line) and the C phase are continuously connected and there is no quantum phase transition between them. Note that the S phase, which is the singlet or the symmetric phase, is also stabilized for $\Delta_{\mathrm{SAS}} = 0$ by finite bias effects. This phase (i.e., the S phase along the $\Delta_{\mathrm{SAS}} = 0$) is the spontaneous interlayer coherent symmetric or singlet phase (the CS phase) and is analogous to the corresponding $\nu = 1$ spontaneous interlayer coherent phase studied extensively [14] in the context of the $\nu = 1$ bilayered quantum phase diagram. There is, however, a fundamental difference between the

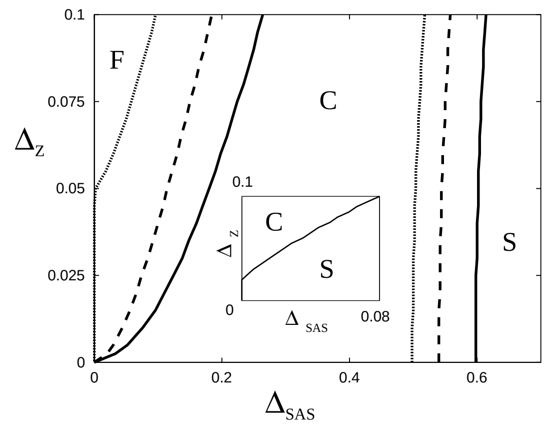

FIG. 1. Calculated Hartree-Fock $\nu = 2$ bilayered phase ($\Delta_{\mathrm{SAS}} - \Delta_z$) diagrams for different bias V_+. Continuous, dashed, and dotted lines correspond to $V_+ = 0$, 0.5, and 0.65, respectively. The inset corresponds to the $V_+ = 1.42$ case. The length and the energy units are the magnetic length, ℓ, and the intralayer Coulomb energy $e^2/\epsilon\ell$. The interlayer separation is 1.

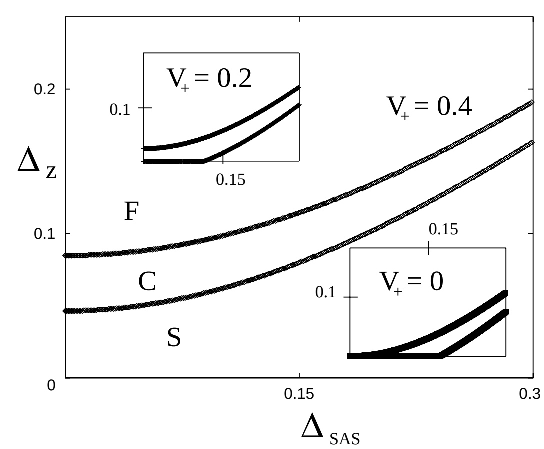

FIG. 2. Phase diagram in the bosonic spin theory for different bias voltages. All the units and parameters are the same as in Fig. 1. (See Ref. [8] for details on the bosonic spin model parameters.)

coherent CS phase for our $\nu = 2$ bilayered system and the corresponding [14] $\nu = 1$ spontaneous interlayer coherent phase; our $\nu = 2$ bilayered CS phase can exist only under a *finite external bias* (the same as our CC phase). Unlike the corresponding $\nu = 1$ bilayered system [14] or the recently studied zero magnetic field bilayered system [15], there is no spontaneous breaking of the pseudospin U(1) symmetry (generated by the interlayer electron density difference) in our $\nu = 2$ coherent bilayered phases which can exist only in the presence of an external voltage bias. We emphasize that there is no analogy to our canted phase (C or CC phase) in the corresponding $\nu = 1$ bilayered quantum phase diagram [14].

We note that the main difference (cf. Figs. 1 and 2) between the Hartree-Fock [1–3,7] theory and the bosonic spin [8] theory is that the Hartree-Fock theory underestimates the stability of the S phase (compared with the bosonic spin theory) at small values of Δ_z. This is a real effect and arises from the neglect of quantum fluctuations in the Hartree-Fock theory which treats the interlayer tunneling as a first order perturbation correction in the S phase. The bosonic spin theory is essentially exact for the S phase and is therefore more reliable near the C-S phase boundary, particularly for small values of Δ_z where tunneling effects are important.

In Fig. 3 we show our calculated quantum phase diagrams in the gate voltage (V_+)-tunneling (Δ_{SAS}) space for fixed values of the Zeeman energy Δ_z (and the Coulomb energies) using both the Hartree-Fock and bosonic spin theories. The phase diagrams in the two theories are qualitatively similar, and the interlayer coherent phases (CC and CS phases) are manifestly obvious in Fig. 3 because the C and the S phases now clearly extend to the $\Delta_{SAS} = 0$ line (the ordinate) for finite bias voltage. In general, the presence of bias therefore allows for six different quantum magnetic phases in the $\nu = 2$ bilayered system: the usual F, C, and S phases of Refs. [1–3] as well as the purely Néel (N) phase [1–3] along the $\Delta_z = 0$ line in Fig. 1 (the F, C, S, and N phases are all allowed in the balanced $V_+ = 0$ situation), and two new (bias-induced) coherent phases (CC and CS) along the $\Delta_{SAS} = 0$ line in Figs. 1–3. The most important effect of the external bias, which is an important new prediction of the current paper, is that it allows for a continuous tuning of the quantum phase of a $\nu = 2$ bilayered system within a single gated sample, as is obvious from Figs. 1–3. The predicted quantum phase transitions can now be studied in light scattering [4,5], transport [6,11], and NMR [13] experiments in single gated samples by tuning the bias voltage to sweep through various phases as shown in Figs. 1–3.

