Observation of Cosmic Acceleration and Determining the Fate of the Universe

Glenn Starkman, Mark Trodden, and Tanmay Vachaspati
Department of Physics, Case Western Reserve University, 10900 Euclid Avenue, Cleveland, Ohio 44106-7079
(Received 10 February 1999)

Current observations of type-Ia supernovae provide evidence for cosmic acceleration out to a redshift of $z \lesssim 1$, leading to the possibility that the universe is entering an inflationary epoch. However, inflation can take place only if vacuum energy (or other sufficiently slowly redshifting source of energy density) dominates the energy density of a region of physical radius $1/H$. We argue that, for the best-fit values of $\Omega_\Lambda = 0.8$ and $\Omega_m = 0.2$ inferred from the supernovae data, one must confirm cosmic acceleration out to at least $z \simeq 1.8$ to infer that our portion of the universe is inflating. If $\Omega_\Lambda < 0.736$ then no present-day measurement can confirm or falsify that inference.

PACS numbers: 98.80.Es, 98.80.Cq

Recent direct [1,2] measurements of the cosmic expansion using supernova at redshifts from $z = 0-1.2$, suggest that the expansion of our portion of the universe is *accelerating*. This supports earlier indirect evidence leading to the same conclusion [3–5]. This is unlike the deceleration expected in a universe dominated by the energy density of ordinary or dark matter. It implies that the expansion is driven by the energy density of the vacuum. This vacuum energy has variously been ascribed to a cosmological constant in the Einstein equations, to the zero-point fluctuations of quantum fields, so-called vacuum energy, to the potential energy density of dynamical fields, quintessence, and to a network of cosmic strings.

In exploring the implications of these explanations, it has widely been assumed that the energy density driving the accelerating expansion is homogeneous. If so, then unless the field dynamics are chosen to avoid it, the universe has entered on an extended period of rapid growth—a new epoch of inflation—in which objects currently within our observable universe will soon begin to leave it. If correct, this result could have dramatic implications for our understanding of fundamental processes underlying the big bang. However, one must be careful to separate the observational results from the assumptions which are built into the standard interpretations of them.

The observational situation can be briefly summarized: measurement of the light curves of several tens of type-Ia supernovae has allowed accurate measurements of the scale factor as a function of redshift $[a(z)]$ out to $z \lesssim 1$. When the supernova data in conjunction with cosmic microwave background radiation (CMBR) anisotropy data are fit to a family of cosmological models parametrized by a homogeneous vacuum-energy density and a homogeneous matter-energy density (characterized by the ratios Ω_Λ and Ω_m of these energy densities to the critical energy density $\rho_c = 3H_0^2/8\pi G$, all at the present epoch), then the measured functional form of a is most consistent with $\Omega_\Lambda \simeq 0.8$ and $\Omega_m \simeq 0.2$. Notwithstanding any debates about systematic uncertainties, for the purposes of this paper let us accept this data as reported. Still, the data do not imply that the observable universe out to the last scattering surface and beyond is vacuum-energy dominated. We could be living in a bubble of high vacuum-energy density surrounded by much lower, even zero, vacuum-energy density. For example, the vacuum-energy density could be due to a scalar field which locally deviates from the minimum of its potential.

The question now arises—will the local vacuum dominated region inflate or will it not? The answer to that question depends on what one means by “inflate.” The effective scale factor of our local corner of the universe *is* apparently experiencing a period of accelerated growth. However, the essence of inflation is not local acceleration, but acceleration over a large enough region to affect the causal relationships between (comoving) observers. In particular, if the acceleration is taking place over a sufficiently large region, then unless the acceleration is halted, comoving observers from whom one was previously able to receive signals, will disappear from view [6]. To be precise, much as if we were watching them fall through the horizon of a black hole, we will see them freeze into apparent immobility, the signals from them declining indefinitely in brightness and energy.

The question we address in the balance of this Letter is how far out must one look to infer that the patch of the universe in which we live is inflating?

In any Friedman-Robertson-Walker (FRW) cosmology, there exists for each comoving observer a sphere centered on that observer, on which the velocity of comoving objects is the speed of light. When sources inside that sphere emit radially inward-directed light rays, the photons approach the observer; when sources outside that sphere emit radially inward-directed light rays, the physical distance between the photons and the observer increases. This sphere is the minimal antitrapped surface (MAS). For a homogeneous universe, the physical radius of the MAS is $1/H$, since the physical velocity v of a comoving observer at a physical distance x is $v = cHx$.

