

Time-Dependent Feshbach Resonance Scattering and Anomalous Decay of a Na Bose-Einstein Condensate

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(Received 9 February 1999)

We study Feshbach resonance scattering in a time-dependent magnetic field. We explain the extremely rapid decay observed in a recent experiment investigating Feshbach resonances in a Na Bose-Einstein condensate. In our picture, the decay is stimulated by the formation of a molecular condensate of quasibound atom pairs. Another essential element is the concept of a global and a local resonance lifetime. The predicted decay rates are large, about 5 orders of magnitude larger than typical dipole decay rates, and 1 order larger than typical exchange decay rates. We point out the possible role of a Josephson-like oscillation between the atomic condensate and a long-range molecular condensate.

PACS numbers: 32.80.Pj, 03.75.Fi, 34.50.-s

A remarkable aspect of the recently realized Bose-Einstein condensates in dilute alkali and hydrogen atomic gases [1] is the prominent role of atom-atom interactions. For instance, the linear dimensions of a trapped condensate may be several times larger than for the quantum mechanical ground state in the trap potential [2], i.e., the state of the condensate without interactions. Other examples are the spin domain structure recently observed in a spinor condensate [3], an amazing and counter-intuitive phenomenon for a dilute system, and the fascinating recent four-wave mixing experiment [4]. There is reason to expect that interactions will also be important for future developments in the direction of coherent matter waves. The fact that such waves can interact may well be one of the most important advantages of atom lasers compared to optical lasers.

Unlike previously studied degenerate systems such as ^4He , the new quantum liquids can be understood quantitatively on the basis of first principles. Most of their properties can be expressed with the aid of a single interaction parameter: the scattering length a . Interestingly, this parameter can be experimentally modified. A promising way to do this relies on the strong variation of a that occurs if a Feshbach resonance is tuned through zero energy by varying an external magnetic field [5]. Such resonances have been observed in Na [6] and in ^{85}Rb [7]. In the Na experiment the scattering length was seen to vary dispersively as a function of the magnetic field as predicted:

$$a(B) = a_\infty \left(1 + \frac{\Delta}{B_0 - B} \right), \quad (1)$$

where a_∞ is the off-resonant scattering length, and Δ characterizes the width of the resonance as a function of B . Feshbach resonances thus offer the possibility to study quantum liquids and coherent matter in widely varying circumstances with positive, negative, and zero values of a in a single experiment. In particular, it should be

possible to see the predicted [8], but still unobserved collapse of a condensate if its scattering length is suddenly shifted to a sufficiently negative value.

Cold atom Feshbach resonances are exceptional also in another sense. For practically realizable time-varying fields, it is possible to change the properties of a scattering process significantly while it is going on. It is the purpose of this Letter to point out that this was in fact an important element in a recent experiment by Stenger *et al.* [9] at MIT. In an attempt to realize a maximum variation of a in an optically trapped Na Bose-Einstein condensate, they observed a strong decay of the condensate when a resonance was approached or crossed with the external magnetic field. The experiment consisted of runs of two types: (1) Runs in which the magnetic field was changed adiabatically from an off-resonant value to a value near a resonance field strength without crossing the resonance; (2) runs in which a resonance was crossed with high ramp speed, beginning and ending with off-resonant fields. A mechanism for the type 1 observations has been proposed by Timmermans *et al.* [10] and will be briefly recapitulated later in this paper. We will focus on the anomalous decay in the second type of experiment. As pointed out by Stenger *et al.*, the experimental data suggest decay rates far larger than expected for any of the known two-body and three-body mechanisms. We will present a new picture based on time-dependent Feshbach resonance scattering.

