Evidence for the Existence of Supersymmetry in Atomic Nuclei

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We found strong evidence for the existence of supersymmetry by studying the odd-odd nucleus ^{196}Au using the ^{197}Au$(\vec{d},t)$, ^{197}Au(p,d), and ^{198}Hg$(\vec{d},\alpha)$ transfer reactions. High resolution ^{196}Pt(p,d) and ^{196}Pt$(\vec{d},t)$ transfer experiments performed in parallel yielded an improved level scheme of ^{195}Pt. Using extended supersymmetry, a single fit of the six parameter eigenvalue expression yielded a complete description of all observed low-lying excited states in the four nuclei forming the supermultiplet. The detailed comparison of the transfer amplitudes the odd-odd member of the supermultiplet ^{196}Au using a semimicroscopic transfer operator provides, then, evidence that this description is correct.

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Symmetry is an important concept in physics. In finite many-body systems, it appears as time reversal, parity, and rotational invariance but also in the form of dynamical symmetries. Among all mesoscopic systems, the atomic nucleus forms one of the best testing grounds for the use of symmetry concepts because a wealth of experimental spectroscopic information can be obtained. In the field of dynamical symmetries, a remarkably versatile model was elaborated in the mid-1970s by Iachello and Arima [1]. This interacting boson model (IBM) considers $2N$ valence nucleons which are coupled to N nucleon pairs, as s ($l = 0$) and d ($l = 2$) bosons. The even-even nucleus is then described in a space spanned by the irreducible representation (irrep) $[N]$ of $\mathrm{U}^B(6)$. The model turned out to be very successful for medium-heavy and heavy nuclei.

For the description of odd-A nuclei, a fermion needs to be coupled to the N boson system. This can be done by a semimicroscopical approach which relies on seniority in the nuclear shell model [2]. An alternative to this interacting boson-fermion approach is the construction of Hamiltonians exhibiting dynamical Bose-Fermi symmetries that are analytically solvable. In both approaches, the boson-fermion space is spanned by the irrep $[N] \times [1]$ of $\mathrm{U}^B(6) \otimes \mathrm{U}^F(M)$ where M is the dimension of the single particle space. A further step towards unification was made in the early 1980s when Iachello and co-workers embedded the Bose-Fermi symmetry into a graded Lie algebra $\mathrm{U}(6/M)$ [3,4]. The supersymmetric irrep $[\mathcal{N}\}$, then, spanned a space that describes both an even-even nucleus with $\mathcal{N}$ bosons and an odd-A nucleus with $\mathcal{N} - 1$ bosons and an odd fermion. In some cases, the dynamical supersymmetry leads to an analytically solvable algebraic Hamiltonian with fixed parameters for both nuclei. If this is the case, one concludes that the nuclei exhibit a supersymmetry.

One successful case is U(6/12) in which the fermion can occupy the orbits with $j = 1/2$, $3/2$, and $5/2$. Considering those as arising from the coupling of a pseudospin part with $s' = 1/2$ with a pseudoorbital part with $l' = 0$ and 2, the following reduction is obtained: $\mathrm{U}^F(12) \supset \mathrm{U}^F(6) \otimes \mathrm{U}^F(2)$ which allows the coupling of the pseudoorbital part with the bosonic generators at the U(6) level [5]. This supersymmetry can, thus, be applied in all mass regions provided that the relevant single particle orbits have $j = 1/2$, $3/2$, and $5/2$. Another one is U(6/4) which uses the isomorphism between the $\mathrm{U}^F(4)$ group, describing the space for a 3/2 fermion, and the bosonic O(6) group [4].

The extended supersymmetry [6] deals with boson-fermion and neutron-proton degrees of freedom, allowing the description of a quartet of nuclei, using the same algebraic form of the Hamiltonian. The quartet consists of an even-even nucleus with $(\mathcal{N}_\nu + \mathcal{N}_\pi)$ bosons, an odd-proton and an odd-neutron nucleus with $(\mathcal{N}_\nu + \mathcal{N}_\pi) - 1$ bosons, and an odd-odd nucleus with $(\mathcal{N}_\nu + \mathcal{N}_\pi) - 2$ bosons and a proton and neutron. Supersymmetry relates the often very complex structure of the odd-odd nucleus to the much simpler even-even and odd-A systems. Besides the possibility of understanding odd-odd nuclei, the best way to study the existence of supersymmetry is accomplished by testing its predictions for odd-odd nuclei [7].