In a matter or radiation dominated epoch, when the dominant energy density in the universe scales with the

 0031-9007/99/83(8)/1510(4)$15.00

scale factor a as $\rho_{\rm dom} \propto a^{-n}$ with $n > 2$, the comoving radius of the MAS grows—the inward-directed photons outside the MAS, which were making negative progress in their journey to the observer, eventually find themselves inside the MAS, and reach the observer. New objects are therefore constantly coming into view. If, on the other hand, the dominant energy density in the universe scales as the scale factor to a power greater than -2 then the comoving radius of the MAS is contracting. For an equation of state with $\lim_{t\to\infty} n < 2$, the MAS contracts to zero comoving radius in finite conformal time, $\eta_{\rm max}$, corresponding to timelike infinity. The interior of the past null cone of the observer at $\eta_{\rm max}$ is the entire portion of the history of the universe that the observer can see, what we shall call hereafter the observer's visible history of the universe (VHU). Since the null cone is contracting, comoving objects cross out of the null cone, and thus disappear from view (see Fig. 1). More precisely, the history of the objects after the time they cross out of the null cone is unobservable. Again, much as when watching something fall through the horizon of a black hole, the observer continues to receive photons from the disappearing source until $\eta_{\rm max}$, but the source appears progressively redder and dimmer, and its time evolution freezes at the moment of horizon crossing.

The comoving contraction of our MAS, and particularly the motion of comoving sources out of our VHU, is the essence of inflation. These sources can never again be seen, unless the equation of state changes so that $n > 2$ and the MAS grows once more, i.e., inflation ends. (Then, the sources never actually crossed out of the VHU, merely out of the apparent VHU—the VHU one would have inferred without the change in equation of state.)

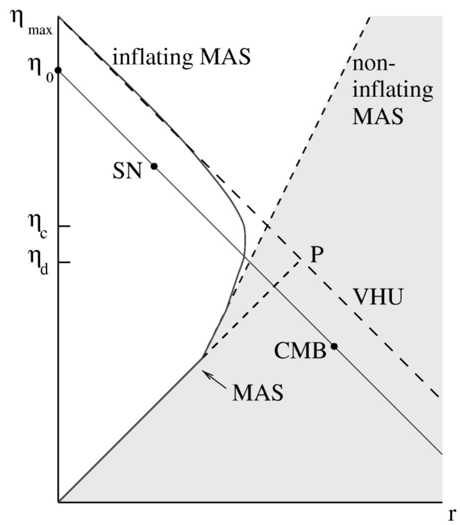

FIG. 1. Spacetime diagram showing the MAS position ($r = 1/aH$) as a function of conformal time η, where the universe eventually does (solid line) and does not (dashed line) undergo inflation. On our present light cone we observe supernovae (SN) and the cosmic microwave background (CMB). If the universe inflates, the MAS curve turns around at η_c and the interior of the past light cone of an observer at future infinity $\eta_{\rm max}$ covers only a portion of the spacetime (triangular area under the dashed line marked VHU). In this case, the first object to leave causal contact with us will do so at η_d through the point P.

By examining the Raychaudhuri equation governing the evolution of the divergence of geodesics, Vachaspati and Trodden [7] recently showed that at any time η_e a contracting antitrapped surface cannot exist in the universe unless a region of radius greater than $1/H(\eta_e)$ is vacuum dominated and homogeneous. (The result depends on the validity of the weak energy and certain other conditions. In the present application, the conditions seem reasonable.) Thus no inflation will have occurred unless a region of size $1/H(\eta)$ remains vacuum dominated long enough for the MAS to begin collapsing. We apply this bound to determine our ability to infer the current and future state of our patch of the universe.

The geometry of a homogeneous and isotropic universe is described by the FRW metric:

$$ds^2 = a^2\left[d\eta^2 - \frac{dr^2}{1 - kr^2} - r^2(d\theta^2 + \sin^2\theta\, d\phi^2)\right], \tag{1}$$

$a(\eta)$ encodes the changing relationship between coordinate (comoving) distances and physical distance. [The conformal time η is related to the proper time t measured by comoving observers by $a(\eta)dt = d\eta$.] The evolution of a is determined by the mean energy density of the universe ρ and by the curvature radius of the geometry, characterized by k, via the Friedmann equation:

$$\left(\frac{\dot a}{a^2}\right)^2 = \frac{8\pi G}{3}\rho - \frac{k}{a^2}, \tag{2}$$

where $\dot a \equiv (da/d\eta)$, and $(\dot a/a^2) \equiv H$ is the Hubble parameter; currently $H \equiv H_0 = 72$ (km/s)/Mpc [8].