Feshbach resonances arise when the total energy of a pair of colliding atoms matches the energy of a quasibound two-atom state, leading to the resonant formation of this state during the collision [11]. Figure 2 of Ref. [12] shows an example of the enhancement of the collisional wave function near a resonance, reflecting the increased amplitude of the admixed quasibound state. Magnetic tuning is possible if the pair of free atoms and the quasibound state have different magnetic moments $\mu_{\text{free}} - \mu_{\text{qb}} = \Delta\mu \neq 0$, giving rise to different Zeeman dependencies. Figure 1 shows the crossings of two Na

quasibound states with the collision threshold at $B = 853$ and 907 G. The corresponding resonances were observed [9] in a condensate of atoms in the lowest hyperfine state $|f, m_f\rangle = |1, +1\rangle$, with $\vec{f} = \vec{s} + \vec{i}$ the total atomic spin. First, we will focus on the resonance at 853 G.

In our picture the extremely rapid loss of the atomic condensate in the high ramp speed experiments is due to the fact that the formation and the decay of the resonance state occur at two different field values as a consequence of the delay caused by the resonance lifetime $\tau_{\text{res},0}$. Whereas outside the resonance the energy of each of the two atoms follows adiabatically the single-atom Zeeman dependence (see schematic inset in Fig. 1), this energy decrease is interrupted during the lifetime of the resonance by a rate of change of energy, different by $\Delta\mu\dot{B}$. The result is a significantly increased kinetic energy of the free two-atom state arising from the decay of the quasibound state. This increase can be estimated as $\Delta\mu\dot{B}\tau_{\text{res},0} \approx k_B[0.5 \text{ to } 3.5 \mu\text{K}]$ (the Boltzmann constant k_B will be omitted in all equations below where energy is expressed in degrees kelvin). Because it is larger than the mean-field energy in the condensate, the accelerated atoms are counted as lost from the condensate when the remaining number of atoms is determined after the sweep [13]. In short, the quasibound pairs start to be formed from the atomic condensate, stimulated by a bosonic stimulation factor, and form a ‘‘molecular’’ condensate [14]. On the other hand, their decay back to the atomic condensate is almost completely suppressed due to the above kinetic energy gain. Instead, a very rapid decay

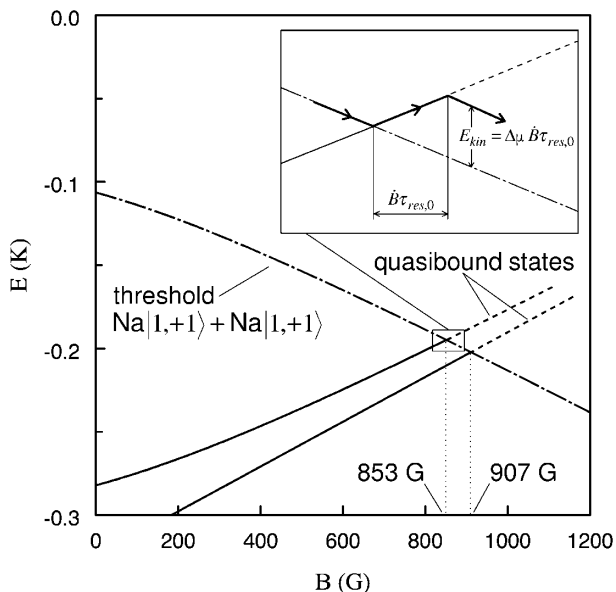


FIG. 1. Crossing of quasibound two-atom states with Na + Na collision threshold at $B = 853$ and 907 G, due to different Zeeman dependencies of quasibound states and state of free atoms. Inset: Schematic illustration of proposed loss mechanism.

takes place to a noncondensed atom fraction, which is effectively observed as a loss process. The possibility that the resonantly formed quasibound atom pairs form a condensate was previously suggested by Timmermans *et al.* [10]. In contrast to their proposal, however, a crucial element of our description is the above decay to a noncondensed atom fraction.