In this Letter, we will provide such a test for the $\mathrm{U}_\nu(6/12) \otimes \mathrm{U}_\pi(6/4)$ extended supersymmetry which is described in Refs. [6–8]. In order to be able to exhibit a dynamical symmetry, a physical system has to fulfill certain conditions. In the case of extended supersymmetries, these constraints strongly limit their occurrence. To be applicable for the $\mathrm{U}_\nu(6/12) \otimes \mathrm{U}_\pi(6/4)$ scheme, the even-even core should exhibit the O(6) symmetry of the IBM. The odd proton has to occupy a dominant $j = 3/2$ orbit and the odd neutron the $j = 1/2$, $3/2$, and $5/2$ orbits. Nuclei, exhibiting the O(6) symmetry, are found near semiclosed shells like the Xe and Pt region [9,10]. An isolated $j = 3/2$ orbit is found only in the case of

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an occupied $2d_{3/2}$ orbit when nucleons form three to six holes below the 82 shell closure. Besides in the *sd* shell, $j = 1/2$, $3/2$, and $5/2$ are occupied together above the 28 shell and below the 126 shell closure. Thus, for nuclei near stability the $U_\nu(6/12) \otimes U_\pi(6/4)$ scheme can occur only in the Au, Ir region for the negative-parity states formed by the $\nu(3p_{1/2}, 3p_{3/2}, 2f_{5/2}) \times \pi 2d_{3/2}$ configurations. It is encouraging that, indeed, the supersymmetry was observed to be approximately valid in ^{198}Au [6,11] and ^{194}Ir [8], the two best studied odd-odd nuclei in this mass region. However, it was realized from the beginning that the ultimate candidate for the test would be the odd-odd nucleus ^{196}Au [6] since the quartet ^{194,195}Pt, ^{195,196}Au contains the nuclei ^{194}Pt-^{195}Pt which are considered to be the best example of the U(6/12) supersymmetry [12].

If the Hamiltonian is built out of Casimir operators of groups forming a group chain, its eigenvalues are analytical as a function of the quantum numbers classifying the irreps. In case of $U_\nu(6/12) \otimes U_\pi(6/4)$ this leads to the expression [6]

$$E = A[N_1(N_1 + 5) + N_2(N_2 + 3)] + B[\Sigma_1(\Sigma_1 + 4) + \Sigma_2(\Sigma_2 + 2)] + B'[\sigma_1(\sigma_1 + 4) + \sigma_2(\sigma_2 + 2) + \sigma_3^2] + C[\tau_1(\tau_1 + 3) + \tau_2(\tau_2 + 1)] + DL(L + 1) + EJ(J + 1) \tag{1}$$

with A, B, B', C, D, and E free parameters and $[N_1, N_2], \langle\Sigma_1, \Sigma_2\rangle, \langle\sigma_1, \sigma_2, \sigma_3\rangle, (\tau_1, \tau_2), L, J$ the quantum numbers, correlated to the irreducible representations of U(6), $\bar{\text{O}}$(6), O(6), O(5), O(3), and spin(3), respectively. The reduction rules, then, lead to the level schemes in the different nuclei. These can be found in [4,5,8]. In addition to the analytic expressions for the excitation energies, the supersymmetric scheme also provides analytic results for the wave functions. These do not depend on the parameters given above and can be tested via the calculation of electromagnetic transition rates and single particle transfer reaction amplitudes. Especially the transfer experiments provide a very stringent test of the existence of supersymmetry via the distribution of single nucleons into the predicted wave functions.