The energy density ρ can receive many distinct contributions, but what is crucial to the nature of the solution to (2) is the fraction of the critical density $\rho_c = 3H_0^2/8\pi G$ comprised by each species, and how each of these contributions scales with a. The two most important possible contributions today, nonrelativistic matter and vacuum energy, scale, respectively, as a^{-3} and a^0.

Since the metric (2) describes a homogeneous and isotropic space, we can choose, without loss of generality, to locate ourselves at the origin of coordinates, $r = 0$. Consider then a source located at a comoving distance r from us. At conformal time η_e the source emits a photon directed toward us; the photon is received, and the source therefore observed, at $r = 0$ at our present conformal time η_0. If the source is a standard candle of known luminosity $\mathcal{L}$, then by measuring the observed flux $\mathcal{F}$ of the source, one can determine its luminosity distance:

$$d_L = (\mathcal{L}/4\pi\mathcal{F})^{1/2}. \tag{3}$$

If one can measure the redshift z of the source, then one can constrain Ω_M and Ω_Λ using the relationship

$$d_L(z) = cH_0^{-1}(1+z)\,|\Omega_k|^{-1/2} \times \mathrm{sinn}\left[|\Omega_k|^{1/2}\int_0^z dz'\,[(1+z')^2(1+\Omega_M z') - z'(2+z')\Omega_\Lambda]^{-1/2}\right]. \quad (4)$$

Here $\Omega_k \equiv 1 - \Omega_M - \Omega_\Lambda$, and $\mathrm{sinn}(x)$ is $\sin(x)$ if $k > 0$ and $\sinh(x)$ if $k < 0$. Repeating this for a variety of sources at different comoving distances allows one to determine Ω_M and Ω_Λ. In Fig. 2, we plot the difference in apparent magnitude between a flat ($k = 0$) homogeneous vacuum-energy dominated cosmology ($\Omega_\Lambda = 0.4, 0.6, 0.8$) and the best fit vacuum-energy-free cosmology, a negatively curved ($k < 0$) universe with $\Omega = 0.3$.

This approach has recently been applied to type-Ia supernovae by two independent groups [1,2]. Looking at tens of supernovae out to a redshift of $z < 0.83$, both groups find that the best-fit values of Ω_Λ and Ω_m are approximately 0.8 and 0.2, respectively.

The physical distance between the emitting supernova and the observer at the time of emission η_e was

$$d(\eta_e) = a(\eta_e)\int_{\eta_e}^{\eta_0} d\eta\,, \quad (5)$$

where, for simplicity, we have now restricted our attention to the case when the geometry of the universe is flat ($k = 0$). We are seeing our MAS if $d(\eta_e) = 1/H(\eta_e)$, i.e.,

$$\frac{\sqrt{1+\alpha(1+z)^3}}{(1+z)}\int_1^{1+z}\frac{dy}{\sqrt{1+\alpha y^3}} = 1\,, \quad (6)$$

where $\alpha \equiv (\Omega_\Lambda^{-1} - 1)$. In Fig. 3, we plot z_{MAS}, the redshift of the MAS, vs Ω_Λ. (In Fig. 2, the dashed-with-circles line shows both the redshift z_{MAS} and the associated magnitude difference from the best-fit negatively curved universe.) We find that, for the best-fit value $\Omega_\Lambda = 0.8$, we will see the MAS if we look out to $z = 1.8$. If observations continue to find cosmic acceleration with $\Omega_\Lambda = 0.8$ out to $z = 1.8$, then we can infer that our MAS is contracting, i.e., our patch of the universe is inflating.

Figure 3 shows the redshifts out to which one must necessarily ascertain cosmic acceleration for any given value of vacuum energy before one can be confident that inflation is a possible fate of the universe.