Interestingly, for the above estimate of the kinetic energy it is not the usual ‘‘global’’ resonance lifetime $\tau_{\text{res}} = \hbar/\gamma$ (where γ is the decay width) that is relevant but the, often much shorter, ‘‘local’’ lifetime $\tau_{\text{res},0} = \hbar/\gamma_0$. This is the lifetime associated with the formation or decay of the resonance in the radial region ($r \lesssim 24$ atomic units a_0) where the quasibound state is coupled to the incoming channel via the exchange interaction. It follows from the energy dependence of the local phase shift δ_0 of the radial wave function at $r \approx 24a_0$ near resonance. With $e^{2i\delta_0} = e^{2i\delta_{0,bg}}[1 - i\gamma_0/(E - \epsilon_{\text{res}} + \frac{i}{2}\gamma_0)]$ [11] we find $\tau_{\text{res},0} = 1.4 \mu\text{s}$ [15]. In this formula $\delta_{0,bg}$ is the background value of the local phase shift, ϵ_{res} is the resonance energy, and $E \approx 1$ nK is the typical kinetic energy for an atom in the condensate. The local lifetime is much shorter than the global lifetime $\tau_{\text{res}} = \hbar/\gamma(E) \sim E^{-\frac{1}{2}}$ of order 1 ms, associated with the energy dependence of the phase shift at a much larger distance ($r > 1000 a_0$). In contrast to an intermediate radial range where quantum reflection takes place [16], a wave packet propagates in the regions at small and large r without significant reflection, so that meaningful concepts of phase shift and delay time are possible. Understandably, the global lifetime is the resonance lifetime that mostly occurs in expressions for resonance phenomena. In our case it is much longer than the local lifetime due to strong quantum reflection.

We calculate the atom loss fraction after the field ramp by considering the atom pairs in their quasibound state as a molecular Bose-Einstein condensate, described by a coherent field $\phi_2(\vec{x}, t)$ in addition to the field $\phi_1(\vec{x}, t)$ describing the atomic condensate. The evolution of the double-condensate system is described by a two-state model, governed by a pair of coupled Gross-Pitaevskii equations [10,17]:

$$\begin{aligned} i\hbar\dot{\phi}_1 &= U_0|\phi_1|^2\phi_1 + 2\alpha\phi_1^*\phi_2, \\ i\hbar\dot{\phi}_2 &= \left(\epsilon_{\text{res}} - \frac{i}{2}\gamma_0\right)\phi_2 + \alpha\phi_1^2, \end{aligned} \quad (2)$$

with uniform amplitudes $\phi_{1,2} = \sqrt{n_{1,2}} \exp(i\theta_{1,2})$ over the volume of the condensate. Here $U_0 = 4\pi\hbar^2 a_\infty/m$ is the off-resonant strength of the atomic condensate self-energy and the α terms describe the process that converts atoms into molecules. In our simulations we omit the influence of the interactions between the molecules during their short lifetime and between those molecules and atoms. The coupling parameter is given by $\alpha = (\frac{1}{2}U_0\Delta\mu\Delta)^{1/2}$, while $\epsilon_{\text{res}}(t) - \frac{i}{2}\gamma_0 = [B(t) - B_0]\Delta\mu - \frac{i}{2}\gamma_0$ is the complex energy of the quasibound state relative to threshold including its decay width. In view of that large decay

width, we treat ϕ_2 by the method of elimination of fast variables, expressing it as $\phi_2(t) = -\alpha\phi_1^2/(\epsilon_{\text{res}} - \frac{i}{2}\gamma_0)$. We then end up with an equation for $\phi_1(t)$ only:

$$i\hbar\dot{\phi}_1 = \left[U_0 - \frac{2\alpha^2}{\epsilon_{\text{res}}(t) - \frac{i}{2}\gamma_0} \right] |\phi_1|^2 \phi_1 = U |\phi_1|^2 \phi_1, \quad (3)$$