Although the negative-parity states in ^{196}Au were unknown, except for the 2^- ground state, some years ago a test of the supersymmetric description of this nucleus was indirectly provided via unpolarized transfer reactions [7,13]. The measured angular distributions of differential cross sections allowed a selective observation of p- and f-transfers which populate those states that are provided by the coupling of a neutron hole, occupying the relevant $p_{1/2}$-$p_{3/2}$ or $f_{5/2}$-$f_{7/2}$ orbits, to ^{197}Au. Since the experimental level scheme of ^{196}Au was still poorly known, an experimental study of ^{196}Au was started in a Fribourg-Bonn-Munich collaboration. The experimental program includes in-beam gamma ray and conversion electron spectroscopy following the reactions ^{196}Pt$(d, 2n)$ and ^{196}Pt(p, n) at the cyclotrons of the PSI (Villigen, Switzerland) and the University of Bonn. At the tandem accelerator of the TU/LMU München, high resolution transfer experiments to ^{196}Au were performed, using (p, d), polarized $(\vec{d}, t)$, and polarized $(\vec{d}, \alpha)$ reactions. In case of the (p, d) and $(\vec{d}, t)$ experiments, the nucleus ^{195}Pt was measured in parallel in order to obtain a reference data set. The nuclei were investigated with 26 MeV protons and with $\pm 60\%$ vector polarized deuterons, having an energy of 25 MeV [for the $(\vec{d}, t)$ reaction] and 18 MeV [for $(\vec{d}, \alpha)$]. The beam intensity was several hundred nA on target. The targets of ^{197}Au and ^{196}Pt had a thickness of approximately 100 μg/cm^2 and the ^{198}Hg target of 37 μg/cm^2.

Because of their excellent energy resolution (4 keV FWHM), the (p, d) transfer reactions were used to provide the energy calibration of the ^{196}Au spectra, using the ^{195}Pt data to establish a correlation between measured channels and excitation energies. The achieved uncertainties of the excitation energies are less than 1 keV. These spectra establish a new and for low excitation energies almost complete level scheme of ^{196}Au. In total, 47 states were resolved for the first time in the energy range of 0 to 1350 keV [14] including the resolved ground state doublet with an energy spacing of approximately 6 keV, as shown in Fig. 1.

Since the energy resolution of the $(\vec{d}, t)$ reaction was worse (7 keV FWHM), the spectra were analyzed using the level energies deduced from the (p, d) data. From the polarized measurement angular distributions of differential cross sections $\frac{d\sigma}{d\Omega}$ and analyzing powers A_y are obtained which provide the l and j values of the angular momentum

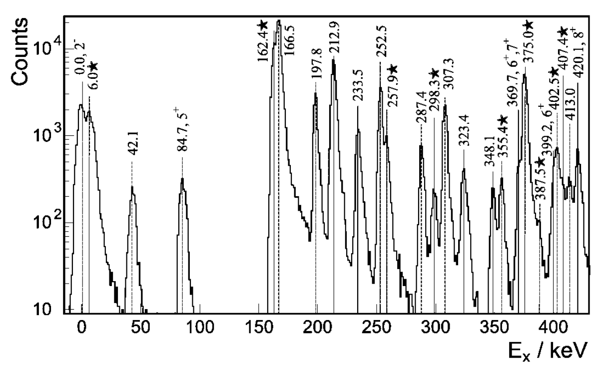

FIG. 1. Part of the ^{197}Au(p, d)^{196}Au spectrum measured with 26 MeV protons on a 67 μg/cm^2 target at an angle of 25°. Shown is the excitation energy range between 0 and approximately 400 keV. Nine newly discovered states are marked by asterisks. All J^π assignments, known from Nuclear Data Sheets [15], are included.