What if observations find a precipitous decrease in Ω_Λ between $z = 0.8$ and $z = 1.8$? Because we are looking along the past null cone, this decrease could be due either to a negative spatial gradient in the vacuum energy ρ_{vac}, or to a temporally increasing ρ_{vac}. In the case of a gradient, we conclude that we are in a vacuum bubble and the universe will not inflate, at least not with $\Omega_\Lambda = 0.8$. However, we cannot exclude the possibility that the vacuum energy was growing with time. It is not easy to get an increasing ρ_{vac}—in the long time limit fields tend to evolve to the minima of their effective potentials. For ρ_{vac} to have increased dramatically between $z = 1.8$ and $z = 0.8$ requires the responsible field to have had sufficient kinetic energy to climb up its potential. But field kinetic energies scale rapidly with scale factor (as a^{-6}). If the kinetic energy was not to have dominated the universe at redshifts greater than 5, that kinetic energy had to have been generated by a recent conversion of potential energy. Thus the "inflaton" must have rolled down one side of a potential well and back up the other. One cannot exclude such a possibility, but the required potentials and the necessary accidents of timing must be carefully tailored to fit the data.

Are the current data on the fluctuations of the cosmic microwave background temperature consistent with us living inside a vacuum bubble? The answer is likely model

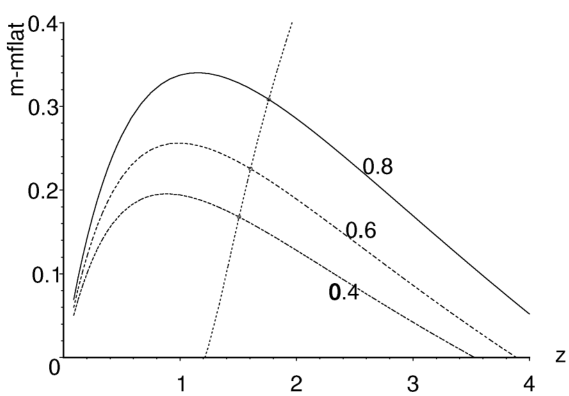

FIG. 2. The difference in apparent magnitude between a flat FRW universe for $\Omega_\Lambda = 0.4$, 0.6, and 0.8, and a $\Omega_\Lambda = 0$, negatively curved universe with $\Omega = 0.3$, vs the redshift z. The other, nearly linear, curve shows z_{MAS} for the different values of Ω_Λ.

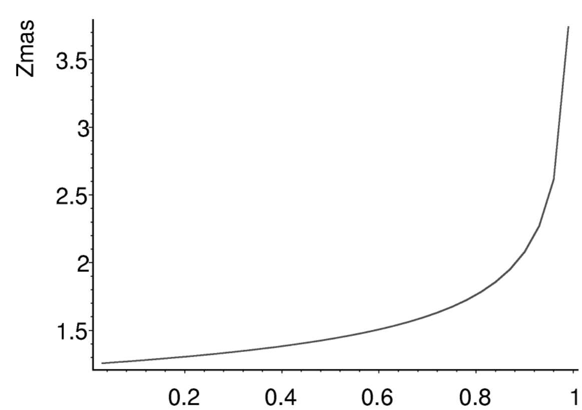

FIG. 3. z_{MAS} vs Ω_Λ. The curve specifies the redshift out to which it is necessary to confirm cosmic acceleration so that it might lead to an inflationary universe.

dependent. We believe the dominant bubble contributions to the CMBR fluctuations will come from the time evolution of the bubble wall and the integrated Sachs-Wolfe effect, and so depend on the wall velocity, energy-density profile, shape, etc. This is a fertile subject for future investigations.

Let us return to Fig. 1 in which, because comoving distance and conformal time are the coordinates, light rays propagate at 45°. We see that during the matter and radiation dominated phases the MAS grows, so that new objects at fixed comoving radius r are constantly entering the interior of the MAS. More importantly, for the observer at $r = 0$, new objects are constantly coming into view. As the universe becomes vacuum dominated, the expansion of the MAS decelerates and it eventually begins to contract. We can readily find the value of the scale factor at which that happens by setting

$$0 = \dot{r}_{\mathrm{MAS}} \propto \frac{da}{d\eta}\frac{d}{da}\left[\sqrt{\frac{8\pi G\rho_c\Omega_\Lambda}{3}}\sqrt{a^2 + \frac{1-\Omega_\Lambda}{\Omega_\Lambda}\frac{a_0^3}{a}}\right]. \tag{7}$$

We have assumed $k = 0$ and ignored the energy density in radiation. Equation (7) is satisfied when $a = a_c$ with

$$\frac{a_0}{a_c} = \left(\frac{2\Omega_\Lambda}{1-\Omega_\Lambda}\right)^{1/3}. \tag{8}$$

We see that the MAS begins to contract at an epoch η_c when $\Omega_\Lambda(\eta_c) = 1/3$. For a present value of $\Omega_\Lambda = 0.8$ this occurs at $a(\eta_c) = a_0/2$.