with initial condition $\phi_1(-\infty) = \sqrt{n_i}$. Apparently, the influence of the resonance during the field ramp has effectively the form of a Breit-Wigner contribution to the condensate self-energy. The off-resonant strength U_0 changes into $U = 4\pi\hbar^2 a/m$, with a complex scattering length of the form (1) with B_0 replaced by $B_0 + \frac{i}{2}\Delta B$, where $\Delta B = \gamma_0/\Delta\mu$. The quantity ΔB characterizes the local width of the resonance as a function of the field. It is approximately equal to twice the width parameter Δ of Eq. (1) in the circumstances of the experiment. More generally, the ratio $\Delta/\Delta B$ is related to the transmission through the quantum reflection region and depends on a_∞ . The imaginary part of U describes the decay of the atomic condensate: $\dot{n}_1 = -G(t)n_1^2$, with the rate coefficient $G(t) = \frac{2\alpha^2}{\hbar}\gamma_0/(\epsilon_{\text{res}}^2 + \frac{1}{4}\gamma_0^2)$. Its maximum value is $8\alpha^2/\hbar\gamma_0 = 4U_0\Delta/\hbar\Delta B \approx 2 \times 10^{-10} \text{ cm}^3 \text{ s}^{-1}$, independent of B . To our knowledge, this value for the instantaneous rate of decay observed in the MIT experiment represents the largest inelastic rate for a cold atom process ever observed, at least 1 order of magnitude larger than typical exchange decay rates ($G_{\text{exch}} \approx 10^{-11} \text{ cm}^3 \text{ s}^{-1}$) and 5 orders larger than typical dipolar decay rates ($G_{\text{dip}} \approx 10^{-15} \text{ cm}^3 \text{ s}^{-1}$). Note that the physics of our picture is very similar to that of *exchange* relaxation in the vicinity of a Feshbach resonance (forbidden in our case). In Ref. [18] we predicted for that process in ${}^6\text{Li}$ - ${}^7\text{Li}$ collisions a relaxation rate with a quantum limit magnitude, at least 10^3 larger than the typical exchange value.

Integrating the rate equation we obtain the fraction of atoms lost from the condensate for a given ramp speed:

$$\frac{n_i - n_f}{n_i} = \frac{n_i \int G(t) dt}{1 + n_i \int G(t) dt} \equiv \frac{p(\dot{B})}{1 + p(\dot{B})}, \quad (4)$$

where $p(\dot{B}) = \pi(U_0 n_i / \hbar)(2\Delta/\dot{B})$. Apparently, the loss is determined by the nonresonant change of the condensate phase θ_1 during the crossing of the resonance. Note that the two-state model predicts the loss fraction to depend on n_i and \dot{B} only via the combination n_i/\dot{B} . The specific properties of the resonance come in only via Δ as was already implicitly assumed in Ref. [9]. While a fast time dependence of the magnetic field is needed for the resonant loss process, the loss decreases with increasing \dot{B} due to the shorter time in which the resonance is crossed. In Fig. 2 we present the loss fraction for $\tau_{\text{res},0}$ equal to the calculated value $1.4 \mu\text{s}$ ($\gamma_0 = 5.3 \mu\text{K}$, $\Delta = 0.0091 \text{ G}$). We obtain rather good agreement with experiment for $n_i = 7.0 \times 10^{14} \text{ cm}^{-3}$. This (uniform) initial density agrees with the experimental value $5.2 \times 10^{14} \text{ cm}^{-3}$ for the mean initial density within the combined experimental

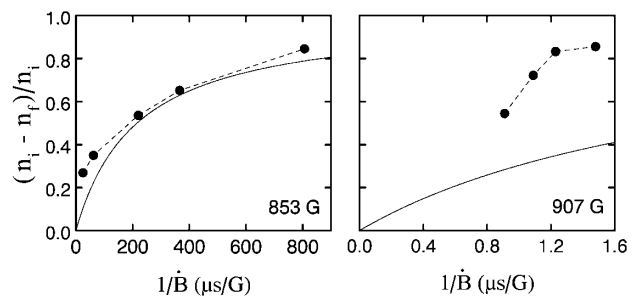


FIG. 2. Predicted (solid lines) and measured (dots connected by dashed lines) fraction of lost atoms after crossing the Feshbach resonance as a function of inverse ramp speed for the resonances at 853 and 907 G.

($\pm 5\%$ statistical, $\pm 20\%$ systematic) [13] and theoretical error bars.