transfers. In the case of ^{196}Au, the neutron shells $p_{1/2}$, $p_{3/2}$, $f_{5/2}$, and $f_{7/2}$ have to be considered in the analysis of negative-parity states while positive-parity states are mainly populated via the $i_{13/2}$ transfer leading to a clear distinction of the two parities. Since the ground state spin of the target nucleus ^{197}Au is $J_A = 3/2$ the angular momenta J_B of the final states, which are observed by the angular momentum transfer j, are in the range $|J_A - j| \leq J_B \leq J_A + j$. Consequently, up to four different j transfers can contribute to the cross section of an excited state in ^{196}Au. The determination of the contributing transfers and the respective spectroscopic strengths G_{lj} was done by a numerical fit of the data using the relations

$$\frac{d\sigma}{d\Omega}(\theta) = \sum_{lj} G_{lj}\sigma^{lj} = \sum_{lj} v_j^2(2j+1)S_{lj}\sigma^{lj}\,, \qquad (2)$$
$$A_y(\theta) = \left(\sum_{lj} G_{lj}\sigma^{lj}A_y^{lj}\right)\frac{1}{d\sigma/d\Omega}\,,$$

with σ^{lj} and A_y^{lj} the normalized angular distributions of distorted-wave Born approximation calculations, S_{lj} the spectroscopic factors, and v_j^2 the occupation probability of the respective neutron orbit j. Using least-squares fitting, spectroscopic strengths G_{lj} for 104 states are obtained in the energy region up to 1350 keV. For the low-spin states of interest here, our extended set of data yields almost complete level schemes of ^{196}Au and ^{195}Pt with spin assignments via the observed j values. Since the same neutron orbits are occupied in the two nuclei, comparisons of experimental properties are possible, for instance the level density or the distributions of transfer strengths. As no major contradiction was found, also in respect to the extensive literature to ^{195}Pt, we are confident in the results for ^{196}Au. Furthermore, the analysis is confirmed by the additional polarized $(\vec{d},\alpha)$ experiment which provided a definite spin assignment for 17 levels.

In order to compare the transfer strengths to the theoretical predictions, one needs to define the theoretical transfer operator. In all calculations made up to now, the transfer operator between nuclei having the same number of bosons N was taken to be the operator $a_j^\dagger$ which creates a fermion in the supersymmetric models. The advantage of this simple operator is that analytic results can be derived [7]. Nevertheless, the transfer operator provides a poor description of the observed fragmentation of the strength [13]. Here, we use a semimicroscopic transfer operator obtained from the mapping of the single-nucleon creation operator onto the boson-fermion space [16] because the experiment deals with the transfer of a single nucleon. This yields in the case of a hole

$$\mathcal{T}^{lj} = \frac{v_j a_j^\dagger}{K_\alpha} - \sum_{j'}\sqrt{\frac{10N_\pi}{(2j+1)N^2}}\,u_j(u_j v_{j'} + v_j u_{j'}) \times \left\langle \frac{1}{2}\,l'j'|Y_2|\frac{1}{2}\,lj\right\rangle s^\dagger(\tilde{d}a_{j'}^\dagger)^{(j)}\frac{1}{K_\alpha}\,\frac{1}{K_\beta}\,, \qquad (3)$$

with $u_j^2 = 1 - v_j^2$. K_α and K_β are normalization constants described in [16]. The semimicroscopic operator contains the simple operator as a first approximation. Both depend on the same number of parameters v_j. These parameters are not free but can be obtained from the experiment. We fixed them by directly using the $^{197}\mathrm{Au}(\vec{d},t)\,^{196}$Au data to be $v_{1/2}^2 = 0.33(5)$, $v_{3/2}^2 = 0.30(5)$, and $v_{5/2}^2 = 0.49(7)$. The second term in Eq. (3) induces the additional fragmentation. Figure 2 compares the theoretical and experimental level schemes of ^{196}Au and ^{195}Pt. The theoretical spectra are obtained from a common least squares fit of Eq. (1) to 8 levels in ^{194}Pt, 31 in ^{195}Pt, 11 in ^{195}Au, and 47 in ^{196}Au and will be presented in more detail in a forthcoming paper [14]. The resulting parameters are $A = 52.3$, $B = 12.4$,