When we look out to z_{MAS}, will we see this contraction? The answer is not immediately clear, since from Fig. 1 we see that, well after η_c, the past null cone of the observer at $r = 0$ intersects the observer's MAS below the turnover. Defining η_v (a_v) to be the time (scale factor) when the turnover in the MAS comes into view, we see that η_v is given by

$$a_c \int_{\eta_0}^{\eta_c} d\eta = H(\eta_c)^{-1}. \tag{9}$$

This can easily be rewritten as

$$\int_1^{a_v/a_c} \frac{dx}{\sqrt{1 + 2/x^3}} = \frac{1}{\sqrt{3}} \tag{10}$$

(including only matter- and vacuum-energy contributions to H), which can be solved numerically to give $a_v \simeq 1.775 a_c$. Combining this with Eq. (8), we see that

$$\frac{a_v}{a_0} = 1.775\left(\frac{1-\Omega_\Lambda}{2\Omega_\Lambda}\right)^{1/3}. \tag{11}$$

For $\Omega_\Lambda \geq 0.736$, $a_v \leq a_0$. Thus only for $\Omega_\Lambda \geq 0.736$ is it possible to look out far enough to infer the contraction of our MAS.

The contraction of the MAS *is* in some sense the onset of inflation; indeed we could reasonably define it as such. This is because it is the projected contraction of our MAS to zero comoving radius in finite conformal time which is the cause of the eventual loss of contact with comoving observers. We see in Fig. 1 that our VHU is an inverted cone whose apex is at the intersection of our MAS with our world line, $r = 0$. However, if the loss of contact with previously visible objects is what we consider to be the defining characteristic of inflation, then this can begin either before or after η_c, depending on the details of the transition from expanding to contracting MAS.

The value of η_d is easily obtained, since $\eta_d = \eta_{\max}/2$. For $\Omega_\Lambda = 0.8$, $\Omega_m = 0.2$, we find $a_0/a_d \simeq 3$. Since this is greater than a_0/a_c, if we look out to z_{MAS} and see the contraction of our MAS, then objects may have already left our apparent VHU.

As we implied above, while the value of η_d may be of some philosophical interest, there is no measurement that one can make that guarantees that objects have left the VHU, since that takes an infinite amount of proper time for one to observe. If inflation is driven by the metastable or unstable potential energy of some quantum field, then the MAS could eventually start expanding once again and objects which were almost out of view could come back into view.

In conclusion, if the present observations of cosmic acceleration with Ω_Λ (suggested to be $\simeq$0.8) do not extend to a redshift of $z_{\mathrm{MAS}}(\Omega_\Lambda)$ (equal to 1.8 for $\Omega_\Lambda = 0.8$), then either we live in a subcritical vacuum bubble which cannot, on its own, support inflation or fine-tuned field dynamics led to a rare period of growth in the vacuum energy immediately preceding its domination of the energy density. If, on the other hand, future observations confirm the acceleration up to and beyond a redshift of 1.8, then the universe is inflating—our MAS is contracting. We will never be able to tell for certain if objects are moving out of our causal horizon, because that would require observations out to infinite redshift. Either way, we will have discovered exotic fundamental physics.

We are grateful to L. Krauss for important discussions. This work was supported by the Department of Energy (D. O. E.) and NSF CAREER grant to G. S.

[1] S. Perlmutter *et al.*, Astrophys. J. (to be published), astro-ph/9812133.
[2] A. G. Riess *et. al.*, Astron. J. **116**, 1009–1038 (1998); P. M. Garnavich *et al.*, Astrophys. J. **493**, L53–57k (1998).
[3] L. M. Krauss and M. S. Turner, Gen. Relativ. Gravit. **27**, 1137 (1995).
[4] J. P. Ostriker and P. Steinhardt, Nature (London) **377**, 600 (1995).
[5] L. M. Krauss, Astrophys. J. **501**, 461 (1998).
[6] L. M. Krauss and G. D. Starkman, Report No. CWRU-99-P1.
[7] T. Vachaspati and M. Trodden, gr-qc/9811037.
[8] B. F. Madore *et. al.*, astro-ph/9812157.