In the case of the 853 G resonance, the quasibound state and the free two-atom state are very weakly coupled. The coupling is more than a factor 100 stronger for the other resonance at 907 G. Figure 2 shows the loss fraction following from the previous expressions for the calculated value $\tau_{\text{res},0} = 0.012 \mu\text{s}$ ($\gamma_0 = 646 \mu\text{K}$, $\Delta = 1.05 \text{ G}$) at the experimental mean density $n_i = 5.7 \times 10^{14} \text{ cm}^{-3}$ [13]. Clearly, for both resonances the order of magnitude of the loss rate is correctly described, confirming the basic mechanism. For the 907 G resonance, however, the theoretical prediction shows a difference with experiment. We believe that this is due to the fact that for the highest experimental \dot{B} values the time dependence of the scattering process is incompletely described in our two-state model. In particular, the finite time needed for the transmission of a wave packet through the quantum reflection region and for the multiple reflections between this region and the origin may play a role, since it turns out to be comparable to the time in which the resonance is crossed with the magnetic field. This suggests that in this regime the time-dependent resonance phenomenon in a condensate is too involved to be described in terms of a combination of an atomic and a molecular condensate wave function.

Instead, a description with a pair condensate wave function $\phi(\vec{x}_1, \vec{x}_2, t)$ in analogy to the independent-pair description for fermionic systems [19] would seem appropriate. This requires a generalization of the coupled Gross-Pitaevskii equations. Remarkably, we found that a simple extension of the above coupled Eqs. (2) with an extra coupling term in both equations produced by an additional reflection, describes qualitatively the behavior of the observed loss fraction for a delay time due to reflection of about $10 \mu\text{s}$. The situation of the quantum reflection region separating two condensates (a condensate of free atoms and a “long-range molecular” condensate) with an initial removal of atom pairs on one side during the crossing of the resonance, is reminiscent of the Josephson effect [2]. In this picture, two types of molecular

condensate play a role: The “long-range” condensate of atom pairs in the initial spin state with interatomic distances up to the quantum reflection region, and the “short-range” condensate of quasibound pairs in the admixed spin state. The latter is only indirectly coupled with the atomic condensate via the long-range molecular condensate. The excess loss observed experimentally would be caused by the additional resonance absorption of the inward atom flux arising from the first Josephson oscillation. The falloff of this excess loss at low $1/B$ would be due to an arrival at $24a_0$ with a $10 \mu\text{s}$ delay of the order of the inverse Josephson frequency, too late to be in full resonance with the quasibound state.

We now turn to the experimental data for the runs of type 1. In this case, the fast-sweep two-body decay mechanism is absent and most of the loss occurs after the field ramp. We follow the treatment by Timmermans *et al.* [10] (see also Ref. [20]), which we recapitulate using the latest Na parameters [15]. They explain the loss as a two-step process: the formation of quasibound pairs during the adiabatic field change, followed by stabilizing collisions of such pairs with third Na atoms with rate coefficient G_{stab} . The density of quasibound pairs follows directly from the static off-resonant equivalent of Eq. (2) without γ_0 , as well as from an explicit wave function calculation: $n_2 = (\alpha^2/\epsilon_{\text{res}}^2)n_1^2$. Assuming that all three atoms are lost, we have the rate equation

$$\dot{n}_1 = -3G_{\text{stab}} \left[\frac{\alpha}{\Delta\mu(B - B_0)} \right]^2 n_1^3. \quad (5)$$

Note that this equation is valid for fields smaller and larger than B_0 . An approximate determination of the effective two-body relaxation constant G_{stab} is possible using Fig. 2 of Ref. [9]. We find good agreement with experiment with $G_{\text{stab}} = 4.10^{-10} \text{ cm}^3 \text{ s}^{-1}$ for the 907 G resonance and $G_{\text{stab}} = 1.10^{-11} \text{ cm}^3 \text{ s}^{-1}$ for the 1195 G resonance, both with an order of magnitude in the range 10^{-9} to $10^{-11} \text{ cm}^3 \text{ s}^{-1}$ estimated by Timmermans *et al.* [10] on the basis of calculations for $\text{H}_2 + \text{He}$ collisions. For the 907 G resonance we can thus compare the decay width $\gamma_{\text{stab}} = \hbar G_{\text{stab}} n_1$ to γ_0 . We find γ_{stab} to be smaller by a factor of order 100 and thus negligible in runs of type 2.