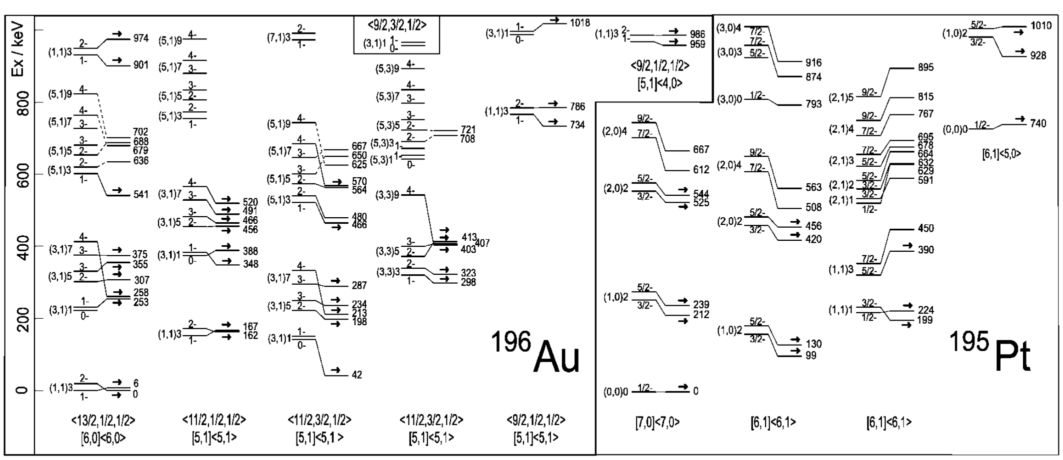

FIG. 2. Supersymmetric level schemes of the nuclei ^{196}Au and ^{195}Pt. Below each band the quantum numbers $[N_1, N_2]$, $\langle\Sigma_1, \Sigma_2\rangle$ and for ^{196}Au, additionally, $\langle\sigma_1, \sigma_2, \sigma_3\rangle$ are given. On the left of each predicted level the quantum numbers $(\tau_1, \tau_2)L$ are shown (in case of ^{196}Au double physical numbering is used and only states lower than 5^- are shown). The arrow on top of an experimental level marks the states well reproduced by calculations with the semimicroscopic transfer operator.

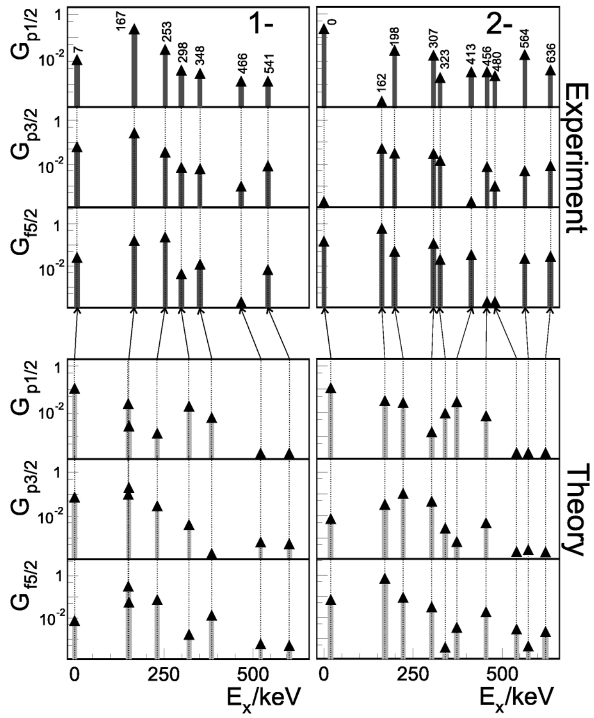

FIG. 3. Comparison of the experimental (top) and theoretical (bottom) absolute transfer strengths for the 1^- (left) and 2^- (right) states in ^{196}Au in the energy range of 0 to 650 keV. The resulting correlations are shown by arrows. Note the logarithmic scale of the plots. The experimental data of the 2^- states reveal also contributions from $2f_{7/2}$ which are outside the model space and, except for the level at 307.3 keV level, rather weak.