The last issue we address here is what one expects to see experimentally in transport and spin polarization measurements, in sweeping through the phase diagram of Figs. 1–3 under an external gate bias. In Fig. 4 we show our calculated results for the variation in the spin polarization of the system as a function of the bias V_+ with all the other system parameters being fixed. As expected the spin polarization is complete in the F phase and remains a constant as a function of V_+ until it hits the F-C phase boundary where it starts to drop continuously through the C phase, essentially dropping to zero at the C-S phase boundary, remaining zero in the S phase. At zero temperature the two phase transitions (i.e., F-C and C-S) are characterized by cusps in the spin polarization (Fig. 4) which perhaps will not be observable in finite temperature experiments. The main features of the calculated spin polarization (per unit area) as a function of bias, as shown in Fig. 4, should, however be readily observable in NMR Knight shift measurements [13]. We have also carried out calculations

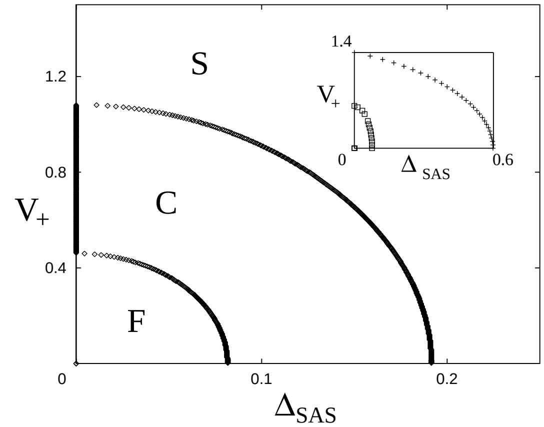

FIG. 3. Calculated phase diagrams in the $V_+ - \Delta_{SAS}$ space for fixed Δ_z. Main figure: Bosonic spin phase diagram for $\Delta_z = 0.1$ (all other parameters correspond to Fig. 2). Inset: Hartree-Fock phase diagram for $\Delta_z = 0.01$ (all other parameters correspond to Fig. 1).

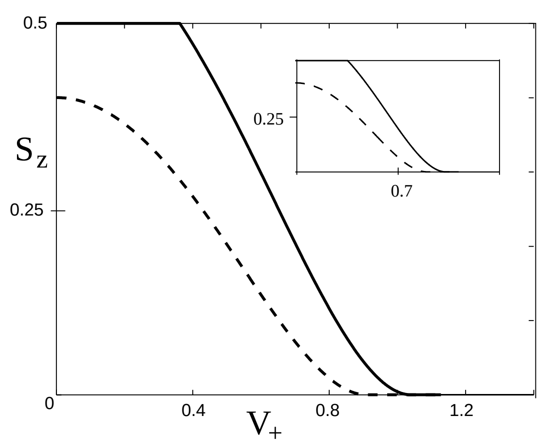

FIG. 4. Calculated z component of the total spin polarization in each of the layers $\langle S_z \rangle$ as a function of the bias V_+ for fixed Δ_z and Δ_{SAS}. This quantity is proportional to the Knight shift [12]. Main figure: Solid line Hartree-Fock theory for $\Delta_{SAS} = 0.05$ and dashed line for $\Delta_{SAS} = 0.1$ ($\Delta_z = 0.01$ throughout); all other parameters correspond to Fig. 1. Inset: Bosonic spin theory for the same parameters.

of the interlayer charge imbalance (which is zero in the F phase and then rises continuously throughout the C and S phases reaching full charge polarization for large V_+ in the S phase) as a function of the bias voltage. There are two cusps in the calculated imbalance as a function of V_+, corresponding to the $F \to C$ and the $C \to S$ phase transitions, which should be experimentally observable (they may, however, be washed out by disorder which is also likely to lead to the spin Bose glass phase as discussed in Ref. [8]). The calculated imbalance therefore looks almost exactly complementary to the spin polarization results shown in Fig. 4. Finally we have also calculated the charged excitation energies within the simple Hartree-Fock and bosonic theories (assuming no textural excitations such as skyrmions or merons), which lead to weak cusps in the activation energies at the phase boundaries. Using the parameters of the samples in Ref. [6], we conclude from our numerical calculations [16] that the phase transition being observed in the $\nu = 2$ bilayered transport experiments of Ref. [6] is the transition from the C phase to the S phase as a function of the density (and *not* from the F phase to the C phase as implied in Ref. [6]). Neither phase in Ref. [6] is a spontaneous interlayer coherent phase (because Δ_{SAS} is finite in the experiment) in contrast to the claims of Ref. [6]. Our results indicate, however, that it should be possible to see all three $\nu = 2$ quantum phases (F, C, and S) in a single gated sample by varying the bias voltage. We hope that the detailed results presented in this paper will encourage future bilayered $\nu = 2$ experiments under external gate bias to explore the predicted rich phase diagram.

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[16] Our numerical Hartree-Fock calculations (not shown here for the sake of brevity) for the samples of Ref. [6] take into account the actual well widths and other sample parameters given in Ref. [6], and also include realistic Landau level mixing effects. We find that the $\nu = 2$ sample (Fig. 4 of Ref. [6]) at the balanced density point to be in the S phase for density $\lesssim 0.5 \times 10\ \mathrm{cm}^{-2}$ and in the F phase for density $\gtrsim 2.8 \times 10\ \mathrm{cm}^{-2}$, and to be in the C phase in between.