We have given an explanation for the losses in the MIT high ramp speed experiment. As a next step, a more detailed understanding on the basis of an extended version of the Gross-Pitaevskii equations would be important, since it should then be possible to make reliable further predictions. For instance, a high ramp speed experiment with a tailored time dependence might lead to the “permanent” formation of a molecular condensate. If followed by a suitable stimulated Raman pulse, this could be converted to a more strongly bound state [14,17]. The resonance at 1195 G seems to be particularly interesting for this pur-

pose. There, the energy of the quasibound state decreases with increasing magnetic field, so that a simple fast sweep from low to high field tends to create condensed molecules rather than atoms with increased kinetic energy. A fast sweep from high to low field across the 853 or 907 G resonance would have a similar effect but has the disadvantage of reducing the condensate fraction when the field is turned on. The dependence on field ramp direction is typical for the fast-sweep two-body mechanism and absent in the three-body mechanism proposed by Timmermans *et al.* Additional high ramp speed observations should be of great importance to test the above n_i/B dependence of the loss for the 853 G resonance, while time-resolved measurements might detect the Josephson-like current-phase effects expected for the 907 G resonance.

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- [1] M.H. Anderson *et al.*, *Science* **269**, 193 (1995); K. Davis *et al.*, *Phys. Rev. Lett.* **75**, 3969 (1995); C.C. Bradley *et al.*, *ibid.* **78**, 985 (1997); D.G. Fried *et al.*, *Phys. Rev. Lett.* **81**, 3811 (1998).
 - [2] F. Dalfovo *et al.*, *Rev. Mod. Phys.* **71**, 463 (1999).
 - [3] J. Stenger, S. Inouye, D.M. Stamper-Kurn, H.-J. Miesner, A.P. Chikkatur, and W. Ketterle, *Nature (London)* **396**, 345 (1998).
 - [4] L. Deng *et al.* (to be published).
 - [5] E. Tiesinga *et al.*, *Phys. Rev. A* **47**, 4114 (1993); E. Tiesinga *et al.*, *ibid.* **46**, R1167 (1992).
 - [6] S. Inouye, M.R. Andrews, J. Stenger, H.-J. Miesner, D.M. Stamper-Kurn, and W. Ketterle, *Nature (London)* **392**, 151 (1998).
 - [7] P. Courteille *et al.*, *Phys. Rev. Lett.* **81**, 69 (1998); J.L. Roberts *et al.*, *ibid.* **81**, 5109 (1998).
 - [8] Yu. Kagan *et al.*, *Phys. Rev. Lett.* **79**, 2604 (1997).
 - [9] J. Stenger, S. Inouye, M.R. Andrews, H.-J. Miesner, D.M. Stamper-Kurn, and W. Ketterle, *Phys. Rev. Lett.* **82**, 2422 (1999).
 - [10] E. Timmermans *et al.*, cond-mat/9805323.
 - [11] C.J. Joachain, *Quantum Collision Theory* (North-Holland, New York, 1972).
 - [12] F.A. van Abeelen, D.J. Heinzen and B.J. Verhaar, *Phys. Rev. A* **57**, R4102 (1998).
 - [13] W. Ketterle and J. Stenger (private communication).
 - [14] P.S. Julienne *et al.*, *Phys. Rev. A* **58**, R797 (1998).
 - [15] Calculated using recently derived interaction parameters: F. van Abeelen and B.J. Verhaar, *Phys. Rev. A* **59**, 578 (1999).
 - [16] P.S. Julienne and F.H. Mies, *J. Opt. Soc. Am. B* **6**, 2257 (1989).
 - [17] D.J. Heinzen, P.D. Drummond, and K.V. Kheruntsyan (unpublished).
 - [18] F. van Abeelen and B.J. Verhaar, *Phys. Rev. A* **55**, 4377 (1997).
 - [19] A. deShalit and H. Feshbach, *Theoretical Nuclear Physics, Part I: Nuclear Structure* (Wiley, New York, 1974).
 - [20] M.W. Reynolds *et al.*, *Phys. Rev. B* **34**, 4912 (1986).