$B' = -58.0$, $C = 50.1$, $D = 6.9$, and $E = 4.4$ (all in keV). They are close to the original prediction which did not consider levels of ^{196}Au in the fit [7]. The main changes can also be understood. The value of A changes due to the fact that our transfer reaction to ^{195}Pt showed the need for a reassignment of the $[N_1, N_2]\langle\Sigma_1, \Sigma_2\rangle = [6, 1]\langle 5, 0\rangle$ states in this nucleus [14]. The lowest of this class of states is now correlated to a $\frac{1}{2}^-$ level at 740 keV (see Fig. 2) instead of a 928 keV level, used in [12] and previously assigned as $\frac{1}{2}^-$ or $\frac{3}{2}^-$ which we found to be a $\frac{3}{2}^-$ state [14]. The values of B' and B obtained in [7] are dependent on a single level in ^{195}Au because all other levels in this fit are only related to $B' + B$ (see also the discussion in [13]). In the present fit, the relative contribution of both terms is fixed by the odd-odd nucleus. For $B' + B$, the new fit yields -45.6 keV which is again close to the old result of -55.6 keV. Using the new classification for ^{195}Pt, a fit of this nucleus together with ^{194}Pt yields $B' + B = -42.2$ keV [14].

Apart from a systematic down-shift of the energy of the 4^- states and two experimentally unobserved levels (which can be part of two closely spaced doublets), a good agreement is obtained for the excitation energies. A further test is provided by the transfer strengths. The spectroscopic factors vary over orders of magnitude and are shown in Fig. 3 for the $J^\pi = 1^-, 2^-$ states in a logarithmic scale. The calculation reproduces the experimental distributions irrespective of deviations in details, probably, due to level mixing. In addition to the prediction of the very rich excitation spectra this further supports that supersymmetry plays a dominant role or, stated otherwise, that for the low energy spectrum of these nuclei symmetry breaking degrees of freedom are remarkably weak.

In view of the extreme complexity of heavy transitional odd-odd nuclei and the few parameters needed to describe simultaneously and nearly quantitatively almost one hundred excited states in four different nuclei, we conclude that the lowest excited states are related by the concept of supersymmetry in atomic nuclei. This conclusion is also supported by the fact that the Au, Ir nuclei are situated in the only region of the nuclear chart where the constraints for $U_\nu(6/12) \otimes U_\pi(6/4)$ are fulfilled. The question is now raised, what is the microscopic basis that makes supersymmetry valid in atomic nuclei? Up to now, only a few arguments have been given [17] and one definitely needs more explanations.

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[1] F. Iachello and A. Arima, *The Interacting Boson Model* (Cambridge University Press, Cambridge, 1987).
[2] F. Iachello and O. Scholten, Phys. Rev. Lett. **43**, 679 (1979).
[3] F. Iachello, Phys. Rev. Lett. **44**, 772 (1980).
[4] A. B. Balantekin, I. Bars, and F. Iachello, Nucl. Phys. **A370**, 284 (1981).
[5] P. Van Isacker, A. Frank, and H. Z. Sun, Ann. Phys. (N.Y.) **157**, 183 (1984).
[6] P. Van Isacker, J. Jolie, K. Heyde, and A. Frank, Phys. Rev. Lett. **54**, 653 (1985).
[7] J. Jolie *et al.*, Phys. Rev. C **43**, R16 (1991).
[8] J. Jolie and P. E. Garrett, Nucl. Phys. **A596**, 234 (1996).
[9] J. A. Cizewski *et al.*, Phys. Rev. Lett. **40**, 167 (1978).
[10] R. F. Casten and P. von Brentano, Phys. Lett. **152B**, 22 (1985).
[11] D. D. Warner, R. F. Casten, and A. Frank, Phys. Lett. B **180**, 207 (1986).
[12] A. Mauthofer *et al.*, Phys. Rev. C **34**, 1958 (1986).
[13] G. Rotbard *et al.*, Phys. Rev. C **47**, 1921 (1993).
[14] A. Metz *et al.* (to be published).
[15] Zh. Chunmei, W. Gongqing, and T. Zhenlan, Nucl. Data Sheets **83**, 145 (1998).
[16] O. Scholten and T. Ozzello, Nucl. Phys. **A424**, 221 (1984).
[17] U. Kaup and P. Ring, Nucl. Phys. **A462**, 455 (1987).